Textile Composites and Inflatable Structures VI
Structural Membranes 2013
Textile Composites and Inflatable Structures VI

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This volume contains the full length papers presented at the VI International Conference on Textile Composites and Inflatable Structures – Structural Membranes 2013, held in Munich, Germany, on October 9-11, 2013.

Previous editions of the conference were held in Barcelona (2003), Stuttgart (2005), Barcelona (2007), Stuttgart (2009) and Barcelona (2011). Structural Membranes is one of the Thematic Conference of the European Community in Computational Methods in Applied Science (ECCOMAS) and is also a Special Interest Conference of the International Association for Computational Mechanics (IACM).

Textile composites and inflatable structures have become increasingly popular for a variety of applications in – among many other fields - civil engineering, architecture and aerospace engineering. Typical examples include membrane roofs and covers, sails, inflatable buildings and pavilions, airships, inflatable furniture, airspace structures etc.

The objectives of Structural Membranes 2013 are to collect and disseminate state-of-the-art research and technology for design, analysis, construction and maintenance of textile and inflatable structures.

Contributions to the Structural Membranes 2013 deal with the presentation of the challenging tasks in the individual design steps of textile composites and inflatable structures. The topics vary from geometrical modelling in the design and construction process, advanced simulation technologies for structural analysis of lightweight structures under various load conditions (e.g. coupled aero-elastic analysis), the description and validation of suitable material laws, methodologies for form finding and patterning, adaptive membrane structures, energetic aspects, testing procedures, maintenance techniques up to manufacturing. Structural Membranes 2013 addresses both the theoretical bases for structural analysis and the numerical algorithms necessary for efficient and robust computer implementation.

The collection of extended abstracts includes contributions sent directly from the authors and the editors cannot accept responsibility for any inaccuracies, comments and opinions contained in the text.

The organizers would like to take this opportunity to thank all authors for submitting their contributions.

Munich, October 2013

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HIGH TENSION – TENSILE ARCHITECTURE

NEW STADIUM PROJECTS

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Key words: FIFA World Cup 2014; stadia; Brazil; Rio de Janeiro; Brasília; Manaus; looped cable roof; new buildings; renovation.

1 INTRODUCTION

Since 2008 several stadia are newly planned and under construction or finished, others will be expanded and modernized for the FIFA 2014 World Cup in Brazil. In this paper three projects are presented. To develop designs for three different stadia at the same time and to represent the designs locally, requires appropriately staffed and experienced teams, but also a clear definition of the design parameters. These parameters differ in each project and so are the resulting designs new and refreshing variations of the familiar theme.

2 MARACANÃ IN RIO DE JANEIRO

The Estádio Mário Jornalista Filho, as it is called correctly, built in 1950 was completely reconstructed for the 2014 World Cup. Besides the tiers and the facilities underneath, particularly the roof construction was affected. The original concrete cantilever roof merely spanned the upper tier of the grandstand and therefore it no longer met the requirements for a modern stadium. During the design process several expansion options were developed to keep the existing roof structure. In the end the poor condition of the structure turned the balance in favour of a complete renewal of the roof.

New specifications included the demand for almost 70 metres roof depth and the continuous use of the listed concrete substructure. After the demolition of the roof the substructure comprised the dominant building supports and the facade columns, as well as a continuous ring beam on eaves height.

To maintain the appearance of the original stadium the roof structure was to be kept low at the inner and outer rim, and the shaped roof was meant to be floating just above the stadium bowl. This has been achieved by the design of a cable-net roof which is based on the design principle of a spoked wheel. A combination of one or more continuous outer compression rings and a pre-strained cable-net, consisting of inner tension rings and radial connection cables, form a stable roof slab which can transfer compressive loads and suction forces, as well as horizontal loads (deriving from wind friction or alternating wind suction and pressure forces).

The design of the Estádio Jornalista Mário Filho is based on a new option of this construction type.
3 MANÉ GARRINCHA IN BRASÍLIA
During the redevelopment of the existing national stadium Estádio Nacional De Brasília
Mané Garrincha in Brasília its capacity has been extended from 45,000 to 70,000 seats. The stadium is located prominently in the western part of the Eixo Monumental. The East-West axis through the city accommodates many renowned buildings by Oscar Niemeyer, e.g. the national congress, the national museum and the cathedral. Against this architectural background characterised by concrete buildings, the natural consequence is to use this material also for the new stadium roof. Hence, all member subjected to high compressive forces are made of concrete.

The design of the roof structure is a combination of the suspended roof and the spoked wheel principle. The suspended roof comprises a lightweight cable and steel structure that is supported by a concrete compression ring with an outer diameter of 309 m. Three concentric circles of columns with 96 concrete columns in each circle support the compression ring. The access structure for the stadium bowl has been integrated in this jungle of columns. A ramp system takes the spectator up to the gallery, the so called ´Esplanade´ to access the upper tier of the stadium stands.
4 ARENA DA AMAZÔNIA IN MANAUS

Amongst other facilities the already existing sports complex consisted of the football stadium Vivaldo Lima and a so called `Sambadromo´ where dazzling carnival parades are held every year.

The football stadium used to be an earth wall stadium; the overhang of the concrete structure covered only a part of the grandstand. Thus, a great many spectators were not shielded from the daily occurrence of heavy showers, typical for the Amazon region. But it goes without saying that rain protection is a requirement in the FIFA regulations. This was one of the reasons to design a completely new stadium building which holds approx. 47,000 people.

During the design of the dominant façade and roof structure the interdisciplinary team thoroughly dealt with the given conditions and parameters, and managed to achieve a perfect symbiosis of architecture, technology and functionality.

Diagonally intersecting beams, with rounded edges in the area of the nodes, generate a diamond-shaped roof area with an organic appeal. The fact that the roof area is visually not separated from the façade faces enhances the architectural and structural integrity of the design.

Figure 5: Rendering of the Stadium in Manaus (gmp Architekten)
### 4 PROJECT DATA

**Maracanã Stadium, Rio de Janeiro**

<table>
<thead>
<tr>
<th>Client</th>
<th>EMOP, Rio de Janeiro</th>
</tr>
</thead>
<tbody>
<tr>
<td>General contractor</td>
<td>Consórcio Maracanã</td>
</tr>
</tbody>
</table>
| Concept and design of roof | schlaich bergermann und partner  
Knut Göppert with Knut Stockhusen  
and Thomas Moschner and Miriam Sayeg |
| Planning of photovoltaics | schlaich bergermann und partner |
| Redesign of bowl | Fenandes Arquitetos / Associados |
| Wind tunnel tests | Wacker Ingenieure |
| Completion | April 2013 |
| Capacity for the World Cup | 81,550 (incl. VIP boxes) |
| Covered gross floor area | 45,500 m² |
| Steel Structure | 2,300 t |
| Cable structure | 410 t |
| Surface of membrane | 46,500 m² |

**Mané Garrincha, Brasilia**

<table>
<thead>
<tr>
<th>Client</th>
<th>Novacap, Brasilia</th>
</tr>
</thead>
<tbody>
<tr>
<td>General contractor</td>
<td>AG + VIA</td>
</tr>
</tbody>
</table>
| Architect | gmp Architekten von Gerkan, Marg und Partner  
Castro Mello Arquitectos, São Paulo |
| Concept of roof and design | schlaich bergermann und partner  
Knut Göppert with Knut Stockhusen  
and Stefán Dziewas and Miriam Sayeg |
| Wind tunnel tests | Wacker Ingenieure |
Completion March 2013
Capacity for the World Cup 70,000
Covered gross floor area 67,000 m²
Steel Structure 2,200 t
Cable structure 410 t
Surface of roof 47,000 m²

Arena da Amazônia, Manaus
Client Manaus Government
General contractor Andrade Gutierrez
Architect gmp Architekten von Gerkan, Marg und Partner
Concept of roof and design schlaich bergermann und partner
Knut Göppert with Knut Stockhusen and Sebastian Grotz and Miriam Sayeg
Wind tunnel tests Wacker Ingenieure

Completion March 2014
Capacity for the World Cup 46,000
Covered gross floor area 25,000 m²
Steel Structure 6,600 t
Surface of membrane roof and façade 31,000 m²
CONSTRUCTION OF GRIDSHELLS COMPOSED OF ELASTICALLY BENT ELEMENTS AND COVERED BY A STRETCHED THREE-DIMENSIONAL MEMBRANE

STRUCTURAL MEMBRANES 2013

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Key words: Gridshell, Composite materials, dynamic relaxation, form-finding, prototypes.

Summary. This document deals with the gridshells built by the Navier laboratory in the last ten years. The numerical conception is developed, from the draft made by architects up to the final structure. To design a gridshell several numerical tasks have to be performed. The geometry of the gridshell is first considered. Then, an iterative step mixing geometry and mechanical considerations is important. In particular, it is explained how the naturally straight beams are bent to form the final shape. This active bending provides many interests like high stiffness for a light weight structure. After the numerical design of the grid, the geometry of the membrane is drawn from the numerical final geometry of the gridshell. The improvements of gridshells, including safety considerations as well as practical considerations are then developed, through the four gridshells recently built.

1 INTRODUCTION

In the last twenty years many applications of composite materials in the construction industry were made. The main field of application concerns the reinforcement of concrete beams with carbon fiber plates [1] or post tension cables. More recently, a footbridge with carbon fiber stay-cable was build in Laroin (France, 2002 [2]), another footbridge, all made of glass fiber composites, was build in Aberfeldy (Scotland, 1993 [3]) and a movable bridge (the Bonds Mill lift bridge in Stonehouse, England, 1995 [4]). Nevertheless applications using composite materials as structural elements remain exceptional. Although the qualities of their mechanicals properties are obvious (low density, high strength and high resistance against corrosion and fatigue), their relatively low elastic modulus is a disadvantage against steel. Indeed most slender structures in structural engineering are designed according to their stiffness and rarely to their strength. In addition, the elastic instabilities depend linearly on the Young modulus, so that again, having a low Young modulus is a real disadvantage when a designer tries to calculate structures based on conventional design structure. In order to take
advantages of every characteristic of composite materials, new structural concepts have to be found.

The Architected Structures and Materials research unit of Navier laboratory is working on the development of innovative solutions for composite material in civil engineering. Four design principles guided the conception of the structures:

- Optimal use of the mechanical characteristics of the fibers;
- Simple connection between components of the structure;
- Optimal design according to its use;
- Use of components already available in the industry for cheap material costs.

Several structures were investigated such as an innovating footbridge [5] and several experimental gridshells [6] [7] [8] [9]. The purpose of this paper is to explain the method used to design gridshells, up to the fabrication of the membrane and then to emphasis the improvements of the building process. The first section gives a proper definition of gridshell and emphasis the specificity of their construction process. Then, the numerical aspects of the project are developed. Finally the steps of construction and the improvements made through the salvo of projects are approached.

2 GRIDSHELLS: DEFINITION AND PROCESS OF CONSTRUCTION

The name of gridshell commonly describes a structure with the shape and strength of a double-curvature shell, but made of a grid instead of a solid surface. These structures can be made of any kind of material - steel, aluminum, wood… Generally, the metallic structures are made of short straight elements defining a cladding made of plane triangular or quadrangular element. The complexity of this geometry requires the development of many clever and expensive assemblies. In order to avoid these complex joints, a very specific erection process was developed using the ability of slender components to be bent [10]. Long continuous bars are assembled on the ground, pinned between them in order to confer on the grid a total lack of plane shear rigidity what allows large deformations. The grid is elastically deformed by bending until the desired form is obtained and then rigidified. With this process, the initially straight beams are bent to form a rounded stiff surface. Only few gridshells were built using this active bending method, among which the most famous are: the Mannheim Bundesgartenschau (arch: Mutschler and Partner and Frei Otto, Str. Eng: Arup, 1975 [11]), the carpenter hall of the Weald and Downland Museum (arch. E. Cullinan, Str. Eng. Buro Happold, 2002) [12] and the Japanese pavilion for the Hanover 2000 Exhibition (arch: Shigeru Ban, Str. Eng. Buro Happold) [13]. In addition, the Navier research unit has already participated to the construction of four gridshells in glass fibre reinforced polymer (GFRP), increasingly large. The gridshell for the Solidsays’ festival was 300 m² large [8] but very recently a 350 m² gridshell called “Cathedrale Ephemere de Creteil” has been constructed to replace the Creteil cathedral during its renovation which should last at least two years.

Construction steps: the main building steps are illustrated figure 1: the grid is assembled flat on the ground (figure 1a), then erected by two cranes (figure 1b) and gets its final form when attached on anchorages.
3 COMPOSITE MATERIALS TAILOR MADE FOR THIS TYPE OF STRUCTURES: FLEXIBILITY FOR STIFFNESS.

Most of the gridshell structures have been made of wood because it is the only traditional building material that can be elastically bent without breaking. This flexibility generates curved shapes which generates structural stiffness. However looking at other industrial fields (sport and leisure, nautical...), it can be noticed that every time high strength and high deformability are required, composite materials is replacing wood (ship masts, skis, rackets). To study accurately the question of the best material for gridshells, the authors adopted the method proposed by M. Ashby [14]. In this method, indicators characterising the object to be designed are defined. In the case of gridshells, it is necessary to have a material with:

- High elastic limit strain in order to be able to bend the element and obtain a curved shape.
- High Young modulus to confer to the gridshell its final stiffness after bracing.

The Ashby method drawn for these two characteristics provided several materials potentially better than wood for the gridshell application. These materials are titanium, CFRP, GFRP and technical ceramics. In addition, this study showed that steel or concrete can not be better than wood for such an application: these materials can not deform as much as wood. To choose between the four families of material, other aspects have to be considered. In particular, materials shall not be too brittle to be easily handled on site by workers and therefore ceramics are not suited. Because of cost limitation, titanium and CFRP can not suit for the gridshell application.

The most valuable alternative to wood is hence glass fibre reinforced polymers (GFRP). They have higher elastic limit strain (1.5 % at best for GFRP and 0.5 % for wood) so large curvature synonymous of freedom of shape is possible. Their Young modulus also is higher (25-30 GPa against 10 GPa for wood). This is an advantage to make a stiff structure. In addition, supposing that for a given geometry, the buckling load of a gridshell is linearly dependent from the young modulus, one can expect the buckling load of a gridshell in composite materials to be 2.5 to 3 times higher than one made of wood. Moreover, as composites are industrially produced, the reliability of their mechanical properties is much
higher than that of natural materials like wood. Finally, while wood beams have to be made of several pieces of wood stuck together, GFRP profiles can be made continuously, as long as necessary.

Concerning costs, if one takes into account the mechanical properties and the ability of composites to be formed into efficient sections like tubes, GFRPs become very interesting challengers, especially if pultrusion production is used. Indeed, hollow sections make possible the use of light beams optimized for each application (stiffness and curvature). Moreover, the polymer chosen for the GFRP can resist to corrosion, UV and other environmental attacks, whereas wood materials need maintenance.

At this point, the gridshell concept is explained. The type of materials chosen is GFRP for flexibility, cost, stiffness and reproducibility reasons. The process of construction can be developed.

4 CONCEPTION OF A GRIDSHELL: FROM A GEOMETRY TO A FINAL SHAPE.

At the beginning of the process, the architects have to define a shape. To be well suited with the process of the gridshell, the shape has to be a rounded shape with curvatures as homogeneous as possible. Then, according to the range of curvatures, the engineers have to choose the geometric properties of the beams (mainly the outer radius – this is explained in equation 1). Then, the engineers have to do first a geometric mesh of the shape before relaxing it mechanically to get the final form of the gridshell. After this form-finding step, the engineer adds the third layer numerically (bracing layer) and evaluates the stresses in the structure under serviceability limit states (SLS) and ultimate limit states (ULS) loads, according to construction codes [15]. They might have to modify the mesh, or the shape of the gridshell, in agreement with the architects to reduce the stresses. Once the form-finding has converged and the stresses are suitable, the engineers can design the membrane according to the three-dimensional shape numerically obtained. The method summarized here is developed in the following.

Geometrical step: The method used for the forming of the grid is "the compass method". This method consists in constructing a network of regular quadrangles on any surface, plane or not. This method was described in IL10 Gitterschalen of Frei Otto 1974 [11]. The figure 2 shows the different steps of the method on surface which can be three-dimensional. The task is to construct a grid using only a compass. First, two curves that intersect each other are laid down on the surface to mesh. Then, a mesh size is chosen and serves as the compass radius. The spacing of the grid is marked along each axis, from the point of intersection of the axes. The knots are determined by the intersection of two circles as shown on the figure 2. Gradually, new points are determined.
So, to generate the grid of the gridshell, a 3D compass method is performed on the surface. Obviously, the grid obtained has no mechanical meaning. The real shape of the gridshell is obtained later when the mechanical properties are considered. This method was used for the design of the gridshell for the Solidays festival (June 2011) and for the one of the “Cathédrale Ephémère de Créteil” (February 2013).

An implementation of the geometrical method has been developed at Navier laboratory, using Rhino NURBS modeller. This modeller makes possible the modification of a surface through control points. This is very interesting because the compass method is also performed under the same numerical environment. Thus, modifications of the surface to mesh - but also modifications of the curves defining the mesh - are easy to do and the process of meshing is immediately auto-updated.

To sum up the process, first a shape of a structure is proposed by the architects. Secondly, the surface is extended and two main axes for the construction of the grid are drawn (figure 3a). Thirdly, the mesh is automatically generated (figure 3b). The mesh might either cover all the functional shape or cover only a part of it. In the first case, the mesh can be more or less homogeneous. Among meshes covering the entire functional surface a mesh has to be chosen. It is then trimmed to get the final mesh (figure 3c).

**Mechanical step**: the final shape is obtained by performing a non linear structural analysis of the structure with real mechanical properties. This non linear algorithm is based on dynamic relaxation algorithm [10], [16].

If the shape proposed by architects is suitable with the gridshell process, the geometry of the grid provided by the compass method is very slightly modified by the dynamic relaxation process. Once the real shape is found, classical structural analyses are performed with the
standard loads. Obviously, the stress due to form-finding is taken into account in the structural analysis: the main source of stress is linked with the bending of the beams during the erection process. In other words, the stress $\sigma$ is proportional to curvature of the beams $1/R$, (equation 1), and the curvature is mainly due to forming: even under critical loads, its shape (and so the curvature of beams, and also stress) does not change significantly. This is the main advantage of the active bending which provides high stiffness in this case.

$$\sigma = \frac{E y}{R}$$ (1)

where $E$ is the Young Modulus and $y$ the outer radius of the beam.

Designing a grid shell is a difficult task. As a guideline, the designer has to check that:
- The curvature in each bar is not too high, to avoid the break of beams even with relaxation and fatigue phenomena. In practice, according to Eurocomp [15], the maximal stress in the bar must not exceed 30% of the strength of the beam. To this limit stress corresponds a limit curvature under which the risk of break is low enough to be acceptable (equation 1).
- The entire surface is meshed
- The mesh does not get too concentrated locally

If the grid is too weak to support the external loads, the designers have to reinforce it by reducing the size of the mesh and/or modifying the geometry of the cross section of the beams. If the outer radius is increased, the stress due to the form-finding gets higher as the maximal stress in a beam is proportional to both the curvature and the outer radius of the cross section of the beam. So the engineers have to be cautious.

The consideration of wind and snow loads presupposes that the gridshell is covered by a membrane. The membrane is made according to the geometry obtained through the dynamic mechanical process. From this geometry, the surface is partitioned in pieces of plane surface (according a tolerance which depends to the material of the membrane). Then the pieces are sewed to form the three-dimensional membrane. Some pieces of membrane (the yellow ones) can be easily seen thanks to colouring, on the gridshell, figure 4. The membranes are PVC coated sheetings.

Figure 4: Long term erected composite gridshell. Some planar parts (yellow) of the membrane can be seen.
5 PROTOTYPES

Prototypes: to demonstrate the feasibility of composite gridshells, four full scale prototypes of composite material gridshells have been built. The two first ones were built on the campus of the Université Paris-Est. The first prototype was a purely experimental structure which was tested under several loading conditions in order to investigate the behaviour of gridshell structures and to compare it with the numerical models (figures 4 and 5a). Detailed results of these tests can be found in [9]. The behaviour of the prototype is very close to simulations performed numerically, with the dynamic relaxation algorithm presented in [14]. This gridshell is now serving as a shed for equipment and has a great importance since it is the one that was erected the first (around five years ago). It is still erected and serves as testing for long term damage (mainly creep and fatigue). The second prototype was built to cover a wind tunnel, figure 5b.

![Figure 5: a. First experimental gridshell under testing. b. Second experimental gridshell used as a shelter for a wind tunnel.](image)

Prototypes sheltering people: as previously written, two gridshells built to house people have been recently made. The first one for the Solidays festival (June 2011) and the last one built to temporally replace the Creteil Cathedral (February 2013 for at least 2 years of use). More details about the context of this gridshell and about the project can be found in [17]. Compared to the two first experimental gridshells, these two are larger and had to take into consideration many aspects for public safety.

These two last gridshells have got several improvements regarding the previous ones. First, their size was larger. So large that most of the tubes of the structure was to be built from several tubes joined up together. Second, the gridshells had to obtain an attestation from administrative authorities to house people for a specified period. This attestation was given after a committee had studied the structures, that is to say the project on the paper as well as the execution on the building site.

The shapes: the Solidays Gridshell was looking like a half peanut (two domes) while the Ephemeral Cathedral looks a little more like a stretched one dome structure. The dimensions of these structures are quite similar: around 7 m high, 25 m long and 15 m wide. They are constituted of about 2 kilometres of pultruded unidirectional tubes from Topglass (polyester resin from DSM + Owens Corning glass fibres) with a Young modulus of 25 GPa and a limit
stress of 400 MPa. The available length and diameter of the tubes are respectively 13.4 m and 41.7 mm; the wall thickness of the tubes is 3 mm.

As said before the stress is limited to 30% of the limit stress to avoid severe creep and damage effects like progressive rupture of the fibres.

Unlike the former gridshells and as written previously, given the short period for the project, the geometries of the membrane of these two gridshells were drawn from the numerical shapes.

**Computation:** the computation has been performed for different sizes of mesh. It appears that a mesh size of one meter was acceptable to resist to the loads studied (dead weight, wind, snow). Under these loads, it is important to check that the stress remains acceptable in all the structure, but since the stress in the bars is mainly due to the form-finding if the structure has been cautiously designed – that is to say if it does not buckle under considered loads - the stress might not reach too high values. In this case, the stress can be drawn from the study of the gridshell without any load applied (figure 6).

![Figure 6: Stress resulting from forming in the Soliday’s gridshell](image)

**Fabrication:** once the form of the structure is defined, the coordinates of the extremities are picked up and precisely reported by geometers on site and stakes are positioned. Then, the grid is assembled flat on the ground: tubes has been cut to the right dimensions with hacksaws and connected to the others with standard swivel scaffolding elements (figure 7a). These scaffolding elements allow rotation around their axis. They are chosen for their low cost due to industrial production.

Then the grid is deformed and shifted by two cranes that hook up the grid in several places (figure 1b). The final form is reached when the extremities of the beams are fixed on the anchorages. The erection phase requires only a few hours work for about ten people whereas the preparation of the grid can take many days.

The following structural step is the bracing. This step is essential, as before bracing, the grid still holds its shear degree of freedom. To behave like a shell, the bracing will transform every deformable quadrangle into two rigid triangles. The third direction of beam is installed as shown in figures 7b and 7c with the use of new scaffolding elements. Once the bracing is
installed, the gridshell gets its full mechanical properties and its stiffness gets about twenty times the stiffness of the grid before bracing [8]. The bracing step does not apparently change the form of the gridshell, but since the bracing can’t be done everywhere in the same time, it may modify the shape a little. This step is the most fastidious one because the third layer of beams has to be set up in the deployed geometry. Thus the operators have to adjust each connector and tighten the beam inside it, generally in a basket, a few metres over the ground.

![Figure 7: a. Joint detail. b. Mesh before bracing. c. Mesh after bracing.](image)

The structural part being finished, the positioning of the membrane can be started. First the PVC coated membrane is pulled above the gridshell (figure 8a). In order to fix the canvas, a girder is set up 10 cm from the soil (figure 8b). The girder follows the contour of the gridshell. For the Creteil gridshell, this girder was a pultruded rod able to support a large amount of shear stress as well as high curvatures (here hollow cross-sections are not suited and the outer radius has to be smaller than for structural beams). The canvas is then positioned and stretched. This step was supposed to be critical as polypropylene-PVC coated canvas is almost not stretchable, and was manufactured according to the geometry of the numerical model. So a mistake during the numerical design or during the building phase could have led to a situation where the canvas would not really fit to the structure. As the gridshells were accurately set up, the canvas fitted to their shape. No wrinkle was observed (figure 8c). The membrane might play a part in the structural behaviour of the grid but given the high dependence to modelling (in particular to friction between beams and membrane and also between connectors and membrane), it is very difficult to evaluate accurately the real stiffening effect of the membrane.

![Figure 8: a. Positioning of the PVC coated membrane (Creteil). b. Continuous beam for border during tightening (Creteil). c. Membrane without wrinkle (Solidays gridshell).](image)

**Improvements:** to deal with the fact that these last gridshells are made to shelter people many improvements have been added to the previous prototypes. In particular, fire,
waterproofness, lightening and thermal considerations have been added to the primary mechanical considerations. Nevertheless mechanical properties have been considered with much more attention to assure the public safety. The reference construction guide - named Eurocomp, for composite materials – guided the construction. In particular, the way of production of beams, their constitutive materials as well as the duration of solicitations acting on the structure were taken into account. On the other hand, many assays were performed to get the real properties of the beams (mean strength, variation coefficient).

In addition, a robustness study has been performed on the Solidays gridshell [18]. This study showed that the gridshell can undergo accidental situation such as vandalism without risking collapse. Indeed, thanks to redundancy, the stress from a break would spread largely and the stress in the neighbouring beams would not get too high. In the same time, large displacement of broken beams would be visible and the evacuation of public could be launched. This kind of ductility is named pseudo-ductility in this case when fragile materials are mainly used. This study has also showed that the buckling of the gridshell must be avoided at any cost: if buckling starts, the curvature of some beams will increase a lot and the stress will increase in the same time, up to breaking and then to the ruin of the structure.

Other improvements were done, relating to the connection of the grid with the soil (figure 9a) and also to assembling of 13 m pipes to form long beams up to 35 m (figure 9b). The difficulty is to make connections able to transmit stress in a way that the assembled beams keeps mechanical properties of the primary GFRP beam. In particular, the joining up of two beams have to:

- transmit normal stress for structural stiffness
- have similar bending stiffness for the continuity of the global shape

Thus, to combine these faculties, the system presented on the figure 9b was chosen. Each beam is assembled with a sleeve slightly larger with three pins. This assembly can undergo axial forces up to 30 kN. This theoretical value (obtained with the help of [15]) has been experimentally validated thanks to a salvo of assays. In addition, some extra glue is put inside the assembly to prevent from relative movements and thus fatigue phenomena. Then, between the sleeves of the two beams to connect, a M20 threaded rod is screwed inside bolts welded to the sleeves. This threaded rod is adapted to behave like the structural beams, under bending stress.

Figure 9: a. Pin anchorage for beams. b. Assembly used to join two beams.
6 CONCLUSION

This article first explained the choice made by the Navier laboratory to use composite materials, and in particular the choice of GFRP. Then, the process of conception of these composite material gridshells is presented. In particular, the active bending of the gridshell is illustrated, on a numerical aspect as well as during the building stage.

The development of the prototypes can be seen as the prototypes got larger, with safety standards getting higher and higher because of their use as a shelter for public and also to make possible longer periods of use. In particular, assembling devices and anchorages have been largely improved. The critical points are also approached. The most fastidious one is the bracing which can last a long time. Another one is linked with the membrane covering the structure, which is sewed according to the numerically obtained geometry. That does not leave much room for error during the construction step.

Nonetheless it is important to remind the qualities of such structures. Due to suitable process of construction, the structures shows light weights (around 5 kg per m²), as well as high strength. Another advantage is the fact that most of the stress in elements is due to forming, and that extra loads like wind and snow add very little stress, under acceptable load cases. This behaviour is observed since in these cases, bucking is not generated. Finally from an architectural point of view, this technology provides new horizons. The gridshell built in Creteil is a good illustration, for the global exterior shape (figure 10a) as well as from an interior view (see figure 10b).

Figure 10: Gridshell built in Creteil. a. Global view. b. Inside view.

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Summary

This paper discusses form-finding and simulation strategies for form- and bending-active hybrid structures, with practical feedback from two realised projects. Next to some general aspects of computational form-finding approaches with focus on finite element methods (FEM), the influence of changing mechanical properties of elastic beams on the resultant form-found hybrid system will be discussed on an umbrella structure with integrated bending-active beam elements. Alongside the question of simulation strategies comes the search for a practical design setup to establish an FEM environment that is cross integrating information from various other modelling environments. This is discussed through the case study project M1 where physical form-finding and vector-based spring methods are utilised to generate input data for the FEM simulation.

1 INTRODUCTION

The flexibility and lightness inherent to bending-active structures [1] integrates well with form-active membrane structures that are themselves flexible and adjust to applied loads [2]. Their particular reciprocal dependency of mechanical properties, pre-stress and form makes them an interesting field of study for exploring computational form-finding techniques and thereby developing new kinds of structure systems.

Functionally, the integration of bending-active elements within a pre-stressed membrane surface offers the possibility of short-cutting tension forces and creating free corner points. The system is stabilised solely by the elastic beams which, in turn, are restrained by the membrane surface. Since buckling of the beam is prevented, slender elastic beam profiles may be used. Formally, adding the elastic beam to the limited catalogue of basic membrane types generates a vast extension to the possibilities of shape generation. The introduction of the elastic beam hybridizes the clearly separated membrane-only types as it may, for example, transform from a curved boundary edge directly into a ridge within the surface. In this hybrid
approach of combining form- and bending-active structures, a new scope of formal possibilities presents itself. We classify this interdependence of form and force of mechanically pre-stressed membranes and bending-active fibre-reinforced polymers as a textile hybrid.

Independent of the exact simulation technique, the form-finding process of such textile hybrid structures may be split into several steps, usually starting with the elastic deformation of the beam elements. The membrane surfaces are then generated on the updated boundary conditions of the beam elements leading to a second form-finding of the combined system. In some cases of low topological complexity, a completely simultaneous form-finding of both systems may also be possible. In this paper both approaches are shown in the discussion of two case study projects.

While standard routines for the form-finding of membrane structures are available in commercial FEM software, the necessity of incremental deformation for form-finding large elastic deformations in the beam elements is accomplished through custom programming.

2 VARIABLES IN THE FORM-FINING PROCESS

The design process of bending-active structures can be summed up as the alignment of mechanical and geometrical variables to generate a structurally and architecturally functioning result from a physically informed deterministic form-finding process. Similar to membrane structures, the built geometry is a result of the erection process, where the structure is tied to its boundary points. The self-defining shape of the structure is previously discretised by a cutting-pattern in the case of a membrane and the unrolled geometry in the case of a bending-active structure. In contrast to membrane structures, the form-finding result does not automatically define a structurally optimised geometry which may be linked to the fact that bending-active structures offer far more variables to influence the form-finding result.

Both form- and bending-active systems undergo large deformations during form-finding, yet there is a fundamental difference in the simulation of their stress states. For form-active membrane elements, the biaxial tension pre-stress is a command variable set prior to the form-finding, whereas stresses in the bending-active beam elements are computed from the deformation resultant strains. This leads to considering material properties differently during form-finding. In the modified stiffness method used in FEM form- for pre-stressed membranes, one simply considers the fact that a membrane only serves to carry tension forces by simulating a surface under pure tension. The actual mechanical material properties of the membrane are, however, not considered since the form-finding is purely based on the equilibrium of tension stresses and only geometrical stiffness is considered while elastic stiffness is temporarily set to zero. In contrast to this, the form-finding of bending-active structures is largely influenced by the mechanical material behaviour of the beam or shell elements. While the geometry of a single and homogeneous elastica curve is independent of size and material, the structurally necessary coupling of several bending-active components
results in a material dependent geometry.

In form-active structures, the surface dimensions are the minimal result defined by the stress state and boundary conditions which are independent of the input geometry. In contrast to this, the form-finding of bending-active structures is largely influenced by the length of a beam or dimension of a surface that is bent as a result of the constraining boundary conditions as well as the mechanical material behaviour of the beam or shell elements. Consequentially, the two fundamental differences in the form-finding of form- and bending-active structures lie in the definition of length and surface dimension and the simulation of material behaviour.

Having factually doubled the amount of input variables in the form-finding process of bending-active structures, as opposed to form-active structures, noticeably complexifies the general design process. Putting these input variables into a functioning relation, which satisfies both mechanical behaviour and architectural specifications, becomes the challenge of this form-finding and the general design process. Because of this unique combination of freedom and complexity, it was found that one computational simulation technique alone does not offer necessary tools for developing textile hybrid structures. The combination and integration of various modelling techniques into a design process is necessary to successfully develop complex textile hybrid structures. These may include physical, behaviour-based, computational and finite element based modelling.

3 FEM FORM-FINDING APPROACHES

3.1 FEM Form-finding of textile hybrids

The form-finding of membrane structures with bending-active support systems necessitates a combined form-finding of the form- and bending-active elements. There are three principal approaches that can be followed to achieve such a combined equilibrium system:

Additive: The form-finding of the bending-active and form-active structures are separated. The two systems are coupled together once the separate entities are form-found. A subsequent equalising calculation of the coupled system, where stress is referenced but no additional loads are applied, will find the system’s final equilibrium shape. This approach is possible for systems where the membrane has a small or predictable influence on the bending-active structure.

Successive: The process is separated into first, the form-finding of an elastically bent beam structure and second, the form-finding of the membrane attached to the beams. Here, the second form-finding step serves to generate an intricate equilibrium system which is based on further deformations in the beam structure.
Simultaneous: Some scenarios also allow simultaneous form-finding of bending-active beam elements and pre-stressed membrane elements. For numerical form-finding, the bending of beam elements requires out of plane forces on the beam; this may be achieved by eccentricities and/or three-dimensional input of the membrane-mesh.

In the projects presented in this paper, the FEM software Sofistik® was used for the form-finding of textile hybrids, which has the advantage that the form-finding and patterning routines for tensile membranes are already included in the software. These routines could easily be combined with any of the above mentioned approaches and a custom programmed load increment loop for the bending-active elements. Therefore, any of these approaches may be chosen depending on the individual nature of the form-finding problem. In addition, a physics-based particle–spring computational modelling environment was used for informed digital form explorations with bending-active and textile hybrid systems in particular.

3.2 Elastic cable approach

As a practical approach for form-finding coupled bending-active systems in FEM, the first author developed a new strategy using contracting cable elements to pull associated points from an initially planar system into an elastically deformed configuration [3]. These cable elements work with a temporary reduction of elastic stiffness which enables large deformations under constant pre-stress. This method was originally developed for the form-finding of tensile membrane structures using, for example, the transient or modified stiffness method [4] and [5]. For the form-finding of coupled bending-active systems, the great advantage is that the cables allow complete freedom of the equilibrium paths that are followed during the deformation process. The pre-stress independent of the change in element length also allows the simultaneous use of several cable elements in the different positions of the system.

This approach enables the form-finding of topologically highly complex systems as represented by the M1 Project discussed below.

3.3 Continuous mechanical description

Form-finding in FEM requires system updates before structural analysis with external loads is performed on the form-found system. In this update, both geometry as well as inner stress states are stored in the stiffness matrices of the model to create an updated reference state of the system that includes all mechanical information. This mechanical continuity of the FEM model becomes particularly important in the aforementioned successive form-finding approach of textile hybrid systems. The fact that the form-found equilibrium state of such systems is only satisfied if both updated coordinates and stored elastic stresses are included in the subsequent static calculations was also discussed by Philipp et.al. [6]. Fig. 1 shows the continuous mechanical description of the structural FEM Model from form-finding to patterning of the M1 Project introduced below. In this example, the form-finding is
successive, starting with the bending-active system where the hybrid with form-active elements is form-found afterwards. For the validity of the equilibrium state form-found with this approach, it is essential that an update of the model always references both coordinates and elastic stresses. This process is continued all the way through to patterning of the membrane, in which compensation is based on the stress state of the membrane surfaces.

4 SPRING BASED MODELLING

Spring-based methods have been developed to simulate and explore complex material behaviours, utilizing vector-based methods combining mass (particle) and momentum (spring) forces within a time-based solver [7]. In the research developed by the second autor, this has been employed for enacting both tensile and bending stiffness. Particular material behaviours are defined in specific spring topologies placing positional constraints on variable
(user-defined) networks of particle-springs. A hierarchy is established where certain springs serve to provide tensile and bending stiffness (in- and out-of-plane forces) while other particle-springs define meshes representative of a physical geometry. In this topological construct, the relationships within the condition of the textile hybrid are relative and not explicitly expressive of select material descriptions.

The approximation of the mechanical behaviour for bending stiffness is captured in particular topological arrangements of springs and the springs’ properties of stiffness. Three primary numerical methods have been established in Computer Graphics: cross-over, vertex position and vertex normal [8]. The vertex normal method has been expanded upon in structural engineering simulating three degrees of freedom at each particle (node) in order to calculate the behaviour of bending-active beam elements [9].

4.1 Interaction

Spring-based methods enable explorations of textile hybrids to advance complexity and specification in their topological relationships via relative descriptions of force characteristics. A certain freedom from specific mechanical descriptions allows for a minimization of the pre-planning effort prior to beginning the form-finding process. Most importantly, housed within a functional modelling environment, relationships can be actively manipulated in topology and behavior, while the time-based solver continually runs. Where the complexity of the textile hybrid belies intuition, iterative feedback through the computational environment elicits knowledge in particular topological and behavioural manipulations. In a programmable modelling environment, such specific manipulations can be encapsulated and embedded in order to advance the potentials for geometric performance. Such was accomplished with a modelling environment programmed in Processing (Java) by the second author.

5 CASE STUDIES

5.1 Umbrella Marrakech

At the Institute of Building Structures and Structural Design, a new type of membrane structure was developed and realised in collaboration with HFT Stuttgart. The project is based on a student workshop that developed shading solutions for an outdoor plaza space at an architecture school in Marrakech, Morocco. The design proposal of a funnel-shaped membrane roof was further developed by the first author with the aim of minimizing anchoring forces to the surrounding buildings. The introduction of a bending-active supporting structure for the free edges of the membrane proved to be a very efficient solution. After a successful test setup in Stuttgart, which took place in June 2011, the structure was mounted by students from Stuttgart and Morocco in March 2012.

The structure features six elastically bent glass fibre rods with a length of approx. 7.5m. The rods push out three additional corner points on both free edges of the structure. The funnel-shaped membrane has a span of approx. 11m x11m and an eaves height of 5.5m resulting in a membrane surface of approx. 110m² (Fig. 2).
For the relatively simple geometry of the umbrella, the simultaneous form-finding approach is used (Fig. 3). Here, controlling the stability of the beam during form-finding is a particular challenge, since the stabilizing effect the membrane has on the beam is only activated in the post form-finding configurations. This necessitates a temporary restraining of the beam perpendicular to the bending plane. This is particularly the case because the utilised FEM Software Sofistik® uses the transient stiffness method where the large deformations which occur during the form-finding are enabled by a temporary reduction of the elastic stiffness in the membrane and edge-cable elements [4]. Floating coupling elements are used to control the off-set distance between the mechanically pre-stressed membrane elements and elastically deformed beam elements.

The influence of the elastic beam’s mechanical properties on the form-finding result becomes particularly visible in this structure where the cantilevering condition of the beam offers little geometrical constraint on the equilibrium shape. Figure 4 shows a comparison of different beam stiffness ratios $n$ in the FEM simulation results from simultaneously form-finding the form- and bending-active elements of the structure.
5.2 M1 Project

The Textile Hybrid M1 at La Tour de l’Architecte showcases the research on hybrid form- and bending-active structure systems. The scientific goal of the project was the exploration of formal and functional possibilities in highly integrated equilibrium systems of bending-active elements and multi-dimensional form-active membranes. The resulting multi-layered membrane surfaces allows not only for structural integration but also serves a functional integration by differentiating the geometry and orientation of the membrane surfaces. The site selected for the design is a historical and structurally sensitive tower in Monthoiron, France. The tower is based on a design by Leonardo Da Vinci from the 16th century, which brought the owners to the idea of making the tower usable for exhibitions. On the basis of a spatial program, a textile hybrid system is developed where short-cutting of forces produces a minimization of the loading on the tower. In the context of this project, the M1 is developed as a representative pavilion.

The scientific goal of this project was a formal and functional exploration of textile hybrid systems through the establishment of an iterative design process, which passes through various modelling environments and finishes with a realised structure (Fig 5).
The M1 structure is comprised of 110 meters of GFRP rods, 45m² of membrane material covering an area of approx. 20m² and anchored to the ground with only three foundations resting against the existing stone structures which neighbour the tower. In total, the textile hybrid structure weighs approximately 60 kilograms (excluding foundations), with clear spans ranging from 6 to 8 meters. Fig. 8 shows the finished structure, elaborating upon the hybrid nature of the system, where the organisation of bending-active beams and tensile surfaces creates moments of long span arches, overlapping grid-shell conditions, and doubly-curved pure tensile surfaces, both at a macro- and meso-scale.

For generative studies, the spring-based modelling environment as described above is utilised alongside exhaustive physical form-finding experiments. The computational modelling allows for complex topologies to be developed and altered, quickly registering feedback from the prototypical physical studies (Fig 6). In particular, this approach was utilised for the form-finding of the secondary textile hybrid system, a series of differentiated cells providing additional structure to the primary envelope and variation to the illumination qualities of the space.

As both a design avenue and method for material specification, FEM is utilised. The parameters of the complex equilibrium system are explored to determine the exact geometry and evaluate the structural viability. The complexity of the M1 geometry necessitates a separation of the form-finding steps and applying the successive approach as introduced above. After the form-finding of the beam elements, the membrane mesh is generated on the given boundary conditions of the edge beams. Because of the general elasticity of the structure, the membrane pre-stress largely deflected some of the beams and therefore has a significant influence on the overall geometry. By means of automatic mesh generation, the membrane surfaces are added and a final form-finding of the fully coupled textile hybrid is undertaken (see Fig. 7). This form-found structural analysis model allows verification of the geometrical shape including its residual stress, as well as analysing the deformations and stress levels under external wind loads. Furthermore, the form-found membrane surfaces are processed directly by the textile module of the software for patterning (Fig 1).
Thus, all three design models, the physical and both generative, and specific simulation techniques informed each other in this iterative design process. The realised structure was fully based on the digital information computed through these modelling environments. The fitting harmonious pre-stress of the patterned membrane surfaces proved the validity of this design and simulation process (Fig. 8).
5 CONCLUSIONS

The two case study projects discussed in this paper showed that a full exploration of the functional and formal potentials of textile hybrid structures necessitates the use and close interaction of various modelling environments.

The Marrakech Umbrella relied on FEM simulation alone and thereby proved the ability of computing complex equilibrium systems with modern commercial FEM software. However this project was not able to explore further new structural and formal possibilities of textile hybrid systems. This aspect was taken up by the M1 project in which a design methodology was established that enabled a cross integration of information from various other modelling environments. On-going research is focusing on refining this process to establish a closer interaction of the physics-based particle-spring computational modelling environment with the FEM simulation.

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FORM-FINDING AND ANALYSIS OF BENDING-ACTIVE SYSTEMS USING DYNAMIC RELAXATION

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Summary. A common challenge for architects and engineers in the development of structurally efficient systems is the generation of good structural forms for a specific set of boundary conditions, a process known as form-finding. Dynamic relaxation is a well-established explicit numerical analysis method used for the form-finding and analysis of highly non-linear structures. With the incorporation of bending and clustered elements, the method can be extended for the analysis of complex curved and bending-active structural systems. Bending-active structures employ elastic deformation to generate complex curved shapes. With low computational cost, dynamic relaxation has large potential as a design and analysis technique of novel large span structural systems such as spline stressed membranes and small scale robotics, bio-mechanics and architectural applications made of novel materials such as electro-active polymers (EAP).

1 INTRODUCTION

Large elastic deformation phenomena are well known to engineers. At a (sub)microscopic level, these phenomena are attractive for their potential as concept generators for micro-lens surfaces and gels [1], [2] nano-tubes [3] and elastic shells [4]. However, in larger scales such as in structural engineering large elastic deformations are considered as failures and are designed against. There are only few examples in the built environment where implementing elastic deformation as a form-generating strategy has been explored [5-10].

Current design theory holds that excessive elastic deformations are undesirable in structures. Our research challenges that philosophy; we believe that large elastic deformations can be successfully modeled, analyzed and interpreted as a form-finding strategy for bending-active systems. By integrating elastic deformations in the form-finding process, novel lightweight spatial structures constructed from flexible yet strong engineering materials can be explored.
This paper focuses on the analysis of bending-active systems using dynamic relaxation. Dynamic relaxation is an established explicit numerical form-finding and analysis method [11]. In Section 2, the method and the element formulations for bending and clustered elements are presented. Section 3 presents the numerical modeling of a dielectric-elastomer minimum-energy structure and a stressed spline membrane. Finally, conclusions are presented in Section 4.

2 THEORY AND ELEMENT FORMULATION

2.1 Dynamic Relaxation Basic Scheme

Dynamic relaxation (DR) traces the motion of each node of a structure for small time increments, $\Delta t$, until, due to artificial damping, the structure reaches a static equilibrium [12]. In form-finding, the process may be started from an arbitrary geometry, with the motion initiated by a stress or force application. For analyses, the process starts with a valid geometry and the motion is caused by a sudden load application. The DR description summarized below is for structures with axial links and assumes a “kinetic” damping of the structural system to obtain a static equilibrium state. When kinetic damping is employed, the motion of the structure is traced and when a local peak in the total kinetic energy of the system is detected, all velocity components are set to zero. The process is then restarted from the current geometry and repeated until the energy of all modes of vibration has been dissipated and static equilibrium is achieved.

Dynamic relaxation is based on Newton’s second law that governs the motion of any node $i$ in direction $x$ at time $t$:

$$ R_{ix}^t = M_i \ddot{v}_{ix}^t $$

(1)

where $R_{ix}^t$ is the residual force (difference between external and internal forces) at node $i$ in direction $x$ at time $t$, $M_i$ is the lumped mass at node $i$ which is set to optimize the convergence and ensure the stability of the numerical process. $\ddot{v}_{ix}^t$ is the acceleration at node $i$ in direction $x$ at time $t$.

Expressing the acceleration term in Equation (1) in a finite difference form and rearranging the equation gives the recurrence equation for updating the velocity components:

$$ \dot{v}_{ix}^{t+\Delta t} = \frac{\Delta t}{M_i} R_{ix}^t + \dot{v}_{ix}^{t-\Delta t/2} $$

(2)

Hence, the updated geometry projected to time $t+\Delta t/2$ is given by:

$$ x_{i}^{t+\Delta t} = x_{i}^{t} + \Delta t \dot{v}_{ix}^{t+\Delta t/2} $$

(3)
Equations (2) and (3) apply for all unconstrained nodes of the mesh in each coordinate direction. Moreover, the equations are nodally decoupled: the updated velocity at a node depends only on the previous velocity and residual force of the same node. Nodes are not directly influenced by updates at other nodes.

The updated geometry is then employed to determine the new link forces and together with the applied load components $P_{ix}$ to define the updated residual forces $R_{ix}$:

$$R_{ix}^{t+\Delta t} = P_{ix} + \sum \left( \frac{F_{i}^{t+\Delta t}}{L_{m}} \right) (x_{j} - x_{i})^{t+\Delta t}$$

where $F_{i}^{t+\Delta t}$ is the force in member $m$ connecting node $i$ to an adjacent node $j$ at time $t+\Delta t$, $L_{m}^{t+\Delta t}$ is the length of member $m$ at time $t+\Delta t$. The procedure is thus time stepped using Equations (2) – (4), until a kinetic energy peak is detected. Velocity components are then reset to zero (with a small adjustment made to the geometry to correct to the true kinetic energy time peak), and the process is repeated until adequate convergence (equilibrium) is achieved.

2.2 Spline Bending Formulation

‘Splines’ originally denoted continuous flexible wooden or rubber strips used by draughtsmen to draw smooth curves for ship lines or railway curves. In this paper, the term refers to tubular structural elements that are bent from an initially straight state. It was shown that the torsional stiffness need not enter the analysis of a bent spline [7]. Therefore, the bending action in the spline is idealized as a series of bending moments between the nodes of the finite elements that compose the spline. The basic idea behind the bending formulation is that the bending moments across the elements result from changes in the curvature engendering shear forces at their nodes. Shear forces are then taken into account in the residual forces in the DR scheme.

Adriaenssens and Barnes [13] proposed a spline type formulation that deals with moments and shear forces in deformed tubular members. The formulation adopts a finite difference modeling of a continuous beam. Figure 1a represents consecutive nodes along an initially straight tubular element, and Figure 1b two adjacent deformed segments, $a$ and $b$, viewed normal to the plane of nodes $ijk$. The two elements are assumed to lie on a circular arc of radius $R$. The spacing of nodes along the traverse must be sufficiently close but the segment lengths need not be equal. The radius of curvature $R$ through $i$, $j$ and $k$ and the bending moment $M$ in the arc can be defined as:

$$R = \frac{L_{c}}{2\sin\alpha}$$

$$M = \frac{EI}{R}$$
where $EI$ is assumed to be constant along the beam, $E$ is modulus of elasticity and $I$ second moment of area. The free body shear forces $S_a$, $S_b$ of elements $a$ and $b$ complying with moment $M$ at $j$ are thus given by:

$$S_a = \frac{2EI\sin a}{l_al_c}$$

$$S_b = \frac{2EI\sin a}{l_bl_c}$$

where $l_a$, $l_b$, $l_c$ are the distances between nodes $ij$, $jk$ and $ik$, respectively. The three non-collinear nodes $i$, $j$ and $k$ define a reference plane $ijk$. The shear forces $S_a$ and $S_b$ are applied at nodes $i$, $j$ and $j$, $k$, respectively and act normal to the links $ij$ and $jk$ respectively and in the $ijk$ plane.

![Figure 1: (a) Consecutive nodes along an initially straight tubular beam traverse; (b) Two adjacent deformed segments, $a$ and $b$, viewed normal to the plane $ijk$.](image)

The calculations and transformations required in the DR scheme are thus rather simple. Nodes along the spline element are considered sequentially in sets of three, each lying in different planes when modeling a spatially curved tubular element bent from an initially straight condition. The formulation is useful for modeling grid shells with continuous tubular members, and also for membranes in which flexible battens are employed to give shape control such as in sails.

### 2.2 Clustered Formulation

Clustered elements describe sliding or continuous tensile elements and were introduced by Moored and Bart-Smith [14]. Clustered elements can group two or more links. Figure 2 (right) illustrates a clustered four-node system, where the clustered element replaces two tensile elements ($links 2$ and $3$). The clustered element can be seen as a cable running over a small frictionless pulley on node $3$. Therefore, $links 2$ and $3$ in the clustered element carry the same tensile force. Additionally, node $3$ in the clustered structure has fewer kinematic
constraints compared with the same node in the un-clustered system (see Figure 1 left) as it can move around its current position.

![Figure 2: Illustration of an un-clustered four-node system (left) and a clustered configuration (right).](image)

A clustered element for dynamic relaxation was proposed by Bel Hadj Ali et al. [15]. Similar to [14], a clustering matrix $S$ is used to link the clustered structure with its corresponding un-clustered configuration. The clustering matrix $S \in \mathbb{R}^{e \times e}$ is defined as follows:

$$S_{ij} = \begin{cases} 
1, & \text{if the link } e_j \text{ is part of the clustered element } e_i \\
0, & \text{if not}
\end{cases}$$

where $\bar{e}$ is the number of elements in the clustered structure and $e$ is the number of elements in the traditional (un-clustered) structure. The equilibrium of the clustered system is thus linked with the equilibrium of the un-clustered system. Element characteristics such as the elastic modulus, the cross-section area, the fabrication length and pre-stress are also linked to the un-clustered system using the clustering matrix $S$:

$$\vec{p} = Sp$$  \hspace{1cm} (9)

where $\vec{p}$ corresponds to a characteristic of the clustered system and $p$ is the same characteristic for the un-clustered system. The internal force in the $m^{th}$ element of the clustered structure at a time $t$ is given by:

$$\vec{f}_m = \frac{\bar{E}_m A_m}{l_{0,m}} \left( \vec{R}_m - \vec{l}_{0,m} \right) + \vec{f}_m^0$$  \hspace{1cm} (10)

where $\bar{E}_m$, $A_m$ and $\vec{f}_m^0$ are the elastic modulus, the cross-section area and initial pre-stress of the clustered member $m$. $l_{0,m}$ and $\vec{l}_m$ are the fabrication length and the current length of clustered member $m$. The internal forces of the un-clustered structure can be related to the clustered-element internal forces through:
\[ f_m^t = S^T \bar{f}_m \]  

(11)

where \( f_m^t \) is the internal force in the \( m^{th} \) element of the un-clustered structure at a time \( t \). Linking the equilibrium of the clustered system with the equilibrium of its corresponding un-clustered configuration allows dynamic relaxation to correctly model sliding or continuous tensile elements while maintaining its computational advantages.

3 FORM-FINDING AND ANALYSIS EXAMPLES

3.1 Dielectric-Elastomer Minimum-Energy Structures

Dynamic relaxation provides an inexpensive alternative for the simulation of dielectric-elastomer minimum-energy structures (DEMES). DEMES are electro-active bending-active structures composed of a prestressed dielectric elastomer membrane adhered to a thin flexible frame [16]. The strain energy of the prestressed membrane is transferred to the initially straight frame deforming the structure until equilibrium. The shape of the structure is controlled by prestress and reflects a minimum energy state in the structure [17]. DEMES have been proposed for shape-shifting applications in various disciplines such as robotics [16], bioengineering [17] and architecture [18]. However, predicting the behavior of DEMES remains a challenging task requiring complex analytical or numerical models.

In this paper, we analyze a DEMES with a rounded triangular shape (Figure 3) using dynamic relaxation and compare the form-found shape with the shape obtained with a physical model. The frame of the model has a length of 52mm and a width of 4mm. A similar structure was studied by O’Brien et al. [19]. The input for dynamic relaxation (nodal coordinates and connectivity) is a planar mesh of links connected with nodes based on the scheme of Figure 3. Nodes at the base of the system are pinned. Clustered elements with an axial stiffness of 0.08N/mm are used to model the membrane while bending elements with a bending stiffness of 7.8mm\(^2\) are employed for the frame.

Figure 3: Illustration of the elements in the numerical model in relation with the physical model.
DEMES equilibrium shapes are controlled by the prestress in the membrane. Therefore, clustered elements are given initial lengths providing the desired prestress. To initiate the simulation in the numerical model, the top of the mesh is given a small initial deformation. The prestress in the membrane induces the bending of the frame until an equilibrium shape is obtained. Figure 4 shows the equilibrium shape obtained with the dynamic relaxation DEMES model in relation with the shape of the physical model. The shape resulting from the application of prestress in the clustered elements of a flat structure is similar to the shape of the DEMES physical model.

The equilibrium shape of the numerical model and the physical model correspond to different prestress states. The membrane in the physical model is prestressed at 200% of its initial length, while in the numerical model clustered elements have a prestress of 150%. This discrepancy is most likely due to uncertainties in the numerical model [20] as well as due to the hysteretic DEMES behavior [21].

3.2 Stressed Spline Membranes

Stressed spline membrane structures are bending-active structures that combine spline elements with a prestressed membrane. Splines need to be sufficiently flexible to be curved into the required shape and to enough strength to resist the forces arising from bending and the loading combinations. Therefore, materials such as Fibre Reinforced Plastics (FRP) that have low Young’s modulus and high strength are favored for spline applications over traditional structural materials such as steel and timber.

In this paper, we analyze focus on a branched spline system in which the splines themselves provide bracing. The structure is based on three arcs of a circle joined to each other at one end at a central height of 3.5m using a branching splice joint while their other end lies on the circumference of a 8m radius circle, 120degrees away from each other (Figure 5). Between the arc ends on the circumference, boundary arches with a central height of 2.3m are inclined at 65degrees from the vertical plane and fixed. The geometry provides a double curvature in the prestressed membrane and therefore increased stiffness.
The apex joint is required to be stiff and flat to be a branched splice. Therefore, it is modeled using splice and virtual elements (Figure 6). Adjacent splines are connected through splices that run from the penultimate node of one spline via the central node to the penultimate node of the next spline. Virtual members are also added to the apex of the structure to keep it horizontal during the form-finding and the load analysis. The additional members link the penultimate nodes of the structural splines triangulating the apex into a rigid joint. In practice, the branched splice might be a stiff casting onto which the splines are slotted.

Splines are composed of tubular Glass Fibre Reinforced Plastic (GFRP) elements with a diameter of 120mm and a wall thickness of 5mm. GFRP tubes have a Young’s modulus of 40000MPa and admissible stresses of 100N/mm² for compression and 700N/mm² for tension as well as bending. The membrane is made out of PVC. It has a warp and weft stiffness of 1MN/m and an admissible strength of 12kN/m. Moreover, a prestress of 1kN/m is applied in the membrane in both directions. The design load cases considered include self-weight along with symmetric and asymmetric snow and wind loading. Four loading cases were analyzed: 1.
asymmetric wind loading and self-weight, 2. asymmetric snow loading and self-weight, 3. asymmetric snow and wind loading as well as self-weight, 4. uniform snow loading and self-weight. Stresses under these load cases remain below element strength (Table 2). Furthermore, although splines tend to straighten out under loading, deformations remain within acceptable levels.

### Table 1: Engineering materials and their properties

<table>
<thead>
<tr>
<th>Loading condition:</th>
<th>Max. stress in the spline [N/mm²]</th>
<th>Max. force in the membrane [kN]</th>
<th>Deflection at the center [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Asymmetric wind loading and self-weight: $p$ and -$4p$</td>
<td>376.20</td>
<td>8.53</td>
<td>81</td>
</tr>
<tr>
<td>2. Asymmetric snow loading and self-weight: $2.5p$ and $p$</td>
<td>394.30</td>
<td>6.79</td>
<td>277</td>
</tr>
<tr>
<td>3. Asymmetric snow and wind loading as well as self-weight: $2.5p$ and -$4p$</td>
<td>381.90</td>
<td>8.67</td>
<td>146</td>
</tr>
<tr>
<td>4. Uniform snow loading and self-weight: $2.5p$ and $2.5p$</td>
<td>443.8</td>
<td>9.51</td>
<td>444</td>
</tr>
</tbody>
</table>

### 4 CONCLUSION

This paper focuses on the form-finding and analysis of bending-active structure. With the incorporation of bending and clustered elements, dynamic relaxation is extended for the analysis of complex curved and bending-active structural systems. The formulation provides a fast and valuable tool to investigate structural behavior in the radial direction of in-plane bending. Two bending-active structures were investigated using spline and clustered elements: a basic DEMES and a stressed spline membrane. When compared with actual physical models, it was found that dynamic relaxation correctly predicts the equilibrium shapes of DEMES. In the stressed spline membrane, the pre-stress in the membrane acts as a continuous restraint for the splines allowing them to be very slender and therefore bent to a tighter radius. The examples show that the presented formulation has great potential for the modeling of bending-active elements that undergo large elastic deformations and opens the door to the development of a whole new realm of novel structural curved systems.

### REFERENCES


Restraining actively-bent structures by membranes

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Key words: active-bending, membrane restraining, non-linear, FEM, hybrid structure

Summary. Membranes can be used to restrain actively-bent structures. The paper explains the structural behaviour behind using membranes to couple actively-bent elements and analyses the influence of different parameters.

1 INTRODUCTION

In actively-bent elements the curved end-geometry is achieved by means of elastic bending. The shape depends on the boundary conditions (e.g. supports, length, cross-section and material properties of the bent element) and normally results in non-constant curvature and stress distributions. By changing the boundary conditions, different curvatures can be achieved. Based on this, initially identical elements can be transformed to achieve different curvatures. As the curvature is inversely proportional to the bending stress of the element, different curvatures lead to different residual stress distributions. In order to minimise the residual stress, the cross-section height of the bent elements needs to be small. However, small cross-sections lead to relatively small stiffness.

One option to create stiff structures with actively-bent elements is to restrain the actively-bent elements using a membrane. The membrane restrains the bent elements to one another. Membrane-restrained structures can be used as standalone systems or as components within arch-, column- or girder-structures.

Figure 1: membrane restrained structures [2], [3], [5]
2 COUPLING OF STRUCTURAL ELEMENTS WITH MEMBRANES

The basic idea of coupling structural elements by means of a membrane is based on a force transfer between the elements. The membrane-coupling results in a multi component cross-section. The membrane is directly involved in the load transfer to the supports and thus is an indispensable part of the structure.

The goal of coupling the structural elements is to transfer shear forces between the elements. The shear stiffness of textile membranes is very low and not sufficient to ensure a shear-resistant structural behaviour. Therefore the shear forces are transferred in another way. The transfer of shear forces can be described by strut and tie forces in a truss model. In a truss as in figure 2, left, the shear forces are transferred by diagonal ties and vertical struts between the upper- and bottom-chord. As membranes can only carry tension forces, the diagonal tension forces can be transferred within a membrane between upper- and bottom-chord. The vertical compression forces cannot be transferred within the membrane but by additional struts. The struts can be integrated into the membrane as sail battens for example.

![Figure 2: parts of truss-models for coupled elements with straight chords (left) or curved chords (middle, right)](image)

If the chords of the truss are curved and both ends supported in vertical and horizontal direction, they will transfer the loads as an arch- or cable structure. Elastically bent beams are generally supported horizontally, since the horizontal supports are needed to maintain the bent shape. Arch and cable structures are much stiffer to take vertical loads compared to beam structures. So the vertical compression component of the shear force in the truss model can be transferred by the arch- or cable elements without the need of additional struts. The diagonal tension forces of the truss model lead to compression forces in the arches and tension forces in the cable.

The ability for an arch or cable structure to carry vertical loads depends on its curvature. Figure 3 shows four arches with the same span but different arch rise and for comparison a linear beam of equal span is also shown. The arches are elastically bent elements and therefore exhibit curvatures consistent with the bending shape. Within this bending shape the curvature is not constant, but zero at the supports and largest at the centre of the span. The beams of all systems have the same rectangular hollow cross-section measuring 50 mm in width and 25 mm in height with a thickness of 1.5 mm. The E-Modulus of the beams is 20,000 N/mm². In load case 1 the load is constantly distributed along the entire span, in load case 2 only the left half of the span is loaded. The maximum deformation occurs in point A for load case 1 and point B for load case 2. The maximum deformation in point A does not occur in the middle of the arches, as the curvature in the middle is higher than near to the ends.
The load-deflection curves are based on the maximum vertical deformation and they show a logarithmic gradient. The gradient of the load-displacement curve correlates to the stiffness of the arch with respect to the maximum vertical deformation. At the load where the curve approaches a horizontal line the structure will snap through. The snap through of the structure causes high deformations, until the structure finds a new equilibrium geometry as a cable structure. This structural behaviour is not shown in the load-deflection curves. The load-deflection curve of the beam has in both load cases an exponential gradient. Unlike the actively bent beam arches, the beam does not snap through since its deformed geometry results in immediate tension force activation from both load cases. As deflections in the beam increase, the transfer of bending forces will decrease while the transfer of tension forces will increase.

![Figure 3: structural behaviour of a beam and different arch structures](image)

The stiffness of the arches increases logarithmically with respect to the arch rise for both load cases. The stiffness of the beam is significantly lower than the stiffness of the arches. This means that a minimum curvature is required to enable membrane coupling. The coupling effect will be more efficient if the curvature is higher.
The principle membrane forces in the membrane occur only in one direction and they are orientated diagonally like in the truss-model (figure 2). If the membrane is not stressed by an additional load (e.g. pre-stress), the uniaxial membrane forces will lead to wrinkles in the membrane. The efficiency of the coupling effect is not only depending on the stiffness of the arch, but also on the E-modulus of the membrane. Therefore even if the level of principle membrane forces in the membrane is low, choosing membranes with high stiffness is advised.

3 COUPLING OF TWO ELASTICALLY BENT BEAMS

The coupling-effect will be exemplified by means of numeric analysis. The following structure consists of two elastically bent beams, which are connected to each other at their ends. At these ends axial and shear forces can be transferred between the two beams. The ends of the beams are connected by a cable to introduce and maintain the pretension in the beams. The profiles selected for the beams are rectangular hollow profiles measuring 50 mm in width and 25 mm in height with a thickness of 1.5 mm. The E-Modulus of the beams is 20,000 N/mm². The two beams behave as upper and lower chords connected by a membrane. The membrane is fixed to the beams in tangential and radial direction and not pre-stressed. The membrane is modelled with two-dimensional, quadrangular elements, which can only sustain tension forces. The specific orthotropic material properties of the membrane are considered in the FE-elements according to Münsch-Reinhardt. The E-modulus of the membrane was chosen as 600 kN/m for both fibre directions and the shear modulus was taken as 40 kN/m. The membrane orientation is turned by 45° to the x-axis. The structure is analysed as two-dimensional system and as such buckling out-of-plane is neglected.

Two different load cases were simulated. The first load case consists of a continuous vertical line load along the bottom chord, the second load case consists of a continuous vertical line load on the left half of the bottom chord. The maximum displacement is in point A for load case 1 and in point B for load case 2. The load-deflection curves show for both load cases a linear gradient for the maximum vertical displacement. The deflections are higher in load case 2 than in load case 1. To determine the influence of the coupling effect, a second system was analysed. Here the membrane is replaced by vertical cables (cable restrained
system). The cross-section and E-Modulus of the cables is with respect to the stiffness equal to the E-Modulus of the membrane in warp- or weft-direction. In load case 1 the deflections of the cable restrained system are lower than the deflections of the membrane-restrained system. As the membrane is turned by 45°, the stiffness of the membrane in vertical direction is lower than the stiffness of the cables. Therefore more loads are transferred by the bottom chord and less by the upper chord. This leads to higher deflections of the bottom chord.

In load case 2 the deflections of the cable restrained system are much higher than the deflections of the membrane restrained system. The cables ensure that both chords transfer loads to the support, but since the forces cannot be transferred in diagonal direction a coupling effect is not possible. This shows that the coupling effect improves the structural behaviour significantly.

Figure 5: structural behaviour of membrane restrained system, cable restrained system and two-chord girder

The membrane and the cable restrained systems were also compared with a third system: a two chord girder (figure 5). Both chords are straight and supported in vertical and horizontal directions at both ends. If a shear force transfer by the membrane is assumed, the two pinned supports at each end will perform like a single clamped support for the girder. Additionally
there is a direct load transfer in the membrane: The membrane forces are orientated diagonally and run from the bottom chord to the upper supports. Due to these two effects near the supports, one might expect the two-chord girder to be stiffer than the other systems. However, the deformations of the two-chord girder are much higher in load case 1. In load case 2 the deformations of the two-chord girder and the cable-restrained system are nearly identically for deformations up to 20 mm. The load-deflection curves of the two-chord girder show an exponential gradient because the system’s stiffness increases when the chords act as cables by transferring tension forces. In the two-chord girder the membrane is not able to couple the chords like in the truss-model, as the stiffness of the chords for vertical loads is not high enough. For the coupling effect it is always necessary to ensure both a membrane for a diagonal load transfer as well as beam curvature for sufficient stiffness.

The principle membrane forces in load case 1 are orientated horizontally and vertically in the middle of the system and diagonally at the ends. The deformations of the chords are higher near the ends than in the middle, as the curvature is smaller near the ends. Near the ends the membrane couples the chords. The membrane is stressed in warp- and weft direction everywhere.

![Figure 6: principle membrane forces](image)

In load case 2 the principle membrane forces are orientated diagonally only. This correlates to the truss model (figure 2). The whole membrane is only stressed uniaxially, so wrinkles parallel to the membrane forces may occur.

The membrane restrained system was analysed for different heights (figure 7). The load deflection-curves show a linear gradient for all heights. In both load cases the deformations decrease with increasing height of the system. This occurs due to the increasing stiffness of the arch and cable structures, when the arch rise and the cable sag increases (chapter 2). The influence of the height to coupling effect can hardly be determined by these results.
Figure 7: structural behaviour of membrane restrained systems with different height

The influence of the coupling-effect can be determined by calculating the quotient of the deformations of the membrane restrained system by the deformations of the cable restrained system. In both systems the stiffness increases with the height of the systems. Whereas in the cable restrained system the stiffness between the chords in vertical direction is higher, the membrane offers diagonal force paths for a coupling effect.

In load case 1 the deformations of the membrane restrained system and the cable restrained system are quite similar. This means that the coupling effect is negligible for the structural behaviour. As the loads are almost affine to the geometry of the chords, they will be transferred to the supports predominantly by the chords as arch and cable structures.

In load case 2 the deformations of the membrane restrained system are significantly smaller than for the cable restrained system. For the systems with a height of 0.20 m, the deformations of the membrane restrained system lie between 70 and 80% of those for the cable restrained system. As the height increases, the difference in deformations between the two systems also increases. The deformations of the membrane restrained system with a height of 0.80 m represent only about 10% of the deformations of the cable restrained system.
with the same height.

Figure 8: comparison of membrane- and cable restrained system

The coupling effect increases with the curvature of the chords. Therefore the curvature plays a significant role in the structural behaviour of these membrane restrained systems.

4 CONCLUSION

The concept of membrane coupling is based on a shear force transfer between two edge-elements. Since the shear-stiffness of textile membrane is negligible, the shear force transfer is achieved by the formation of tension ties in the membrane and not by the shear stiffness of the membrane itself. The edge elements must be curved in order to provide sufficient stiffness to take vertical forces. When using a truss model analogy, the formation of tension ties in the membrane act as the diagonal tension members of a truss while the curved edge beams act as the vertical compression struts of a truss. Membrane coupling does not provide a completely shear resistant connection, but instead provides force paths which offer a limited shear load transfer between the chords. The shear transferring connection in combination with the upper and lower chords form a multi-part cross section which increases the stiffness and the load-bearing capacity significantly compared with an uncoupled system.

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ADAPTABLE LIGHTWEIGHT STRUCTURES TO MINIMISE MATERIAL USE

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Key words: Adaptive Structures, Structural Optimization, Flexibility, Dynamics, Actuators, Whole-life Energy Savings.

Summary. The optimal use of materials is highly necessary and a key issue for the near future. World’s current demand on resources and waste production has an enormous impact on the environment, which will even dramatically increase when future’s explosion on population takes place. The high contribution to this by the building industry gives it a big responsibility to reduce the concerning figures of material use.

This paper describes an approach in which structures are optimised with the use of adaptability. Structural optimisation is normally only done for static structures, while they are loaded in a dynamic way, meaning loadings which are changing in time, level and location. When structures are made adaptable to these dynamic environmental conditions the adaptability can significantly decrease the structural materials needed by increase of its efficiency.

Within this paper the structural adaptability is categorized in passive and active adaptability; passive adaptability using a higher deformation acceptance (flexibility) and active adaptability actively controlling the structure with actuators under different circumstances in a static or dynamic adaptive way.

The required higher deformation acceptance for passive adaptable optimisation, and the non-efficient influences of actuators for permanent loadings, like self-weight, does focus the research within the field of lightweight structures. Within the chair ISD of the Eindhoven Technical University ‘adaptable lightweight structures’ is one of the key research topics.

1 INTRODUCTION

The building industry must change. At the moment the building industry is responsible for more than half the mass waste production worldwide and uses a large percentage of the total used energy. Because of the increase of the number of people on this planet this problem of mass consumption will dramatically increase in the future. The population has grown in the last 80 years by a factor of 2,5 to about 7,5 billion people. Added to this the (additional)
growth of world’s population, the lifestyle of the current less developed countries will increase, claiming more space and luxurious materials. The impact of this increase of population and on the other hand the shortage of materials is critical to the environment and cannot become reality.

This means there is a big responsibility within the building industry to reduce these concerning figures of material use and mass waste. The current status however is, that in general buildings are build unsustainable and inefficient. Stiff and static objects are built with a lot of materials while the external environment acting on it is highly dynamic and changes in time and loading type, -level and -location. When structures are made adaptable to these variable environmental conditions the adaptability can significantly decrease the structural materials needed by increase of its efficiency.

The generally used optimisation of static structures is therefore extended by passive adaptive and active adaptive optimisation. Structures with higher deformation acceptance can react in a passive adaptable way which can reduce the load impact and the occurrence of internal forces. With active adaptability the structural behaviour can be specifically controlled and further optimised. Combining the passive and active adaptability can result in an even further structural optimisation allowing the structure to deform in a controlled way as a reaction to avoid or absorb the impact of the varying external environment.

Figure 1: reduction in material use in an arch by passive adaptive, active adaptive and passive + active adaptive behaviour
2 PASSIVE ADAPTIVE STRUCTURES

Structural deformation due to passive adaptable behaviour is often seen as undesired behaviour and in practice limited by building codes. But nature educates us about the advantages of flexibility. Loads are varying constantly. The wind’s velocity, its direction and gust location changes and turbulence occurs, but even then organisms, such as plants, survive with minimum of damage despite their lightness. Important for their capability to survive is their flexibility and compliance.

But copying nature is not possible and not correct; there are other demands amongst deflections, isolation, cost, safety and many more. But by abstracting the main principles of the use of flexibility will result in better, lighter and more efficient design solutions.

By using this compliance passive adaptable structures can avoid the impact of the governing loading by adapting to it. Reduction of loaded surface, a better form factor and less friction are part of this principle. The so called Fluent Structural Interaction (FSI) plays an important role here.

Passive adaptability can also improve the efficiency of the material by changing its geometry and therefore its stiffness and/or the internal load-path (i.e. increase in curvature of a loaded membrane).

Allowing larger deformations in a structure will influence the dynamic behaviour. It can introduce dynamic absorption of forces and thus reducing peak loadings in time. For instance, this is the basis for the design of a bomb-blast resisting façade. But depending on the load spectrum and the vulnerability to excitation of the structural system, the dynamic behaviour can also increase the internal forces by resonance. For this reason active adaptability is researched not only as a static active adaptation, but also as a dynamic active adaptation to be able to control the dynamic behaviour (chapter 4 and 5).

Roofs and (second) skin facades can play an important role in the study of flexibility within structures. The environmental loadings will act on this skin. When the skin will be designed in a way that larger deformation acceptance can be used for passive adaptable structural optimization the skin can improve the load absorption and transformation through the building to the foundation.

A brief study of the structural behaviour of the static and passive adaptable arch in figure 1 shows that the static tube arch of 2m in diameter spanning 67m with a height of 33m and a deformation limitation of h/200 can be reduced in weight by about 50% when the steel stress of 355 N/mm² is the limiting factor and not deformation.
3 STATIC ACTIVE ADAPTIVE STRUCTURES

In contrast with passive adaptability, active adaptability needs actuators and therefore input of energy to influence the structural behaviour. When using structural active adaptability, structures will mostly behave in a conventional passive way up to a certain level of loading or deformation. Above this threshold the structure will be active controlled by actuators, so called ‘active load-bearing capacity’.

These high loadings, which are often the dominating design parameter, occur very rare, and cause the structure to be overdesigned for most of its working life. When these high loadings above the threshold occur these actuators modify the pattern of internal forces – ‘load path management’ - to optimize the internal load distribution towards a more homogenous stress pattern and/or reduce the governing deformation. This results in an active controlled design with minimised structural elements without losing structural capacity in strength or stiffness.

The actuators can influence different variables within a structure. They can introduce counter forces, change geometry or adjust the section or even material properties. The aim is to maximise the structural improvement while minimising the effort of required actuation. Studying variations within the different active adaptable variables will unveil their impact towards an increase of structural efficiency and their optimum under different circumstances.

Considering the active adaptable arch in figure 1, the active rotation of the supports can reduce about 50% of the steel weight within the same limitation in stress as well in deformation as the static arch.

3.1 Load path management

The Load Path Management procedure as proposed by Teuffel\(^1\) consists of three principal steps. First, the load path is optimized while ignoring the geometrical compatibility equations. During this step the axial forces in the elements as well as their section areas are optimised for minimal volume. However, the optimised distribution of forces and section areas is non-compatible due to neglecting compatibility. The constraint forces that arise in the structure by applying the constraints and external loadings must be compensated by the actuators to induce the optimal load path. These constraint forces are calculated by performing a static analysis for all load cases on the optimised structure using the minimised cross sectional areas. The difference between the incompatible optimal load path and the constraint forces must be accounted for by the actuators (eq. 1).

\[
\Delta N = N_{\text{opt}} - N_{\text{c}}
\]

Figure 3: Four bar structure with optimal load path and constraint forces for the first load case.
As example a four bar structure\(^1\) is taken (fig. 3). The first load case consists of a horizontal nodal force and the second load case consists of a vertical nodal force.

The minimum number of actuators needed to induce the axial forces \(\Delta N\) in general is equal to the degree of indeterminacy of the structure. The optimal locations of the actuators are determined with the use of a sensitivity matrix \(S_n\) (eq. 2). Each element is subjected to a unity length change and the internal forces that arise in all elements due to the unity length change in the considered element are calculated. The vector \(e_i\) describes the efficiency of an element per load case (eq. 3), and \(E_i\) defines the efficiency of the elements considering all load cases at once (eq. 4).

\[
S = \begin{bmatrix}
\frac{\partial N_1}{\partial l_1} & \cdots & \cdots \\
\vdots & \ddots & \vdots \\
\vdots & \cdots & \frac{\partial N_3}{\partial l_3}
\end{bmatrix}
\]

\[
e_i = \sum_{i=1}^{n_{LC}} \left( S \cdot \Delta l_i \right)\frac{\Delta N_i}{\Delta N}
\]

\[
E_i = \sum_{i=1}^{n_{LC}} (e_i)
\]

During the adaptation process, the length changes of the actuators are determined while taking the compatibility equations into account. The length changes needed to control the load path and to create the optimal force distribution in structure follow from:

\[
S^a \cdot \Delta l_{act} = \Delta N
\]

The adaptive four bar structure with the actuators marked in red is shown in figure 4.

In figure 5 the deformations and stresses due to external loading (load case 1) and the internal actuation are shown as well as the final optimal force distribution. The stresses in the adaptive structure in the passive state subjected to the external loading are higher than the allowable stress. However, by using the actuators the load path can be altered resulting in an optimal force distribution where all elements are 100% utilized in all load cases.
Figure 5: Static results of the adaptive four bar structure for the first load case

The same goes for the adaptive structure subject to the second load case.

Figure 6: Static results of the adaptive four bar structure for the second load case

4 DYNAMIC ACTIVE ADAPTIVE STRUCTURES

When reducing structural material by optimisation its vulnerability to dynamic excitation can increase. This can limit the desired level of structural optimisation or will demand additional dynamic control systems.

For this reason, the same actuators used for static adaptive optimisation are used to control the dynamic excitation by superimposing additional small length changes based on the nodal velocities. This requires the actuators to be designed in such a way that they can sustain high static loadings while at the same time producing smaller but highly dynamic loadings. So by using the actuators not only for internal force control but also for control of deformation and dynamic excitation, the structural optimization can be highly efficient and further extended towards all boundaries set for strength, deformation and acceleration. Active adaptability will control the internal load path, deformation and dynamic excitation, while passive adaptability can increase the structural efficiency by increasing the threshold level of active interference.

5 CASE STUDIES OF DYNAMIC ACTIVE ADAPTIVE STRUCTURES

To accomplish a better understanding on how the dynamic behaviour can be actively controlled by actuators, two case studies are presented. The first study presented is a dynamic analysis on the adaptive four bar structure shown before which is followed by a dynamic study on a trussed arch structure.
5.1 Case study 1: Dynamic analysis of the four bar structure

For comparison a static and dynamic active adaptable four bar structure is analysed. They are subjected to harmonic resonance loadings corresponding to the two static load cases used before (horizontal and vertical nodal loading).

First the natural frequencies and the corresponding mode shapes are determined by solving the eigenvalue problem (eq. 6). Where \( M \) is the mass matrix, \( K \) is the stiffness matrix and \( \omega_n \) is a vector containing the natural frequencies of the structure. The natural frequencies and corresponding mode shapes of the four bar structure are shown in figure 7.

\[
-M \ddot{x} + K x = 0
\]  

(6)

During the dynamic analysis energy dissipation due to various mechanisms, such as friction and material damping, must be taken into account. Therefore, a damping matrix is defined by performing a modal analysis. Modal analysis is a method to rewrite the mass and stiffness matrix in a new modal basis by using the orthogonality conditions so that the system of equations is decoupled.

Once the equations are decoupled the modal damping parameters can be defined using the modal mass and stiffness parameters (eq. 7). Where \( m_j \) is the modal mass parameter, \( \omega_j \) is the corresponding natural frequency and \( \zeta_j \) is the desired damping ratio. In this case a damping ratio of 2% is used considering that both the passive and adaptive structures are made from steel. Rewriting in the real basis again the matrix becomes full resulting in the damping matrix \( C \) which is used during the dynamic analysis.

\[
c_j = 2m_j \omega_j \zeta_j
\]  

(7)

5.1.1 Dynamic analysis of the four bar structure with static active adaptability

First the static active adaptive four bar structure is loaded with a horizontal harmonic loading while imposing the desired length changes of the actuators for the static load case. The excitation frequency of the harmonic loading is equal to the first natural frequency of the structure corresponding to the first mode shape where the structure is vibrating horizontally. The magnitude of the horizontal harmonic nodal loading ranges between 90% and 100% of the static horizontal loading.

In figure 8 the horizontal displacement of the upper node and the stresses in the four elements are plotted against time. Initially, the amplitude of the horizontal displacement
increases very fast but after a couple of seconds a dynamic equilibrium establishes due to the applied 2% damping. Clearly, the weakly damped structure is resonating. Also the amplitudes of the stresses increase very fast initially until equilibrium establishes. The stresses increase up to 700 N/mm\(^2\) which is much higher than the allowable stress of 300 N/mm\(^2\).

5.1.2 Dynamic analysis of the four bar structure with dynamic active adaptability

Again, a dynamic analysis is performed on the adaptive four bar structure subject to a horizontal harmonic resonance loading, but this time the actuators are used to control the structure dynamically. The desired length changes of the actuators for the static horizontal load case are imposed and 2% damping is applied. The dynamic control is introduced at each time interval by superimposed additional length changes based on the current relative velocity of the start and end nodes connected to the actuators.

In figure 9 the horizontal displacement of the upper node and the stresses in the four elements are plotted against time. Now, there is no increase in the amplitude of the displacement nor in the amplitudes of the stresses. Immediately a dynamic equilibrium is established and no resonance-effects occur. The dynamic active adaptive structure behaves more like a critically- or over-damped structure.
5.2 Case study 2: Dynamic analysis of a trussed arch structure

A more complex trussed arch structure is studied (fig. 10). The trussed arch is 8 meters high and spans 40 meter with a structural height of 1 meter. The considered load cases are dead load combined with a static symmetric or asymmetric snow load or a harmonic wind load.

5.2.1 Load path management trussed arch

The adaptive trussed arch structure is designed according to the Load Path Management procedure outlined in 3.1. In this case 13 actuator locations are selected and the desired static length changes of the actuators for each load case are calculated.

![Figure 10: The adaptive trussed arch structure with 13 actuators](image)

5.2.2 Static analysis of the trussed arch with static active adaptability

A static analysis is performed on the adaptive trussed arch structure subject to the static load combination of dead load with wind, while imposing the corresponding length changes of the actuators. The results of the static analysis are shown in figure 11.

![Figure 11: Static stress results of the adaptive trussed arch structure under dead- and wind loading](image)

From figure 11 it can be concluded that the adaptive trussed arch structure is not capable of fully utilizing all cross-sectional areas as not all elements are fully stressed. Unlike the adaptive four bar structure the adaptive arch is unable to fully exploit all elements, because the trussed arch structure does not satisfy equation 8:

\[
  n_{el} - n_{bc} \cdot (n_{Dof} - n_{bc}) \geq 0
\]

where \( n_{el} \) is the number of elements, \( n_{bc} \) the number of load cases \( n_{dof} \) the total number of dof and \( n_{bc} \) the number of constrained dof.

\[8\]
5.2.3 Dynamic analysis of the trussed arch with static active adaptability

Following the same procedure as before, the natural frequencies of the adaptive trussed arch structure have been found by solving the eigenvalue problem (eq. 6). In this case there are 36 eigenmodes as there are 36 unconstrained degrees of freedom. In figure 12 the first and second mode shapes of the adaptive trussed arch structure and their corresponding natural frequency are given.

A dynamic analysis is performed while applying 2% damping. First, the static adaptive structure is subjected to a harmonic resonance wind loading while imposing the desired length changes of the actuators for the static load case. The excitation frequency of the harmonic wind loading is equal to the first natural frequency of the structure where the arch structure is swaying horizontally. The magnitude of the harmonic wind loading ranges between 90% and 100% of the static horizontal wind loading.

In figure 13 the horizontal displacement of the unconstrained nodes and the stresses in the elements are plotted against time. The amplitudes of the displacements and stresses increase very fast due to resonance. After 120 seconds equilibrium establishes due to the applied material damping limiting stresses to 719 N/mm².

Figure 12: First and second natural frequencies and mode shapes of the adaptive trussed arch structure

Figure 13: Results of the static active adaptive arch structure subject to a harmonic resonance wind loading
5.2.4 Dynamic analysis of the trussed arch with dynamic active adaptability

The dynamic analysis of the trussed arch has been performed again under the same conditions, except this time not only using the actuators in a static way to alter the load path, but also in a dynamic way to control the dynamic excitation of the adaptive arch structure. Though, controlling the relative nodal velocities - in which the velocity of the start node relative to the end node of the actuated element is controlled - is inefficient in this case. In case of the four bar structure the absolute velocities of the unconstrained degrees of freedom are controlled indirectly because one node of the element is supported, but this is not the case for the trussed arch structure. Therefore the actuators are now used to control the absolute nodal velocities instead of the relative nodal velocities.

The dynamic analysis on the trussed arch structure is performed under the same conditions, in which all unconstrained degrees of freedom are controlled by superimposing additional length changes based on the absolute nodal velocities. Considering the results in figure 14 one can see that resonance effects have been reduced. This time the displacements are smaller and the stresses are limited to 403 N/mm². Using the actuators to control the unconstrained degrees of freedom seems effective.

![Figure 14: Results of the dynamic active adaptive arch structure subject to a harmonic resonance wind loading (controlling all degrees of freedom)](image)

The control system needs time to collect and process the gathered information, to determine the response and to activate the actuators. This will introduce a delay in the response. A first preliminary study in which a short delay of 250 ms is taken into account, shows similar results as in figure 14 for the active adaptive structure with no time delay. But the delay time relates to the equipment used and the impact of the delay will relate to the structural behaviour of the system, to the dynamic loading acting on it and to the excitation frequency.
6 CONCLUSION

Adaptable structures can optimize the structural behaviour by adjusting to varying environmental conditions. This direct relation between loading and structural performance shows strong possibilities towards structural material savings.

Active adaptability is presented for static and dynamic load cases. The static Load Path Management shows strong opportunities towards a more homogenous distribution of internal forces reducing the often governing peak loading in all elements. Depending on the goal of the activation, also the deformations can be controlled. The reduction of the used structural material by using passive and static active adaptability can result in structures more vulnerable to dynamic excitation. By not only using the actuators in a static way but also in a dynamic way to control the dynamic behaviour of the structure, the structural optimization can be highly efficient and further extended towards all boundaries set for strength, deformation and acceleration.

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RESPONSIVE TEXTILES

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SUMMARY

The goal of this research is to investigate kinematic textile membrane strips configurations that serve as responsive second building skins. The movement of these adaptable material systems will be defined by a dynamic response to the environment, balanced with the direct response to the user’s influence on the façade. With use of the integration of sensors and actuators, as well as computation and material responsive enhancers, the end result of the research is aimed to be a self-actuating system that interacts with its surroundings and that is characterized by its behavioral adaptability.

Therefore the user does not control the technology, but represents a change in the containing environment, and form is therefore not afflicted upon the material from the outside, but it represents, together with its behavior over time, an emerging phenomenon arising through the programming of a reactive environment.

The focus falls on tensile architecture in motion, exploring basic responsive capabilities to address local environmental conditions. The goal will be the abilities of the system, and not its image, which represents a shift in perspective: formal becomes behavioral and motion takes over image, as self-actuation means property change and its entails the ability of the material to convert energy to motion. This foreseeable scope will translate into a system that includes the inherent instability given by the multitude of parameters able to transform the morphology of the form, remaining open and indeterminate. The system will perform a continual adaptation and self-organization with a dynamic and changing context.

1. INTRODUCTION

1.1. Research goal

While documenting the possibility of adaptive reactive geometries, the formulation of the assignment and the research expectations is possible:

As part of the research of adaptive multifunctional textile facade systems, how can textile envelopes be configured as responsive user-interaction oriented arrangements? How should the façade respond to user control, what should be the balance between emergent behavior and direct response to user desires? What are the parameters of the façade behavior?
Second skin envelopes are used more and more in recent projects, and commonly serve singular purposes, such as shading. In more recent approaches, facades are also thought of as responsive systems, that adapt to user requirements and that are directly controlled, through sensors and actuators, by the users in order to achieve specific states that respond to diverse needs. (Privacy, shading, rain protection, wind protection) In most cases, these needs are determined after the designers analyze the building context, the natural environment and the user expectations. This set of requirements is fixed and known throughout the design process and is considered not to change during the life span of the building. The only changing parameters are the natural environmental ones, however this change is expected and the results in the change of the facade are foreseeable.

Therefore the question has to be formulated: could a facade system be designed, that responds to a set of predefined requirements but also has the ability to take in new input and adapt to a changing context? Could such a facade respect the typical boundary conditions (busy urban areas and their traffic and accessibility consequences, big city user density, small
built areas, versatility of space usage requirements) and respond to the usual needs of a building (fire resistance, wind resistance, shading necessity, visibility rules, etc.)?

As represented in Fig. 2, the investigation can start by primarily dividing the inquiry into the following topics: user movement and interaction, façade element configuration, behavior of the façade in different weather conditions and effects on the building.

1.2. Search space delimitation: boundary conditions and building typologies

What are the usual building typologies that come into question for such projects? Here a few diagrams that represent some common profiles of buildings in large cities that might benefit from a multifunctional adaptive facade system:

1. Retracted upper stories (for the creation of a terrace)
2. Basic cross section
3. (Double) curved geometry
4. Interior Courtyard type
5. Retracted lower stories (for the creation of a visible access area)
1.3. Realistic approach:

The project strives to apply and integrate a textile façade system to a residential building, with the goal of creating shading that allows user interaction and control, while at the same time obeying rules implied by visibility needs, fire resistance, wind resistance etc.

2. ABSTRACTION

The issue of an adaptable multifunctional textile envelope that should serve as more than a shading system is raised.

With the purpose of search space delimitation, this work will only investigate textile façade systems with a strip-like configuration. The length of the elements must suffice to apply to two stories and the connection to the roof and console must not represent an impediment to the elastic system kinetics, these self-imposed boundary conditions are to be seen as conventionally set limitations.

3. INVESTIGATION PROCESS

Seen analytically, such façade strips might have the configuration seen in Fig. 5 that was used as a guideline for the model building phase. The sample configuration has vertical, horizontal and diagonal elements with semi-rigid joints. For some of the actuation options, some of these elements were not built, a total of 10 different configuration variations being used for the first model phase, as shown in Fig. 7. and exemplified in Fig. 6. The connections to the roof or handrail were not yet defined, as the goal of this phase was to generally determine the actuation mode with the greatest kinematic amplification.
3.1. Actuation possibilities:
As initiation of the investigation process, an option list is developed, that categorizes the movement types possible: As a main diagram, a separation is made between reactions, or effects of the movement, and actuation, or cause of movement, this being the basic structure by which system options could be differentiated.

Actuation can be external or internal: external actuation could be separated into translation and rotation and internal actuation can be divided in contract, expand and bend.

3.1.1 External actuation:
As a convention, external translation actuation is defined as movement of either a single point, a linear element or possible combinations of these moving elements (such as a point and a linear element, two points, two linear elements and so on). The movement takes place in the initial plane of the element, prior to the movement.

- Movement of one point
- Movement of the horizontal frame element
- Movement of the side bar
- Movement of two linear elements.
- Combination of a point and linear element movement

External rotation actuation refers to the circular movement of one linear element or several, outside of the initial element plane.

- Rotation of the side bar
- Rotation of the horizontal frame element

External actuation is applied on the outer frame of the textile strip.

3.1.2 Internal actuation:
Internal actuation is structured into contract, expand and bend.

Contract: The first type of internal actuation can be applied to either a diagonal line in the configuration, a vertical or a horizontal one. The components do not have to start or end on the external frame of the textile strip, and do not have to actually consist of material elements; the movement refers to a line included in the configuration. The actuation translates into diminishing the length of the specific line, by bringing the two limit points closer together.

- Horizontal line
- Vertical line
- Diagonal line

Expand: This second type of internal actuation is the inversion of the contraction movement and can be applied to the same components: vertical, horizontal or diagonal. The limit points are taken farther apart.

Bend: The third internal actuation option is “bend”, which refers to curving a material element in the configuration, be it vertical, horizontal, or diagonal.

- Horizontal element
- Vertical element
- Diagonal element
The investigation process will start with the analysis of examples of each actuation possibility (model building, pictures, 3d models) and the classification of the effects in the geometry.

3.2. Reaction options

Reactions can be either global or local, as follows: global translation, global rotation or local contraction, local expansion, local bending.

After the development of the main analysis procedure, the examples will be compared with the secondary standards and further structured. An important aspect of this step is checking the options against the above described boundary conditions and establishing their fitness in reference to these standards. The goal of this continued classification is the expansion of the geometry option list.

3.3. Association options: physical models, examples, 3d models

3.3.1. Physical models:

The investigated geometries differ from the point of view of the actuation type (and therefore also from the point of view of actuation effects on the outlook), but for the sake of the extensive coverage of the search space, the differentiation will go on to a more detailed level. For each actuation mode possible, a series of 10 possible structural configurations and for each of those 2 possible membrane layouts are built and analyzed (the type of membrane used also differs from the point of view of its elongation tolerance, some being built of plain weave glass fiber mesh, other of elastic textile material). Each of the 16 built models is then actuated. The models are built in scale 1:10, having a total length of approximately 60cm. The actuation is schematically simulated by either introducing elastic elements that pull joints together, or rigid ones that push farther apart (Fig. 6 and Fig. 7)

The geometries are however only investigated for local actuation modes, as the research is intended as an alternative to a global actuation concept. This work also intends to show the diversity allowed by a system not through the diversity of the actuator types or their complexity but through its intrinsic behavior (determined by the interaction between membrane, outline structure, bending-active elements, the proportions of the stripes and the frame of the built environment on which the stripes are attached).

The local actuation modes investigated are contract, bend and expand. The effects contraction and bending have on the geometry outlook are comparable; therefore the more easily buildable option is preferred, which would then be contraction. (A tension and release system is more energy efficient than a flection or expansion one). The expand motion, although possible conceptually is limitedly viable for the fore defined configurations, as it would require an extremely flexible membrane and large actuators. This possibility has however been tested on the configurations with pre-stressed membrane and the conclusions are, predictably, that the amount of force needed to deform the geometry in such a way makes this actuation mode undesirable from the point of view of actuator complexity.

The investigation will therefore be focused on local actuation, and specifically on the contraction option. The 16 built configurations are subjected to contraction of horizontal elements, vertical ones and diagonal components. The first step is to investigate the effect of single element deformations (each of the 3 afore mentioned possibilities), in different relative
positions within the stripe. Combinations of 2 elements follow, either both elements actuated simultaneously or gradually. The last step is combinations of 3 elements, with simultaneously applied deformations or layered phased ones. The results are to be seen in the diagrams and pictures of the models.

The goal is finding the options with the highest degree of kinematical amplification but with the lowest degree of actuator complexity and diversity. Therefore the diagonal contractions, applied in layered phases to configurations with pre-stressed membrane are to be investigated further, first digitally and then with the help of mock-ups. Fig. 7 exemplifies some of the plain weave glass fiber mesh models, locally actuated and linearly and rigidly connected to the roof edge, in front view. The lower connection allows freedom on the OY and OZ axis and permits the rotation of the strip.

3.3.2. Digital models: Digital representation of deformed geometries

Results acquired with the help of the physical models (scale 1:10) are used to guide the modeling of the digital simulations and to identify the parameters that influence the relaxation. The stiffness of the structural elements is one of the main factors responsible for the degree of shape adaption. The membrane pre-stress factors as well as the way it is fixed to the structure are equally important parameters.

The following computational tools were used to simulate and parametrically control the behavior of the system:
Grasshopper: In order to simulate the GFK elements, their stiffness and their bending behavior, a convention is made to represent them as trusses. The grasshopper configuration is fed with two curves, that is then transforms to polylines. The number of control points (also division points) is set with the help of two number sliders: one that represents the number of transversal elements between the 2 curves and the other that represents the number of division points between the transversal elements (the higher the number, the closer the polyline follows the initial curve curvature). The trusses are then built, with as many frames as total number of division points (transversal number * number of division in between).

Kangaroo: The Kangaroo components set transforms the lines (members of the truss) into springs, appoints them stiffness and damping values and uses the anchor points to fixate the general geometry. Forces are either defined as vectors, such as gravity or as attraction stress between two points on the different longitudinal elements. Bending of the linear elements can additionally be simulated by a vector based method that computes the shear forces at a node into bending moments. Diagonal elements are created and enumerated, as to have the possibility to create the movement by determining the numbers of the diagonals to be actuated.

In order to check the validity of the resulting deformed geometries, they are also built by bending the elements that represent the GRFP longitudinal beams. Another way used to confirm the validity of the geometries is building models in 1:10 scale and mock-ups 1:5. Fig. 7 first exemplifies the configuration and applied actuation, then the digital results and then the 1:5 Mock-up results. The strips are connected linearly both to the handrail and to the roof edge. The upper connection is to a rail that can be driven forward or backward, as seen in Fig. 8. The bending-active GFRP elements that constitute the frames of the strips, as well as the transversal elements, both insure that the membrane is only slightly wrinkled but not folded and also that the strips spring into the initial shape after the actuation forces are released.

Fig. 8 also demonstrates the two extreme states of the system, i.e. the neutral (right side of the picture) and the storm state (left side of the picture)
Figure 7: Comparison: digital and physical models 1:5

Figure 8: Mock-up side views
3.4 Computational investigation:

After determining the parameters and limitations that define the geometry (given either by the actual physical constrains of the system, or by the material specific properties) and determining a set of general rules of response to a basic set of natural environmental changes (sunlight, wind, rain) and the consequences of these rules on the system, the computational process begins. This long list of parameters and general geometric rules allows for a multitude of associations, which translates in a diverse set of system configurations. A library made up of all possible deformed geometries is created.

Instead of deciding on an image, and then controlling the appropriate parameters in order to achieve that image, the system is only given general impulses in specific locations. These impulses are given through data sent to the actuators by either one “brain” or by several centers of computation.

The entire algorithm was programmed in Rhinoscript, with additional Grasshopper and C# definitions. The information exchange between the main algorithm and Ecotect (used to evaluate weather conditions effects on the geometry) was also automatic.

The weather information is a critical part of the façade development process. All the geometries in the created library are evaluated for the current weather conditions. These are fed into the algorithm as criteria after being extracted, with the help of Grasshopper, Geco and Ecotect from a weather file. Weather information, such as wind speed, wind direction, solar irradiation, humidity levels and temperature are compared to predetermined values that represent average, highs and lows for the place of construction of the façade and the specific time of year. According to their values and position in a predetermined order of importance, each weather criterion is assigned an importance factor or a weight. Each geometry is then evaluated for each weather criterion (for some criteria the evaluation function is an overly simplified version, to be used as a place holder), and the separate grades are factored by the respective criterion’s weight in order to get the final grade for each geometry, for the current weather conditions.

With reference to the wind analysis, a conventionally simplified version is simulated. Geometries are basically evaluated by the area of the surface normal to the wind vector, the bigger this area, the higher the resistance to wind force, factoring in the main wind vector direction and the wind speed that determine the importance that the results of this simplified analysis will have. The higher the wind speed, the more essential the wind factor become (very high speeds mean storms) and so the higher the weight of the results of the evaluation, this being the principle used for the scaling of the weight of each factor (wind speed, wind direction, solar irradiation, humidity levels and temperature).

Analysis of the temperature and humidity factors shows if there is snow, in which case the geometries with the least horizontal surface (on which the snow would gather) are evaluated the highest. This simplified perspective is developed in order to allow the uninterrupted development of the method, an actual snow and wind load static evaluation to be written upon further study and with the help of specialist. In case there is rain, the options

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1 Grasshopper is an algorithm editor for Rhino 3D
2 Geco is a set of components which connects Rhino/Grasshopper and Autodesk Ecotect
3 Autodesk Ecotect Analysis is a program that performs simulation and building energy analysis
with the largest horizontal surfaces (that would offer shelter) are rated highest. The result is a hierarchic list of all geometries, starting with the fittest one and ending with the one that would behave worst in the current conditions. An initial façade is created, one composed of stripes that find themselves actuated in the form of the fittest geometry.

The user additionally has the ability to interact with the façade and select a desired effect: light/shadow and size of the desired effect: local/entire room/multiple rooms. This selection is then represented as a surface projection. If the desired effect is light, then the geometry that leaves the least shadow on this surface projection is searched and if the effect is shadow, then the geometry that leaves the largest shadow on the surface is to be found. The connection with Ecotect is again created, in order to calculate the shadows of all possible deformed geometries in the current weather conditions and their intersections with the effect surface projection. The order in which the geometries are investigated is the one resulted from the evaluation to the weather conditions, therefore the end result is a façade made up of stripes deformed to states that best behave in the current weather conditions and of stripes that are as close as possible to that goal and at the same time respect the user’s choice of façade effects. Material behavior will also be periodically analyzed and sent as input data to the computational brain. Static analysis results could also be performed through strategically placed sensors and be used as input information. The system therefore interacts with an algorithm that learns the users’ preferences, material behavior and attrition and that continuously adapts and responds to changes and updates.

3.5. Computational brain

The responsive façade interacts as shown above with the environment and the user simultaneously, process that allows the gathering of data about the resulted geometry outlooks. The weather situations are stored and averaged and the user preferences are also remembered and developed into a neuron that allows the façade system to foresee the desired effects in weather conditions met before. During a certain time interval, for example one month of façade functionality, the algorithm keeps gathering information and therefore completes itself, knowing what the user has been desiring for the mentioned time interval. The initial façade created hence includes not only the fittest option in reference to the current weather conditions, but also a suggestion that the algorithm makes after evaluating what the user had desired in similar situations. This learning phase of the algorithm is permanent; the
data base that is used as statistical source keeps growing during the entire life span of the façade.

4. CONCLUSIONS

This study focuses on the vast number of possibilities that a seemingly basic façade geometry offers if its transformation is specific to the employed materials. The kinetic amplification is possible due to the successful collaboration of the membrane material and the fibre reinforced composite bending-active elements, connected with semi-rigid joints.

The present work is to be seen as a conceptual method investigation, in which more detailed functions are to be integrated. The method created represents a structured framework that insures the easy implementation of specialists (electrical engineers, construction engineers, etc.) written functions in the right positions. Therefore the parts of the algorithm used to evaluate snow and wind loads as well as material attrition are to be understood as place holders, overly simplified variations that allow the method to be understood as a whole. Further and more detailed investigations are to be made upon the static characteristics and limitations of the structure, the goal of the present work cornering elastic kinetics being the discovery of the intrinsic transformation capabilities of a constructive ensemble and investigate them analytically.

The algorithm and method created are innovative in terms of extending the adaptability and flexibility of the user-façade interaction. One does not overpower the other; the surrounding environment is never compromised for the sake of user control. All factors with which the façade comes in contact influence it to appropriate degrees and it permanently learns what the user encourages and what façade outlooks are fittest in different scenarios.

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The "Magical" Lleida Technological Park: as an example of environmental building rehabilitation strategy

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ABSTRACT

Keywords: Textile roof, Structural membrane, Industrial, Flexible, Demountable, Sustainable, Rehabilitation

In the present demand-driven market, when time plays a key role, the adopting of a flexible strategy easy to implement may be the answer to create an architecture that meets all requirements. The adaptation with membranes in a rehabilitation, in this case, is an economic way in which this complex was rethought for the new requirements. The membrane contributes towards the stability of the structure, that is compose out of frames, that bear the load of the membrane material, fixed between the buildings, thus creating new spaces and new possibilities. By introducing this new element with its properties, we do not only reshape the complex in a functional way, but we also make it sustainable in connection with the past and the present, creating a new way for the future, in this demanding ever changing market. Therefore the textile structure that covers the access area from the "Magical" Lleida Technological Park is driving this process forward with its persistent commitment to researching and developing new practical design concepts, functions and construction solutions methods. This new approach has a better environmental behavior based on its efficient use of resources, and its easy maintenance and renewal, thus moving it ahead from other standard solutions, making it a so-called pioneer of its generation.

The aim of this paper is: a) to show that there are other methods apart from the traditional ones, for a construction and in some cases more suitable; b) to describe the main characteristics of the "Magical" Lleida Technological Park, with emphasis on the closing textile structure; c) to explain the closings capacity of sustaining itself by responding to the environmental conditions.

The strengths of our proposal for this rehabilitation are related to the use of a textile structure, both in the creation of the facade and in the generation of the closing for the access. Key issues in this process are associated with:

1. The superior quality of form: It should be mentioned that for us the concept of "quality", can't be defined as just one attitude towards architecture, a style or an operating mode over the surroundings, but rather as a way of thinking and a correct approach towards the existing. 2. Adaptability. 3. Simplicity. 4. The quality of the space, generous for human activities. 5. Energy conservation and efficient use of resources. 6. Weight. 7. Durability
1. SUSTAINABILITY CRITERIA AND ENERGY EFFICIENCY AS AN AIM

The strategy set out in the project design, includes a methodology for applying the sustainability aspect, within technical and economic limits, because buildings in general are the largest consumers of energy worldwide and reduce environmental pollution. Consequently, this strategy seeks to optimize the passive behavior of the building, by reducing the environmental impact and energy demand but also look for more efficient systems, supported by renewable energy sources to suit the remaining demand. Sustainable construction aspects proposed are not meant, only to reduce CO2 emissions, but also include other aspects such: the selection of materials and construction solutions which seek to reduce the environmental impact of materials used, incorporating its lifespan criteria, impact of its production, assembly and recycling. Membranes are a building material with a big potential as examples so far have shown, can be used as self-supporting structures which are more adaptable than glass solutions, in terms of both their price and flexibility.

2. THE "MAGICAL" LLEIDA TECHNOLOGICAL PARK

The new test center for audiovisual production is located in the area occupied by the former infantry barracks and the current Science Technological Park Gardeny, in the city of Lleida. This infrastructure begins with the creation of the MAGICAL, a complex with facilities and services which follows the creation of audiovisual content, intended for various fields and market strategies (Figure 1).

The project, motions the rehabilitates of the existing buildings, comprising the adjacent main square. To complete the center's functions, we chose to construct, a third volume, as a large container, to house a film set in the southern corner of the square (Figure 2). A tensioned fabric, covers this container and spreads similar to a fog that embraces and connects the three buildings, defining a concavity towards the square, where there will be established the access to the complex. This material originates under, an open space for reception, waiting, cafeteria, poured into the plain landscape. Textile technology introduced into the building, opposed to the ponderosity of the historical buildings, allows us to see the great possibilities of this new building material.

3. THE TECHNOLOGY IMPLEMENTED FOR ACHIEVING "OUR STRATEGY"

The areas on which we operate with textile material from this complex are: the film set building
because, the facade is exposed to pollution and to solar radiation and in the same time it represents a very important part, and the main entrance space, by enlarging it, and making it suitable for other activities. However, we mentioned before our aim was not only to reshape the complex for functional reasons, but also to make it sustainable and energy efficient.

3.1. The film set:

"A light-able TEXTILE FACADE' for Lleida"

The buildings films set, was conceived as a large container of 600 sqm, 7m height and 8.70 m below the grill and a series volumes in which are installed controls, stores, services and general facilities of the complex.

This new building has been covered with a fabric envelope designed for the dual purpose of creating a space that works as a heating pad, protecting the set of solar radiation excess, and of achieving a new image able to illuminate or project images on the surface.

The textile facade, a single membrane out of Teflon coated fiberglass (PTFE), has been developed in collusion with the industry, by analyzing the pattern, making it resistant to mechanical and climatic actions, outside and inside having good behavior (Figure 3).

![Figure 3- textile facade](image1)

![Figure 4- metal structure](image2)

![Figure 5](image3)

The textile facade allows images to be projected both on the outside and the inside, creating the possibility to advertise the Magical in special moments. Due to reasons of saving, the lighting projection time is kept to a minimal, but it does have the possibility to become a great "Firefly" on Gardeny Hill.

The film set, the largest set of the complex, is a structural box with closing, so it can work under specially controlled light and sound (Figure 4, Figure 5). At the basic insulation it was added envelopes: in the exterior, fabric taut skin that acts as a facade, while inwards, a sound absorbing lining that can maintain optimum reverberation time. In addition it is installed a cyclorama that helps achieve the desired visual effects.

3.2. The main entrance:

The entrance cover, a ports the maximum technological innovation in this project and is truly unique, due to its design and composition (Figure 6).

From the new building, the textile cover is "projected" towards the centers square and along the existing buildings facades, creating a textile canopy that connects and defines the whole lobby. In this access space the internal comfort conditions are improved, using natural air-conditioning systems with cross ventilation ascending, from north to south, with vents and openings used, during hot
weather, and efficient heating systems with radiant floors, for cold weather. The main aim was that this space operates most of the time with no energy support.

The multi-layer membrane consists of, Teflon coated fiberglass (PTFE-Poly Tetra Fluoride Ethylene) in the exterior and fiberglass with silicone in the interior: between the two layers, there is a translucent insulation, named MONIFEX and a ventilated closed air chamber. MONIFLEX made out of layers of cellulose diacetate, folded and laminated transversely, is used mostly for thermal isolation but it is also lets light to pass through. Silicon is applied to enhance its fire resistance, durability, waterproofing and self cleansing.

The space that is generated under this envelope is used for the more public functions such as: reception, waiting areas and bar. The textile canopy is connected to the square and with the access, through a glazed surface that runs along the curve and defines it. On the south side is set the bar area with an outside terrace from which it can be admired the landscape of Lleida.

**Multi-layer translucent membrane advantages**

Various documented studies have shown the benefits of bright spaces, benefits which include improved learning environments for schools, improved productivity for offices and manufacturing facilities.

- By isolating the cover we can increase the original cover's thermal insulation performance by 4+ times
- Energy efficient layers and promotes free natural day lighting.
- Sound and acoustical insulation properties.
- Stain and fade resistant.
- Extreme flexibility.
- Reduces energy consumption, requirements and costs.
- Does not deteriorate over time or resists natural compression.
- A1 fire-resistance level
4. ENERGY SAVING

4.1. The protection of the heat acquired and its management through suitable materials

We paid a great deal of attention to the interactions between the physically determined energy requirements of our textile membrane and the energy installed building services, in order to find the most advantageous combination of passive and active technologies (Figure 7). This cover is an energy efficient design because it balances all aspects regarding the achievement and the saving of resources, due to its multi-layers composition. Functionally, the system provides significantly reduced costs at lighting systems, reduced energy consumption in air-conditioning, by this providing an optimized mix of passive solar strategies and energy-efficient materials.

By using a multi-layered membrane with an air chamber, we create a better controlled inner climate with efficient use of resources, easy maintenance and renewal character. Textile fabric is perhaps the most lightweight and simple construction material to create habitable spaces, linked to constant changing spaces, but it emerged in a permanent space where all the necessary comfort conditions were achieved. The textile cover, translucent with an closed air insulation that moderates the outer conditions, thus becoming an isolate translucent enclosure, has resulted in an efficient solution (Figure 8, Figure 9).

![Figure 8 - main access perspective textile envelope](image1)

![Figure 9 - supporting structure](image2)

As we know an efficient solution must be implemented well in a given tissue to have a favorable behavior. Therefore, we thought thoroughly the details regarding the connection of the membrane with the existing structure. Thermal bridging appears when materials have low thermal insulation capacity, allowing heat to flow through them, thus generating a problem that we tried to prevent. The bridges have to be eliminated, rebuilt with a reduced section or with materials that have better insulating properties (Figure 11, Figure 12, Figure 13).
1-exterior insulation Gutex 80mm; 2-aluminum plate; 3- insulation 40mm; 4- electro-fused metal framework; 5-Teflon coated fiberglass; 6-air chamber; 7-translucent insulation MONIFEX 60 mm; 8-fiberglass with silicone textile; 9-steel support structure; 10-aluminum channel; 11-supporting metal structure of the cover; 12-tirant; 13-extruded polystyrene insulation 40 mm; 14-profile IPE300.

1-Teflon coated fiberglass PTFE; 2-fiberglass with silicone textile; 3-insulation 70mm; 4-aluminum profile; 5-fireproof metal piece; 6,7-aluminum protection against water infiltration.

In general all textiles have easy maintenance, under tension or in the tensile state, these fabric membranes offer durability and fire resistance to meet building codes. PTFE fabric membrane provides exceptional strength and durability. waterproof, resists UV rays and is chemically inert. As a result, it is exceptionally stain resistant and easy to clean. PTFE fabric can reflect as much as 60 percent of visible light to make it a cost-effective, low-maintenance option for building owners and developers who demand a material that stays looking great for many years of service. These features allow the ability to push the design further on.

4.3. Natural ventilation
As we already established, the main goal was the provision of a good environmental behavior in accordance with the changing seasons. The double textile cover acts as a regulator between the interior and the exterior. Fresh air that enters in the lobby trough the openings of the envelope is channeled causing an air stream that allows natural air conditioning. The envelope of the lobby has practicable openings at the highest point. Together with the openings in the north square and the south cornice, it allows the natural ventilation of this space. In addition, this space is be provided with vegetation, so that during the warmer months the atmosphere can cool, by evapotranspiration from plants.

4.3. Day lighting
We studied the compositions of the sun protection elements in order to prevent direct access of solar
radiation and allow adequate light for indoor use. In this sense we have thoroughly explored the possibilities of textile walls, beyond its shading properties. In this way, we obtain a waterproof and berating spaces simultaneously, a thermally insulating and translucent multilayer ventilated enclosures. We always took into account that a textile structure that has one single layer has poor acoustic and thermal insulation properties, but a double or multilayer system is much better.

5. BUILDING REFERENCES


REFERENCES

CODE VERIFICATION EXAMPLES OF A FULLY GEOMETRICAL NONLINEAR MEMBRANE ELEMENT USING THE METHOD OF MANUFACTURED SOLUTIONS

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Key words: Code Verification, Benchmarks, Method of Manufactured Solutions, MMS, Nonlinear Membrane Element

Abstract. This paper presents an effective method to perform Code Verification of a software which is designed for structural analysis using membranes. The focus lies on initially curved structures with large deformations in steady and unsteady regimes. The material is assumed to be linear elastic isotropic. Code Verification is a part of efforts to guarantee the code’s correctness and to obtain finally predictive capability of the code. The Method of Manufactured Solutions turned out to be an effective tool to perform Code Verification, especially for initially curved structures. Here arbitrary invented geometries and analytical solutions are chosen. The computer code must approach this solution asymptotically. The observed error reduction with systematic mesh refinement (i.e. observed order of accuracy) must be in the range of the formal order of accuracy, e.g. derived by a Taylor series expansion. If these two orders match in the asymptotic range, the implemented numerical algorithms are working as intended. The given examples provide a complete hierarchical benchmark suite for the reader to assess other codes, too. In the present case several membrane states were tested successfully and the used code Carat++ assessed to converge - as intended - second order accurately in space and time for all kind of shapes and solutions.

1 INTRODUCTION

In the context of quantitative accuracy assessment of computer codes and their results Verification and Validation activities (V&V) are inevitable. They provide evidences for the
code’s and the results’ correctness. V&V activities can be separated in three parts: Code Verification, Solution Verification, and Model Validation. Code Verification represents the mathematical procedure to demonstrate that the governing equations, as implemented in the code, are solved consistently [17]. Solution Verification, often seen as part of validation, is the assessment of the correctness and accuracy of obtained solutions for a real (physical) problem if interest and Model Validation finally is the assessment of the model accuracy by comparison with experimental measurements. The relation between these activities and the different model parts are shown in figure 1. More detailed explanations of the topics of V&V and their most common definitions can be found in [10, 13, 15]. Roache gave a much simpler, but also very popular definition of Verification and Validation in [15]. He states that Verification is the purely mathematical discipline to assess that a code is “solving the equations right” and Validation is the discipline to assess that the code is “solving the right equations”.

2 CODE VERIFICATION

Code Verification can be performed in many different ways. As a stair of increasing rigor Oberkampf names the following methods [13]: simple tests, code-to-code comparisons, discretization error quantification, convergence tests, and order-of-accuracy tests. The first two are the weakest methods but they don’t need exact solutions of the problem. The latter three methods need exact solutions [13]. This paper focusses on the last method, the order-of-accuracy tests. Here one assesses, if the discretization error reduces with the formal order of the used discretization schemes. For an exact error evaluation of simulations the availability of analytical solutions is a prerequisite for the method. Unfortunately only a limited number of analytical solutions are available. The available test cases are usually much simpler then the application of interest, which reduces their practical value. This is especially the case for initially curved and prestressed, dynamically loaded structures. All other available, highly accurate solutions are insufficient to evaluate the exact discretization error of the simulations. Hence the Method of Manufactured Solutions (MMS) is used.
3 METHOD OF MANUFACTURED SOLUTIONS

Within the order-of-accuracy tests, the MMS provides a framework to generate artificial analytical solutions. It has been used in fluid dynamics [3, 5, 10, 13, 15–17], but also in structural stress computations [2], or recently in monolithic fluid-structure interaction computations [6]. The method can be used as a toolbox to assess implemented discrete procedures for the solution of differential equation(s). A major advantage of the method is the independence of the numerical approach: the method formulates the equations of state in the continuum, therefore it is independent of the discretization method (e.g. Finite Elements, Finite Differences, ...) or the solution procedure (direct solution, fix-point iteration, Newton-Raphson, ...). The main concept of MMS is that an own chosen target solution $\hat{d}$ represents an analytical (but not necessarily a physically realistic) solution for the primary variables $d$ of the differential equations in the continuum. After insertion of the target solution into the differential equations a source/force term remains, as the equilibrium no longer holds for the arbitrary solution $\hat{d}$. The continuous source/force term can be obtained by hand or using symbolic manipulation software like Maple®. If the discrete schemes are correctly implemented one can observe that $d$ tends towards $\hat{d}$ for systematically refined meshes. The difference between $d$ and $\hat{d}$ represents the exact error of the individual calculation. The development of the error with mesh refinement (mesh refinement factor $r = \frac{h_{\text{coarse}}}{h_{\text{fine}}}$ with a characteristic element size $h$) gives the observed order of accuracy $\hat{p}$ in equation 1.

$$\hat{p} = \frac{\log \left( \frac{E_{\text{coarse}}}{E_{\text{fine}}} \right)}{\log (r)}$$

(1)

If the formal order of accuracy $p$ (cf. chapter 6) matches $\hat{p}$ in the asymptotic range of the solution, the following parts of the code have been verified [15]: all coordinate transformations, the order and the programming of the discretization, and the matrix solution procedure. The procedure with all its parts is shown in detail in chapter 7.

If the two orders do not match, there can be many reasons, e.g. programming errors, insufficient grid resolution, singularities, etc. A complete list of reasons is discussed in [14]. MMS is not able to verify individual terms in mixed-order methods. Additionally it is not able to judge the efficiency of a solution process, e.g. the speed of convergence of a nonlinear problem or the iterative convergence rate of steady-state calculations [15]. The prerequisites for the application of the MMS can be found in detail e.g. in [17]. The basic requirement is that the target solution $\hat{d}$ represents a smooth analytical function with a sufficient number of derivatives. Additionally it should not contain singularities in the function or its derivatives. Furthermore the solution $\hat{d}$ should be general enough and well balanced to activate all terms of the governing equations. In this paper the given solutions give a stair of complexity or even a benchmark series to assess all containing parts step by step. This stair can be very helpful to localize coding or other mistakes [10]. Therefore each solution should be as simple as possible but as complex as necessary.
4 ERROR DEFINITION

Error in the context of this paper means the error between the exact solution of the continuum formulation and the approximated equation and solution using numerical techniques. The general sources of error in computer simulations are: physical modeling errors, discretization and solution errors, programming errors, computer round-off errors [12]. In this paper, we concentrate on the mathematical exercise and assessment of correctness of the code; therefore the physical modeling errors can be left aside. The computer round-off and iterative error shouldn’t affect the procedure. For this purpose the solution tolerance should be near to round-off [4] or the error out of it should be at least 100 times smaller then the discretization error [16]. The remaining errors (originating from discretization and of programming) can be assessed with the MMS. To calculate an error of a complete field of a variable in the domain \( \Omega \) one can use error norms. The continuous \( L^2 \) norm for the variable \( d \) compared to an exact solution \( \hat{d} \) can be seen in equation 2. If we assume a discrete solution (e.g. from a Finite Element solution) and an equidistant domain discretization with \( N \) elements or nodes one can reformulate equation 2 in the discrete \( L^2 \) norm as seen in equation 3. Besides the \( L^2 \) norm, the \( L^1 \) norm or the infinity norm are often referenced. Especially in the context of Finite Elements the error is often measured in the energy norm [18, 21].

\[
E_2 = \| d - \hat{d} \|_2 = \sqrt{\frac{1}{\Omega} \int_{\Omega} (d - \hat{d})^2 d\omega} \tag{2}
\]

\[
E_2 = \| d - \hat{d} \|_2 = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (d_n - \hat{d}_n)^2} \tag{3}
\]

5 THE FULLY GEOMETRICAL NONLINEAR MEMBRANE ELEMENT

This paper concentrates on a fully geometrical nonlinear membrane element for large deformations and small strains with a linear elastic isotropic material behavior and plane stress assumption.

5.1 Equilibrium

The equilibrium equations hold in general for the differential/strong form. It states the conservation of momentum in every point of the continuous structure. It can be shown that the equilibrium w.r.t. the current configuration (equation 4) and the initial configuration (eqn. 5) are equivalent [19]. In equations 4 and 5 \( d \) represents the field of displacements, \( \rho \) the density, \( t \) the time, \( f \) and \( F \) the forces on the current and the initial configuration, respectively. \( \sigma \) represents the Cauchy and \( P \) the first Piola-Kirchhoff (PK1) stress tensor [19]. It can be shown that the presented equilibrium in the strong form can be transformed into the commonly known weak forms of equilibrium (e.g. with the principle of virtual work in eqn. 6) [9, 19]. This equivalence of the weak and the strong form is the most important requirement for the applicability of the MMS. In equation 6, \( \Omega \) represents
the initial domain and $\hat{T}$ the surface forces on the boundary $\Gamma$ of $\Omega$. $S$ represents the second Piola-Kirchhoff (PK2) stress and $E$ the Green-Lagrangian strain tensor. With the goal to perform MMS simulations, all necessary terms of the strong form equilibrium (eqn. 4 resp. 5) have to be determined, completely independent of the implementation of the equilibrium (e.g. eqn. 6).

$$-\rho \frac{\partial^2 \mathbf{d}}{\partial t^2} + \nabla \cdot \mathbf{\sigma} + \rho \mathbf{f} = 0 \quad (4)$$

$$-\rho \frac{\partial^2 \mathbf{d}}{\partial t^2} + \nabla \cdot \mathbf{P} + \rho \mathbf{F} = 0 \quad (5)$$

$$\delta W = -\int_{\Omega} \rho \frac{\partial^2 \mathbf{d}}{\partial t^2} \delta \mathbf{d} \, d\Omega + \int_{\Omega} S : \delta \mathbf{E} \, d\Omega - \int_{\Gamma} \hat{T} \delta \mathbf{d} \, d\Gamma = 0 \quad (6)$$

5.2 Kinematics

The kinematics of the element (eqn. 7) are shown in figure 2. Capital and lower case letters indicate that quantities belong to the initial (e.g. $\mathbf{X}$) and the current/deformed configuration (e.g. $\mathbf{x}$), respectively. From equation 7 the covariant base vectors in the initial ($G_\alpha$) and the deformed configuration ($g_\alpha$) can be derived. $\theta^\alpha$ with $\alpha = 1..2$ are the surface parameters along $G_\alpha$. The base vectors $G_3$ and $g_3$ are constructed as normalized cross-product of the first two base vectors. Using the base vectors, the covariant metrics $G_{ij}$ and $g_{ij}$ can be evaluated by $g_{ij} = g_i \cdot g_j$ ($G_{ij}$ analogously). The calculation of contravariant base vectors can be performed with the aid of the contravariant metric $(g^{ij} = (g_{ij})^{-1})$ with the rule $g^i = g^{ij} g_j$. The deformation gradient tensor $\mathbf{F}$ is calculated in equation 9, where $\otimes$ represents the dyadic product [8,19]. The Green-Lagrangian strain tensor $\mathbf{E}$ is calculated using $\mathbf{F}$ and the unity tensor $\mathbf{I}$ in equation 10.
\[ x = X + d = X^i e_i + d^i e_i \] (7)

\[ G_{\alpha} = \frac{\partial X}{\partial \theta^\alpha}, \quad g_{\alpha} = \frac{\partial x}{\partial \theta^\alpha} \] (8)

\[ F = g_i \otimes G^i \] (9)

\[ E = \frac{1}{2} \cdot (F^T F - I) \] (10)

5.3 Material

The linear elastic isotropic material combined with the plane stress assumptions can be computed with the aid of modified Lamé parameters \( \lambda_m \) and \( \mu_m \) [11, 20] in equation 11 with 12. Using the material tensor \( C \) and the strain tensor \( E \), the PK2 stress tensor \( S \) can be calculated (equation 13).

\[ C = C^{\alpha \beta \gamma \delta} G_{\alpha} \otimes G_{\beta} \otimes G_{\gamma} \otimes G_{\delta} = \lambda_m \cdot G^{\alpha \beta} G^{\gamma \delta} + \mu_m \left( G^{\alpha \gamma} G^{\beta \delta} + G^{\alpha \delta} G^{\beta \gamma} \right) \] (11)

\[ \lambda_m = \frac{E \cdot \nu}{(1 - \nu^2)}, \quad \mu_m = \frac{E}{2 \cdot (1 + \nu)} \] (12)

\[ S = C : E \] (13)

5.4 Forces

Additionally to the stresses caused by strains, membranes are in general prestressed. The element of interest has its prestress defined in the initial configuration and the prestress tensor \( S_{ps} \) is therefore added to the PK2 stress tensor [11]. As the equilibrium of momentum (eqn. 4 and 5) contains the Cauchy resp. the PK1 stress tensor, they can be evaluated from the present PK2 stress tensor with equations 14 and 15 [8, 19].

\[ \sigma = \frac{1}{\det(F)} F (S + S_{ps}) F^T \] (14)

\[ P = F (S + S_{ps}) \] (15)

The equilibrium forces required to reach a prescribed deformation \( d = \hat{d} \) in the context of MMS are shown in equations 16 and 17. For the calculation of the forces it is recommended to use symbolic computation software like Maple\textsuperscript{®}. For ease of understanding the stress tensors are shown as a function of the target displacements \( \hat{d} \) (e.g. \( \sigma = \sigma(\hat{d}) \)). The general stress tensor components \( \sigma^{ij} \) or \( P^{ij} \) can be reduced to the in-plane stresses \( n^{\alpha \beta} \) resp. \( N^{\alpha \beta} \) in the membrane theory [1]. \( n^{\alpha\beta}_{\mid\alpha} \) represents the covariant derivative of \( n^\alpha \) [1, 19].

\[ \hat{f} = \rho \frac{\partial^2 \hat{d}}{\partial t^2} - \nabla \cdot \sigma(\hat{d}) = \rho \frac{\partial^2 \hat{d}}{\partial t^2} - n^{\alpha \beta}_{\mid\alpha} = \rho \frac{\partial^2 \hat{d}}{\partial t^2} - (n^{\alpha \beta} g_{\beta})_{\mid\alpha} \] (16)

\[ \hat{F} = \rho \frac{\partial^2 \hat{d}}{\partial t^2} - \nabla \cdot P(\hat{d}) = \rho \frac{\partial^2 \hat{d}}{\partial t^2} - N^{\alpha}_{\mid\alpha} = \rho \frac{\partial^2 \hat{d}}{\partial t^2} - (N^{\alpha \beta} g_{\beta})_{\mid\alpha} \] (17)
The calculated forces in eqn. 16 or 17 are volume forces. To generate an area force acting on the midplane of the thin membrane they have to be integrated over the thickness $b$ resp. $B$. It has to be stated again, that the calculated forces ($\hat{f}$ or $\hat{F}$) represent the equilibrium forces of the problem for a desired or given displacement field $\hat{d}$. That means that this force ($\hat{f}$ or $\hat{F}$) has to be applied to the code indifferently of the variational method of the implemented equilibrium. All these approaches and their developed equilibria are based on the strong form equilibrium (eqn. 4 resp. 5). Thus, the generality of the method is evident.

5.5 Boundary and Initial Conditions

One can directly determine the Dirichlet and Neumann boundary conditions (BC) for the deformed and the initial configuration with equations 18 resp. 19 [19]. $N$ and $n$ represent the in-plane normal vector, $T$ and $t$ are the traction vectors on the edges of the initial ($\Gamma$) and the deformed ($\gamma$) configuration, respectively. In steady-state computations the initial conditions (IC) are set to a nonbalanced state (e.g. to zero). In transient problems the IC have to be set to the target value of the variable. At $t = t_0$ the deformed matches the initial configuration (eqn. 20 for both configurations).

$$d_{\gamma} = \hat{d} \quad t = \sigma n$$  \hspace{1cm} (18)
$$d_{\Gamma} = \hat{d} \quad T = PN$$  \hspace{1cm} (19)
$$d(t = t_0) = \hat{d}(t = t_0)$$  \hspace{1cm} (20)

6 FORMAL ORDER OF CONVERGENCE

The discretization method and the chosen form functions for spatial and temporal discretization determine the formal order of convergence [2, 18, 21]. The approximation of the continuum, meaning the geometry, the boundaries, the solution fields and the integrals cause the discretization error in a simulation. One can determine the formal order of convergence $p$ of an approximated expression by comparison with its Taylor series expansion. The first term of the Taylor series which is not approximated is the leading error term within the asymptotic range of the solution. According to [21] the error for an isoparametric polynomial approximation of order $j$ is $O(h^p) = O(h^{j+1-m})$. $h$ states the characteristic mesh size and $m$ the magnitude of the $m$th derivative of the primary variable. For instance, this means for linear form functions ($j = 1$) that the displacement (primary variable, $m = 0$) converges with a formal order of $p = 2$ while stresses and strains ($m = 1$) converge with a formal order of $p = 1$. The formal order can be reached as long as no other errors (e.g. geometry approximation or integral approximation) with smaller convergence rate occur. A detailed discussion, especially about Variational Crimes can be found in [18].
7 PROCEDURE

The procedure applying the MMS is stated in the following. The notation is based on
the momentum equilibrium in the initial configuration (cf. eqn. 5, 17, 19 and 20).

1. Invention of a manufactured solution \( \hat{d} \)
2. Derivation of the equilibrium forces \( \hat{F} \) using eqn. 17
3. Derivation of the IC (eqn. 19) and BC (eqn. 20)
4. Application of the BC, the IC and the forces \( \hat{F} \) as input of the code to be assessed
5. Performing the simulation with a resulting field \( d \)
6. Error evaluation in \( d \) with the aid of \( \hat{d} \) (e.g. eqn. 3)
7. Repetition of steps 4-6 on systematically refined meshes
8. Calculation of the error development with refinement and evaluation of \( \hat{p} \) (eqn. 1)
9. Comparison of the formal order of convergence \( p \) to \( \hat{p} \)

There are two different ways to assess the transient functionality of a code: one option is
to assess the temporal discretization scheme independently of the spatial discretization
scheme. To do so, one has to isolate the time discretization error from the spatial discretization
error. This can be done either with a 0-dimensional problem where spatial
discretization doesn’t play any role or with a spatial field where the geometry and solution
exactly can be represented by the used shape functions. The second option is to assess
the temporal and the spatial discretization schemes together. Therefore one has to refine
both (time and space) with the same factor \( r = r_s = r_t \) in the refinement process. The
second procedure is also applicable, if the spatial discretization doesn’t match the formal
order of the temporal discretization (here, \( r_s \neq r_t \)) \[10\]. The error evaluation in step 6
always has to be at the same locations of the mesh and in time (e.g. the midpoints of
the coarsest simulation) \[10,12,13,15\]. This guarantees the comparability of the different
resolved solutions.

8 BENCHMARK EXAMPLES

All functions and variables of the following examples are listed in a table, such that
the reader is able to construct the force term \( \hat{F} \), the BC and the IC. The choice of the
BC type (Dirichlet or Neumann on each edge) is left to the reader. Performing own MMS
calculations is therefore possible with the given examples and the procedure of chapter
7. Remember: MMS starts at given equilibrium equations with all its assumptions. It is
not necessary that the used parameters are in the range of applicability of the equations.
This means that e.g. in the first example the membrane has a thickness \( B \) of 0.25m
Figure 3: Example 4: Initial (analytically parametrized with parameter lines) and deformed configuration

Figure 4: Example 4: Error development and observed order of accuracy $\hat{p}$ over refinement

over an area of only $1m^2$ although this wouldn’t make sense in physical applications of the membrane model. For a better understanding of the domain, the parametrization and the geometry are shown for example 4 in figure 3. The logarithmic error and the observed order of accuracy development with refinement for this example are shown in figure 4. The different graphs show the $L_2$ error norms of the different displacement directions $\mathbf{d} = (du, dv, dw)^T$. The negative inclination - with almost a straight line in the more refined area - of the log-log diagram of figure 4 indicates the error tending against machine accuracy and therefore convergence of the variables. The inclination of the error graph is calculated at each refinement step from a pair of errors (using eqn. 2) and is drawn as a graph in figure 4, right. The calculated inclination states the observed order of accuracy $\hat{p}$. In order to judge an example successful, the observed order of accuracy must reach the formal order of accuracy with refinement.
8.1 Example 1: Plane Structure, In-Plane Deformation steady-state

The simplest example is a plane rectangular membrane under pure tension in a steady-state calculation (cf. table 1). The example assesses the pure normal force action with prestress in the fully geometric nonlinear environment.

<table>
<thead>
<tr>
<th>initial configuration</th>
<th>deformation</th>
<th>material</th>
<th>element properties</th>
<th>domain size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \theta^1$</td>
<td>$d_x = 0.1 \cdot \sin(\theta^1 \pi)$</td>
<td>$E = 70000$</td>
<td>$B = 0.25$</td>
<td>$\theta^1 \in [0..1]$</td>
</tr>
<tr>
<td>$y = \theta^2$</td>
<td>$d_y = 0$</td>
<td>$\rho = 0$</td>
<td>$S_{psu1} = 25000$</td>
<td>$\theta^2 \in [0..1]$</td>
</tr>
<tr>
<td>$z = 0$</td>
<td>$d_z = 0$</td>
<td>$\nu = 0$</td>
<td>$S_{psu2} = 25000$</td>
<td>steady state</td>
</tr>
</tbody>
</table>

8.2 Example 2: Plane Structure, Out-of-Plane Deformation, steady state

Example 2 is a membrane which gets deformed steady-state out-of-plane (cf. table 2). It additionally assesses the geometric transformation through the out-of-plane deformation, shear force action, and the full material law with Poisson’s effect.

<table>
<thead>
<tr>
<th>init. conf.</th>
<th>deformation</th>
<th>material</th>
<th>element properties</th>
<th>domain size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \theta^1$</td>
<td>$d_x = 0$</td>
<td>$E = 1000$</td>
<td>$B = 0.001$</td>
<td>$\theta^1 \in [0..1]$</td>
</tr>
<tr>
<td>$y = \theta^2$</td>
<td>$d_y = 0$</td>
<td>$\rho = 0$</td>
<td>$S_{psu1} = 5$</td>
<td>$\theta^2 \in [0..1]$</td>
</tr>
<tr>
<td>$z = 0$</td>
<td>$d_z = 0.25 \cdot \sin(\theta^1 \pi) \cdot \sin(\theta^2 \pi)$</td>
<td>$\nu = 0.3$</td>
<td>$S_{psu2} = 5$</td>
<td>steady state</td>
</tr>
</tbody>
</table>

8.3 Example 3: Plane Structure, Out-of-Plane Deformation, unsteady

Example 3 is a plane membrane which will be deformed out-of-plane in an unsteady calculation (cf. table 3). This example additionally assesses the mass/inertia contribution and the time integration/discretization.

<table>
<thead>
<tr>
<th>init. conf.</th>
<th>deformation</th>
<th>material</th>
<th>element prop.</th>
<th>domain size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \theta^1$</td>
<td>$d_x = 0$</td>
<td>$E = 1000$</td>
<td>$B = 0.001$</td>
<td>$\theta^1 \in [0..1]$</td>
</tr>
<tr>
<td>$y = \theta^2$</td>
<td>$d_y = 0$</td>
<td>$\rho = 0$</td>
<td>$S_{psu1} = 25$</td>
<td>$\theta^2 \in [0..1]$</td>
</tr>
<tr>
<td>$z = 0$</td>
<td>$d_z = 0.25 \sin(\theta^1 \pi) \sin(\theta^2 \pi) \sin(t \pi)$</td>
<td>$\nu = 0.3$</td>
<td>$S_{psu2} = 25$</td>
<td>$t \in [0..1]$</td>
</tr>
</tbody>
</table>

8.4 Example 4: Curved Structure, Out-of-Plane Deformation, unsteady

The last given example is a plane membrane which will be deformed out-of-plane in an unsteady calculation (cf. table 4). This example additionally assesses the geometric approximation of the initial configuration. The initial and the deformed configuration...
of this example are given in figure 3. The error and the observed order of accuracy $\hat{p}$ development are shown in figure 4.

<table>
<thead>
<tr>
<th>init. conf.</th>
<th>deformation</th>
<th>material</th>
<th>element prop.</th>
<th>domain size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \theta^1$</td>
<td>$d_x = 0$</td>
<td>$E = 1000$</td>
<td>$B = 0.001$</td>
<td>$\theta^1 \in [0..1]$</td>
</tr>
<tr>
<td>$y = \theta^2$</td>
<td>$d_y = 0$</td>
<td>$\rho = 1000$</td>
<td>$S_{ps_{si}} = 25$</td>
<td>$\theta^2 \in [0..1]$</td>
</tr>
<tr>
<td>$z = \theta^1 - \theta^1 \theta^1$</td>
<td>$d_z = \frac{\pi}{4} \sin(\theta^1 \pi) \cos(\theta^2 \pi) \sin(\frac{\pi}{2} t)$</td>
<td>$\nu = 0.3$</td>
<td>$S_{ps_{s2}} = 25$</td>
<td>$t \in [0..2]$</td>
</tr>
</tbody>
</table>

9 CONCLUSIONS

The paper presents an effective method for detailed assessment of all functionalities of a membrane element within steady-state and transient Finite Element codes, containing curved geometries, prestress, and Poisson’s effect on curved geometries. Compared to other code verification methods, the MMS represents a very extensive testing method which provides an exact error evaluation. The generality of the MMS method made it very attractive to the authors. The present software code Carat++ could be tested successfully and the formal order of convergence of 2 was confirmed through the performed tests. In addition to the demonstrated application of MMS to structural dynamics with membranes, the methodology can be applied to create benchmarks and to assess other structural dynamics problems, fluid dynamics or even fluid structure interaction software environments, independent of the code structure and the discretization methods.

REFERENCES


CONCEPTION AND DESIGN OF MEMBRANE STRUCTURES CONSIDERING THEIR NON-LINEAR BEHAVIOR

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Key words: architectural membranes, verification, safety factors, hybrid structures, bending active, non-linear behavior, semi-probabilistic approach, reliability, Eurocode.

Summary. The lack of unified verification approaches and standards like the Eurocodes for various materials is a limiting factor to further propagation of architectural membranes. This paper will discuss the possibilities and challenges of integrating the design and verification of membrane structures into the Eurocodes’ philosophy. Therefore an overview of existing guidelines will be given, followed by a discussion of the underlying principles of the Eurocodes. Especially the non-linear behavior of architectural membranes distinguishes them from other structures. Therefore the focus of this contribution is to discuss the implications of this non-linearity on verification approaches. Theoretical considerations as well as in-depth examples help to clarify the necessary basis. Finally the consequences of non-linearity on the verification of the primary structure and hybrid structures are presented.

1 INTRODUCTION AND MOTIVATION

Architectural membranes provide minimal use of material combined with an attractive and impressive language of shapes. These shapes are – in contrast to most shapes in civil and structural engineering – directly mechanically motivated: based on the chosen prestress level and the boundary conditions, form finding determines the shape of equilibrium that allows the membrane to act in pure tension. The algorithms and approaches for successful computation of membrane and cable net structures exist [1-5] and are widely used. In contrast, the lack of consistent standards for verification and unified codes still is a limiting factor for further realization and success of architectural membranes.

In the following sections, steps of the conception and design of tensile structures under special consideration of their non-linear behavior shall be discussed. In section 2, a short overview of verification codes that are nowadays applied to membrane structures will be given. Since the Eurocodes generally provide the central framework of today’s verification procedure in Europe, section 3 will discuss the inscription of architectural membranes’ design in the existing codes, mainly characterized by their non-linear behavior. The problem of verification in the non-linear context increases, when different structural members are mixed. This is the case for primary structures for membranes in general and especially for hybrid structures, where the supporting system undergoes large displacements. The problems arising through this combination shall be addressed in section 4. Finally, concluding remarks will
provide a summary and give an outlook on future research activities towards a unified, consistently based verification standard in alignment with the Eurocodes’ approach.

2 VERIFICATION STANDARDS FOR MEMBRANE STRUCTURES

In contrast to most other materials used in the building and construction industry, currently there is no unified code for the verification of architectural membranes. Some codes and design guides exist on national level, like for example the ASCE 55-10 [6] (USA), the ITBTP design guide [7] (France) or the German practice, combining the DIN 4134 [8] and the dissertation of J. minte [9]. Most of these codes and guidelines are based on a stress factor approach that compares the results of an analysis with characteristic (i.e. unfactored, representative actions) loads to a permissible strength.

As an example one may take the approach from the ITBTP guide [7],

\[
T_C \leq T_D = \frac{k_q \cdot k_e \cdot T_{m}}{\gamma_t} \cdot T_{m} / \gamma_{stress},
\]

where the design strength $T_D$ is derived from the (characteristic) tensile strength $T_{m}$, reduced by the factors $k_q$ and $k_e$, as well as the so-called safety coefficient $\gamma_t$, taking into account the environmental degradation. The design strength $T_D$ represents the permissible strength that is ultimately assessed against the calculated tensile force $T_C$ under the respective load combination, assuming characteristic values for the actions. The quality factor $k_q$ shall adjust the member capacity to the execution quality; the scaling factor $k_e$ reflects the increased risk of critical defect with increasing surface area. For the sake of comparison, the individual factors – $k_q$, $k_e$ and $\gamma_t$ – may be summarized in one stress reduction coefficient $\gamma_{stress}$ (often termed “stress factor”), as demonstrated in equation 1.

Though the various codes and guidelines show differences in their respective prescribed load combinations and the way in which the stress reductions are applied, they can basically be compared to the procedure described in equation 1, summarizing the respective factors and coefficients to the overall stress factor $\gamma_{stress}$. As stated in different publications [10,11] the mentioned guidelines agree on comparable “levels of uncertainty”, reflected in the different stress factors. These reduction approaches are schematically represented in table 1:

<table>
<thead>
<tr>
<th>Standard</th>
<th>Factors</th>
<th>Incorporated influences</th>
<th>$\gamma_{stress}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASCE 55-10 [6]</td>
<td>$L_t$, $\beta$</td>
<td>life cycle factor, strength reduction based on different load combinations</td>
<td>4.0 – 7.8</td>
</tr>
<tr>
<td>ITBTP Design Guide [7]</td>
<td>$k_q$, $k_e$, $\gamma_t$</td>
<td>execution quality, scale factor, environmental degradation</td>
<td>(4.0) 5.0 – 7.0</td>
</tr>
<tr>
<td>German practice, based on DIN 4134 [8] and J. Minte [9]</td>
<td>$A_{res}$ ($\gamma_0$, $\gamma_M$, $A_i$)</td>
<td>loading uncertainties, material safety, test scaling, time influence, environmental degradation, temperature</td>
<td>2.9 – 6.4</td>
</tr>
</tbody>
</table>

In summary one may conclude that permissible stresses are obtained by reducing the
characteristic strength of the textile by a reduction factor $\gamma_{\text{stress}}$ in the order of 4.0 to 7.0 (the extreme values of 2.9 and 7.8 from table 1 are rather rare cases).

3 ARCHITECTURAL MEMBRANES AND THE EUROCODE REGULATIONS

In Europe, the design of structures generally is codified in the so-called Structural Eurocodes (EC). These have been introduced for the most commonly used materials like steel (EC 3), concrete (EC 2) or wood (EC 5). As mentioned above, such a unified standard does not exist for membrane structures up to now. Based on first attempts towards a unified design and verification approach like the TensiNet Design Guide [10], CEN250 Working Group 5 has initiated the development of a new Eurocode. This code shall specifically be applicable for membrane and tensile structures and provide guidance for their very special design and simulation demands. In the next sections, a short overview over the Structural Eurocodes’ underlying principles will be given; in the following, the challenge of incorporating the non-linear behavior of tensile structures will be discussed.

3.1 Eurocode regulations

Like all codes, the Eurocodes tempt to provide the necessary verification and assessment procedures to guarantee “safety” of the structures in scope. The underlying principle as described in the EC 0: “Eurocode – Basis of structural design” [12] is based on reliability theory. The main idea behind the semi-probabilistic approach is to define a probability of failure $P_f$ that represents an acceptable level of safety (cf. Fig. 1, right). This probability of failure can be linked to a reliability index $\beta$. To give an order of magnitude, for “usual” buildings (reliability class RC2) an annual failure probability in the order of $10^{-6}$ is deemed acceptable. All further verifications of certain limit states – leading to the term “limit state design” (LSD) – as prescribed in the different Eurocodes are designed in such a way, that they guarantee this level of probability of failure.

![Figure 1: Sketch of the global safety factor concept and definition of the descriptive parameters: the mean value $\mu_E$ and $\mu_R$, the standard deviation $\sigma_E$ and $\sigma_R$, the realization probabilities $p_R(R)$ and $p_E(E)$ and the defined factors of safety, $\gamma_{\text{nom}}$ and $\gamma_0$ (left); Failure probability $P_f$ (volume under the dark grey area) as a function of the variations of effects of actions $E$ and resistance $R$, failure boundary (green) separating the failure domain (right).](image)

In the context of LSD, two major limit states can be distinguished, the Serviceability Limit State (SLS) and the Ultimate Limit State (ULS). While the first group (SLS) is focused on
functioning, comfort and appearance, the second group (ULS) concerns the safety of people and of the structure. Both verifications are based on the definition of relevant design situations and load cases [12].

In earlier design approaches, a global safety factor concept has been used instead of the reliability based approach of the Eurocodes (cf. Fig. 1, right): Assuming both the loads (and with them the effects of action E like e.g. the resulting stress in a member) and the resistance R being subject to statistical variations, the definition of mean values (e.g. the mean stress $\mu_E$) and fractile values (e.g. the 95% fractile of the stress, $E_{95\%}$) were used for verification. As described in Figure 1, this concept led to two values for the quantification of safety, the central factor of safety $\gamma_0$, and the nominal factor of safety $\gamma_{nom}$.

The concept of distributed probabilities for loads and resistances is still at the basis of Eurocodes: Fractile values are used to define the “characteristic” values for the actions and resistances, $F_k$ and $R_k$, respectively. These characteristic values are directly used to verify for the Serviceability Limit State.

The verification for the Ultimate Limit State is based on a comparison of a design value of an effect of action, $E_d$, and a design value of the corresponding resistance, $R_d$. This basic verification concept can be written as

$$E_d = E_d\{G \oplus Q\} \leq R_d = \frac{R_k}{\gamma_M}, \quad (1)$$

where $R_d$ is defined through the characteristic resistance $R_k$, divided by a partial factor $\gamma_M$ that reflects the uncertainties in the definition of the material properties (the better material properties can be predicted, the smaller $\gamma_M$ can be assumed, e.g. $\gamma_{M,steel} \approx 1.0$ to 1.1). The design value of the effect of action, $E_d$, is the outcome of a load combination of permanent and variable actions $G$ and $Q$, respectively. These actions are collected in load combinations (sign “$\oplus$”) that shall reflect different relevant scenarios the structure may be faced with during its projected lifetime. In addition, partial factors $\gamma_F$ are applied to the respective loads in order to account for the uncertainties in the load values; combination factors $\psi$ represent the probability of occurrence in the respective load combinations (e.g. a combination of dead load of the structure, wind and traffic load). In the specific Eurocodes (depending on the materials used in their construction) detailed instructions for the assessment of structures are given, as well as specific values for the various partial factors.

Besides the lack of a material-specific Eurocode for architectural membranes, another important problem can be identified in advance: Due to the non-linear behavior of tensile structures, the influence of single actions on the effects of actions cannot directly be identified, thus opposing the concept of applying factored loads in order to account for a certain level of uncertainty. Hence for non-linear structures more detailed considerations are necessary, since only few indications are given in the Eurocode 0.

### 3.2 Non-linear behavior of tensile structures

As it is widely known, architectural membranes and other prestressed, tensile structures...
draw their load-bearing capacities out of their shape – generally double curved – and their ability to undergo large deformations that allow providing considerable geometric stiffness. In order to reliably simulate these large deformations, the need for a geometrically non-linear analysis is obvious. This need for non-linear analysis has important consequences on possible verification approaches.

In the context of the present contribution, consequences on load combinations shall be mentioned; consequences for the determination and proceeding of “design loads” as presented in the previous section will be discussed.

As mentioned above, a core ingredient for the Eurocodes’ philosophy is the concept of load combinations. At a first glance, it is important to note that without linear behavior of the structure, the widely used superposition approach is not applicable any longer. While this may at first be considered a minor inconvenience, the complete consequences are much more important: a major simplification made in the Eurocodes is to state that for the determination of the effects of actions (e.g. deflections or resulting stresses in the structure) – in most cases – a factoring of the action is equivalent to factoring the effects of this action, expressed in formula (6.2) of the Eurocode 0 [12] as

\[ E_d = \gamma_{\text{sd}} \cdot \{ \gamma_{f_i} \cdot F_{\text{rep}} \cdot a_d \} \]

where \( E_d \) is the effect of action due to the action \( F_{\text{rep}} \) (\( i \) is the summation index for different loading actions) applied to the design geometry \( a_d \). \( F_{\text{rep}} \) is the representative value of the action that is multiplied by the partial factor \( \gamma_f \) for possible unfavorable deviations of the representative value. At the left, the resulting effect of actions \( E \) is factored by a partial factor \( \gamma_{\text{sd}} \) for the uncertainties in modeling, at the right the underlying actions are factored by \( \gamma_{\text{sd}} \) directly. This concept of factoring the loads allows calculating with factored actions in order to obtain the design values of the effects of actions, which seems quite attractive for the verification of structures.

For the analysis of tensile structures with their large deformations, this concept has some major deficiencies: Since the stress state is strongly connected to the shape of the structure, a factoring of the load would also lead to an “unrealistic” deformation (cf. also section 4).

The Eurocode 0 addresses this problem of non-linearity by indicating a distinction between two different types of non-linear behavior in paragraph 6.3.2(4) “Design values of the effects of actions”, represented graphically in Figure 2: A distinction is made between structures where the effect of action, \( E \), increases more than the representative value of the action, \( F_{\text{rep}} \), (category a) respectively less (category b). The behavior characterized by category a) often is termed “over-linear” while category b) describes “under-linear” behavior. 

Note: for alignment with the commonly used terms in the following no distinction will be made between the characteristic value \( F_k \) and the representative value of the action, \( F_{\text{rep}} \). Only one single action \( F \) will be used.

The simplified representation in Figure 2 shows the difference between the two types of behavior. As mentioned above, for the case of a linear behavior of the structure, the two cases coincide, thus equation (2) becomes valid and the simplification can be applied.
In case of a non-linear structural behavior, it is important to correctly classify the type of structures to one of the above categories. This can be problematic, since the direct output of a non-linear simulation based on non-factorized characteristic actions $F_k$ is only the dimensioning point $(F_k, E_k)$, not a complete graph as shown in Figure 2. In a more abstract sense, this classification of the non-linear behavior represents the determination of the inclination of the $F$-$E$-graph. Two related approaches are briefly discussed in the following.

Since the inclination of the graph is not needed in an analytical, continuous sense, but in a reasonable surrounding of the dimensioning point, it would be sufficient to obtain one more point in addition to the dimensioning point. This determination of an additional point in order to approximate the graph’s evolution can be compared to classical sensitivity analysis.

Another approach is based on the fact that often non-linear simulations use path-following methods like load control [13]. With these methods, equilibrium configurations on the path are available, that allow approaching the graph’s inclination.

3.3 In-depth example of a prototype structure of a reduced hypar

The presented approaches apply to structures with non-linear behavior in general. For the case of architectural membranes the Eurocode 0 gives an indication considering their behavior: “Except for rope, cable and membrane structures, most structures or structural elements are in category a)” [12], and in consequence cable and membrane structures are in category b). In order to underline this assumption and demonstrate some effects of non-linearity, a reduced model of a classical hypar (cf. Fig. 3), will be discussed as a prototype structure. The simplifications taken from the hypar membrane to the model of two prestressed truss members (single degree of freedom (DOF) system) allow keeping the derivations intelligible.
In order to analyze the non-linear behavior of this structure, its residual force equation is derived, based on the principle of virtual work w.r.t. to the displacement variables $u$:

$$-\delta W = -(\delta W_{\text{int}} + \delta W_{\text{ext}}) = \left(\frac{\partial W_{\text{int}}}{\partial u} + \frac{\partial W_{\text{ext}}}{\partial u}\right) \cdot \delta u = (R_{\text{int}} - R_{\text{ext}}) \cdot \delta u = 0 \quad (3)$$

Here, the residual force vector $R = R_{\text{int}} + R_{\text{ext}}$ reduces to a scalar for the 1-DOF-system. In case of conservative loading, the external residual force $R_{\text{ext}}$ is equal to the load $F_{\text{ext}}$. The internal virtual work of a single member $i$ can be written as

$$-\delta W_{\text{int},i} = \int_{0}^{\delta u} \left[ (S_{11,i} + S_{0,i}) \cdot \delta \varepsilon_{\text{GL},i} \right] dV = A_i \cdot L_i \cdot \left[ (S_{11,i} + S_{0,i}) \cdot \delta \varepsilon_{\text{GL},i} \right], \quad (4)$$

where $L_i$ and $A_i$ are the length and cross section of the member $i$, respectively. In the members we assume constant strains and stresses along the elements. The stresses from elastic deformation ($S_{11}$) and prestress ($S_0$), measured as 2nd Piola-Kirchhoff stresses (PK2), are energy conjugate to the Green-Lagrange strains $\varepsilon_{\text{GL}}$. For truss members, the strains $\varepsilon_{\text{GL}}$ can be expressed as a function of the reference length $L$ and the current length $l$:

$$\varepsilon_{\text{GL},i} = \frac{1}{2} \frac{L^2 - L_i^2}{L_i^2}, \quad \text{and consequently:} \quad \varepsilon_{\text{GL},1} = \frac{1}{2} \frac{u^2 + 2h_i u}{L_1^2} \quad \text{and} \quad \varepsilon_{\text{GL},2} = \frac{1}{2} \frac{u^2 - 2h_i u}{L_2^2}. \quad (5)$$

Rewriting the virtual strains $\delta \varepsilon_{\text{GL},i}$ leads to

$$\delta \varepsilon_{\text{GL},i} = \frac{\partial \varepsilon_{\text{GL},i}}{\partial u} \cdot \delta u, \quad \text{and thus} \quad \delta \varepsilon_{\text{GL},1} = \frac{u + h_i}{L_1^2} \cdot \delta u \quad \text{and} \quad \delta \varepsilon_{\text{GL},2} = \frac{u - h_i}{L_2^2} \cdot \delta u. \quad (6)$$

When introducing the simplifying assumptions of equal height $h_i = h$, initial length $L_i = L$, cross section $A_i = A$, and prestress $S_0,i = S_0$, the expression of the internal residual forces $R_{\text{int}} = \Sigma R_{\text{int},i}$ can be written as:

$$R_{\text{int}} = \frac{A}{L^3} \left( u + h \left[ \frac{1}{2} E \cdot (u^2 + 2hu) + S_0 \cdot L^2 \right] + (u - h) \left[ \frac{1}{2} E \cdot (u^2 - 2hu) + S_0 \cdot L^2 \right] \right) = \frac{EA}{L^3} \cdot \left( u^3 + 2h^2 u \right) + 2 \frac{u}{L} S_0 \cdot A \quad (7)$$
Additionally assuming linear elastic material, the elastic stresses $S_{11}$ have been replaced by $S_{11} = E \cdot \varepsilon_{GL}$, introducing Young’s modulus $E$.

For the evaluation of internal forces as effects of actions, the internal forces $N_1$ and $N_2$ of the members can be written as

$$N_i = \frac{\ell_i}{L_i} \cdot A_i \cdot S_{pk_{2,i}} = \frac{\ell_i}{L_i} \cdot A_i \cdot \left( E \varepsilon_{GL,i} + S_{0,i} \right) = \frac{\ell_i}{L_i} \cdot \left[ \frac{EA}{2} \cdot \frac{\ell_i^2 - L_i^2}{L^2} + S_0 \cdot A \right], \quad (8)$$

which can also be formulated for the individual members as a function of the displacements (the current length $\ell_i$ is a function of $u$). In this example, the factor $\ell_i/L$ represents the transition from the reference configuration (and with it the reference orientation) and the current configuration, oriented in the member’s current direction. In all presented developments, a deformation of the section $A$ is neglected (equivalent to Poisson’s ratio $\nu=0$).

With these formulations at hand, three selected parameters are analyzed w.r.t. to their non-linear evolution regarding their possible verification according to the approach of Eurocode 0: (i) the displacement $u$ and (ii) the normal forces $N_1$ and $N_2$. For these selected parameters, a classification according to the Eurocode’s proposition discussed in section 3.2 will be made.

![Figure 4: Representation of the selected effects of actions resulting from the action $F_{ext}$; the distinction of the Eurocode’s categories of non-linearity can be made by comparison with a fictive linear relation between $E$ and $F$ (grey straight lines). For the normal force $N_2$, the factoring of the load must not be applied, as $N_2$ is reduced by increasing $F_{ext}$ (in the surrounding of $F_k$). For the displacement $u$, a SLS verification is applied (no factoring).](image)

The first examined parameter $u$ serves as example for a verification of the Serviceability Limit State. Obviously the question of increasing the load magnitude by a factor is rather artificial in this case, since the SLS has to be verified with characteristic values: only “as-realistic-as-possible” predictions of the deformations to be expected are of value at the design stage. Nonetheless it is considered an effect of action $E$ and thus is plotted in the graph in
Figure 4. For the effect of action $u$ it can be observed, that it increases less than the action $F_{ext}$ itself, thus – theoretically – classifying the structure in category b) (cf. Fig. 2 and Fig. 4).

The question of correctly applying the load factor to the respective value – the action $F_k$ or the effect of action $E_k$ – is more important for the Ultimate Limit State verifications, examined here for the member forces $N_1$ and $N_2$. From the plot in Figure 4 one can observe that for the upper member, the normal force $N_1$ increases less than the action $F_{ext}$. Hence the ULS verification of $N_1$ would be placed in category b), which is in accordance with current design practice for membrane structures: the stresses are calculated based on characteristic loads, these stresses are then assessed against admissible stresses. For the second member, the question of assessing in the loaded state – and in consequence the question of factoring the load $F_{ext}$ or its effect – is irrelevant as the unloaded state ($F_{ext}=0$) represents the most demanding situation for the member. This phenomenon can obviously be explained via the prestress that’s reduced as the member is compressed by the increasing load.

One can conclude that current design’s practice – application of the factors on the effects of action rather than on the action itself – complies with the basic instructions for non-linear structures of EC 0. Nevertheless an important problem arises from non-linearity: Though in the design guides and codes factored load combinations are prescribed, their effect is very delicate to be judged. If one would apply these factored loads in the non-linear calculation, not only effects of action with a magnitude differing from the considerations above may result, but these values are also doubtable as they are based on “exaggerated” displacements, which usually are considered large for tensile structures. This will make it very difficult to judge about the true value, as one cannot be sure whether these values are conservative.

4 TENSILE STRUCTURES AND THE PRIMARY STRUCTURE

The problem mentioned above is even more accentuated, when it comes to the interaction of different types of structures. For membrane structures, this is the case, as they all rely on some kind of primary structure, supporting the textile membrane. This primary structure, often made of steelwork, has to be verified on its own, applying its specific code (for the case of steelwork this would be the EC 3). These codes prescribe the use of adapted load combinations with individual partial factors for the different actions. Here the problem becomes obvious: When for the membrane the load factor is applied on the stresses based on characteristic loads, the necessary individual load factors cannot be applied anymore.

4.1 Interaction of textile membrane and the primary structure

When looking at the load transfer from textile membrane to the underlying structure two aspects may be observed that are closely related yet of individual importance. As the membrane transfers the surface loads to the primary structure through tensile forces, this transfer includes both the magnitude of the force as also its orientation (cf. Fig. 5). The question of the force orientation may seem of little importance, but when considering the large deflections that may occur, it might be of interest for the dimensioning of the primary structure. Thus, the question is rather twofold: which load from the membrane to apply in which orientation on the primary structure? As stated above, the approach usually taken for
the membrane – i.e. simulating the membrane with characteristic loads and applying the load factor on the effects of actions – leads to reasonable values for the membrane design. This approach should be continued for the primary structure. In addition to the load, the effects of the deformed geometry and with it the altered orientation of the interaction forces between membrane and primary structures have to be examined.

In Figure 5 a schematic representation of the load transfer from membrane to primary structure is presented. In order to determine the design value \( M_{\text{steelworks}} \) between the fixation profile and the general steelworks, the tension from the membrane has to be multiplied with its respective lever arm. It is obvious that even when assuming the same tensile force \( n \), the moment is also dependent on the lever arm \( \Delta x \). While even for the design geometry an eccentricity \( \Delta x_{\text{design}} \) must be taken into account, this \( \Delta x \) may increase during deformation.

### 4.2 Hybrid structures

In most cases, the primary structure is considered stiff compared to the membrane. In consequence it is often treated as a fixed support; the loads from these imaginary supports are then verified for the primary structure in a separate assessment. This approach may be justified, when the structure can be considered very stiff.

If the supporting structure is too weak to be considered a fixed support, it has to be
included in the non-linear simulation of the membrane. These structures, where both the form found membrane and the elastic supporting structure have to be taken into account in one integrated computation approach are called “hybrid structures” [14-16].

Due to the large displacements the supporting structure may be subject to, the problematic of correctly computing and applying the forces as well as the geometry is even more accentuated for hybrid structures.

5 CONCLUDING REMARKS

This paper has presented the possibilities and challenges of integrating architectural membranes with their non-linear behavior into the reliability based design approach that represents the underlying principle of the Structural Eurocodes. The verification approaches currently in use and different codes and design guides that are applied have been presented, especially focusing on their verification approach. As basically all guidelines are based on a permissible strength (or allowable stress) design, the stress factors $\gamma_{\text{stress}}$ have been compared.

A general overview over the Eurocodes’ design philosophy, the definition of “safety” by a failure probability $P_f$, has been presented and compared to the global safety factor concept. The limit state design approach and the respective effects of actions are of major importance in this context. The possibilities of integrating non-linear structures in the Eurocodes’ design concept have been discussed. In order to characterize and demonstrate the consequences of this non-linear character of tensile structures, a small scale example of prestressed cables has been discussed in detail, especially focusing on possible verification approaches.

An outlook on the interaction of the membrane with the primary structure has been given, taking into account the consequences of the non-linear behavior of membrane structures. Especially for comparatively weak primary structures – as it’s the case for hybrid structures, where the elastic supporting elements create the need for an integrated simulation approach – this interaction is of importance for possible verification approaches. Examples for these hybrid structures have been given.

To conclude we may state that – though important progress has been made concerning the unification of levels of “safety” and stress factors [10] – the non-linear behavior or architectural membranes presents a major challenge on the way towards a consistent verification methodology. These challenges may be subdivided in three major categories: (i) Still the material properties of textile membranes are far from being consistently derived and widely accessible: it is still very difficult for designers, to obtain reliable data on stiffness, creep or even material strength. This is accompanied by the need for consistent material models on the simulation side. (ii) As discussed in this paper, the determination of loads and load combination as well as the respective partial factors are still subject to current research and code development. The complex curved geometries of membrane structures make it very difficult to estimate some load cases like snow, their light weight in addition makes them prone to wind excitation. This aspect is part of the research efforts invested in Fluid-Structure-Interaction and Computational Wind Engineering [15]. (iii) Last but not least the non-linear character of architectural membranes as well as the design tasks (form finding, cutting pattern generation,...) make the simulation of membrane structures a challenging task for methodological research and software development. Even for well-defined examples,
available software environments provide very different results [17]. In summary it can be stated that the development in the field of tensile structures is far from being finished, important research is needed to solve the mentioned problems and face the arising challenges.

REFERENCES

EXPERIENCE ON THE IMPLEMENTATION OF A NONLINEAR MATERIAL MODEL FOR MEMBRANE FABRICS IN A FINITE ELEMENT PROGRAM

MEMBRANES 2013

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Key words: Membrane Analysis, fabrics, warp fill behavior.

Summary. This paper describes the implementation of a nonlinear material law and presents examples to nonlinear warp fill behavior and wrinkling.

1 INTRODUCTION

Since 1996 SOFiSTiK provides finite element membrane analysis with a simple linear elastic orthotropic material model. The only material nonlinearity was that it could not carry compression and so simulates wrinkles.

Based on a paper in Tensinews [1] we implemented a real nonlinear material model. The model can be calibrated to experimental results and can handle the yarn parallel warp and fill relationship.

In this paper first experiences with this model in complex finite element calculations will be presented. Also the effect of additional shear stiffness and the problem of restressing after wrinkle occurrence will be discussed.

2 IMPLEMENTATION

The nonlinear behavior in [1] is expressed as a stress-strain relation. This means that for a given stress $\sigma_w$ and $\sigma_f$ (w=warp direction, f=fill direction) it gives a corresponding nonlinear strain $\varepsilon_w$ and $\varepsilon_f$. The values of the stress-strain matrix depend on the ratio of $\sigma_w$ to $\sigma_f$ using the factors $\gamma_w$ and $\gamma_f$:

$$\gamma_w = \frac{\sigma_w}{\sqrt{\sigma_w^2 + \sigma_f^2}} \quad \gamma_f = \frac{\sigma_f}{\sqrt{\sigma_w^2 + \sigma_f^2}}$$

$$\begin{bmatrix} \varepsilon_w \\ \varepsilon_f \end{bmatrix} = \begin{bmatrix} \frac{1}{E_w(\gamma_w)} & -\nu_{wf} \\ -\nu_{wf} & \frac{1}{E_f(\gamma_f)} \end{bmatrix} \begin{bmatrix} \sigma_w \\ \sigma_f \end{bmatrix}$$
The finite element program implementation uses a quick internal iteration for the inverse problem, analyzing the stress for a given strain of the actual finite element iteration.

In [1] some typical material parameters are shown, e.g. for Mehler Texnologies Valmex FR700(I):

\[
\begin{align*}
E(1:1)_w &= 653.2 \text{ kN/m} \\
\Delta E_w &= 521.2 \text{ kN/m} \quad \Delta E_f = 403.7 \text{ kN/m} \\
v_{wf} &= 0.327
\end{align*}
\]

As we use isoparametric finite shell elements, also a shear modulus \( G \) is included and represents the stiffness against yarn shear distortion. If no further information is available we recommend a value of 1-3 % of the E-modulus for textile products.

3 NONLINEARITY

The major nonlinear effect occurs in case of decreasing the stress in one direction. The following animation illustrates the interaction of warp and fill yarns in this case:

Figure 1: animation for increasing warp (blue) and decreasing fill stress (yellow)
Starting with an isotropic prestress the blue yarns (warp direction) are further stressed, the yellow yarns (fill direction) are released. As the blue yarns get straight, the yellow yarns must go up and down much more due to geometric nonlinear effects and shorten the material in the fill direction. The deformation in fill direction (yellow) is now disproportionately higher than calculated with simple poisson ration $\mu$. The same example analyzed with the nonlinear material law for quad elements also show this nonlinear effect (quad compressed in fill direction – plotted in red):

![Figure 2: one single finite quad element under fill compression](image)

![Figure 3: fill stress – fill compression strain](image)

With decreasing fill stress (vertical PY) the stain in fill direction changes disproportionately high. On other words: decreasing the fill deformation (fill compression) the stress in the fill direction will not decrease as fast as in a linear analysis.
4 WRINKLES

The main problem is the wrinkling. A further compression in fill direction only creates wrinkles but will not change the stress in warp direction (that occurred at the start of wrinkling)! This means that the material behavior changes completely because no more poisson ratio effect exists: further eps-f compression has no effect on the now uniaxial stress state:

Figure 4: further wrinkle compression (yellow) may not change the force in the stress direction (blue)

On the other hand a fill-wrinkled membrane under following fill tension must act without stress change until the wrinkle deformation is restored. Please notice that a real quad finite element also has a little shear modulus G and may have main stresses sig-I and sig-II that differ from the warp and fill direction. So the following wrinkle procedure also works for isotropic materials (PVC). The program is based on the real total deformation and strain and works as follows:

- compute the linear main stress sig-I and sig-II from given total strain (linear law)
- if both stresses are positive -> standard case -> compute nonlinear warp-fill stresses
- else:
  - calculate main tensile direction beta
  - calculate strain in this direction eps-I and transverse eps-II
  - assume wrinkle transverse to main tensile direction = assume sig-II=0
  - set material law E for this stress assumption and rotate to beta
  - iterate wrinkle starting strain eps-II-0 to achieve sig-II=0 with this material law E
  - if eps-II < eps-II-0 a real wrinkle occurred with wrinkle (damage) strain
damage = stressfree wrinkle strain = eps-II - eps-II-0
  - if created sig-I is negative -> wrinkled in both directions
  - transform stresses sig-I and sig-II back to local quad coordinate system
  - in the general case with shear modulus G not 0.0 this gives sig-x, sig-y and sig-xy
5. EXAMPLES

A prestressed roof with plus-minus curvature is loaded with wind from below. For linear elastic material behavior the stress in warp direction (short span) decreases rapidly and on a downside air pressure of 0.32 kN/m² first wrinkles occur (warp stress gets zero). Using the same material with the new nonlinear material law the wrinkles occur a little bit later on a downside air pressure of 0.37 kN/m²:

Figure 5: roof with wind loading from below

Another example is a membrane structure with a highpoint where wrinkles occur later using the new implemented nonlinear material law. For high upside wind the tangential prestress falls out nearly completely and the membrane carries the load by only radial warp stress:

Figure 6: left: wrinkles with linear material, right: with new nonlinear warp-fill law (both 0.40 kN/m² windload)
The air supported cushion form [2] shows an extreme curved wrinkle zone on the lee side:

6 CONCLUSIONS

The paper reports of the implementation of a nonlinear membrane material law. Specifically for high wrinkle effects, special algorithms were tested to fulfill the requirement of constant stress in case of increasing wrinkles.

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**From NURBS to Textile Architecture**

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Key words: nurbs, splines, geometry, formfinding, membrane cladding

Summary. NURBS models are used in the architectural design to develop new geometries. This document describes the process from the pure geometry to a feasible membrane shape, which is still compatible with the design intend of the architect.

1 INTRODUCTION

More and more design tools like Rhino are used in the development of geometry driven architecture. Very often the intent is to use textile materials to realize these shapes. Especially monolithic geometries modelled as NURBS surfaces have typically synclastic curvature, while the use of textiles, unless they are not inflated, requires typically anticlastic curvature. But even anticlastic shapes, generated with NURBS surfaces have typically less restrictions than membrane shapes generated with a normal formfinding process.

2 TUBALOON, KONGSBERG JAZZ FESTIVAL

Since 1965 in July, one of the most important Jazz festivals in Norway takes place, in Kongsberg, 80km west of Oslo.

The shape of the stage roof is inspired by the inner ear and by wind instruments. In the initial design the perimeter beam of this structure was made as Tensairity beam. Due to the shape of the beam high bending and torsion moments occur, so that a slender steel arch was used for the final concept. The pneumatic ring was kept in the design, and helps to prestress the closure flaps.

The shape was developed with geometric design tools. The structure is sitting on 2 nodes, has one big arch over the stage, one small backstage arch and a "flying ring" sitting on the small arch.

Based on the architectural design we have started a first formfinding, assuming a reasonable stress ratio in the prestress stage. This led to a much wider neck and a higher saddle. In an iterative process we have adjusted the shape to be best fitting with the architectural concept.
Figure 1: Initial design

Figure 2: First formfinding commented by the architect

Figure 3: Improved geometry with further comments
It was then decided to vary different parameters in the formfinding process to obtain a shallow neck and a low saddle. The ring inclined backwards led to a high saddle, so that we checked also different ring inclinations and their impact.

Within these parameter studies we ended up with a satisfactory shape. The stress strain ratio is varying over the whole structure to allow the slender neck, but we had still enough width to realize the structure with continuous membrane strips.

![Figure 4: Rendering with the final membrane shape](image)

![Figure 5: Cutting Pattern of the main membrane](image)
Figure 6: prestress in warp direction

Figure 7: prestress in weft direction

Figure 8: installed structure
3 ZÉNITH DE STRASBOURG

Close to Strasbourg the 18th Zénith of France is situated. The Zéniths are concert halls for «musique populaire» from rock to pop up to musicals.

The inner part of the building is not disclosed at first sight. It consists of 30 cm reinforced concrete which is formed by the lines of different curve radii to achieve an optimization of maximum capacity and best view. The reinforced concrete was selected to have the best possible control over the acoustic.

The oval form was chosen as sculptural element, its monumental volume gets some easiness by the ellipses of the steel structure. 20 steel girders form a sort of access balcony around the massive core and build the primary structure for the façade structure which carries the membrane. 5 horizontal steel rings with a tube diameter of 50 cm enclose the whole building. Like the orbits of the planets they have different distances and inclinations. This leads to a dynamic appearance which is also emphasised by the displacement and rotation of the ellipses.

Figure 9: Rendering by the architect

Figure 10: Rhino model
The geometry developed by the architect was based on the idea of having the same membrane length at every point between two rings, so the wide areas were flat, and the small areas were curved.

In the wide areas this would give a reasonable shape, but in the narrow areas the difference of the two radii was approximately 90 m to 2 m. To get an equilibrium, we would need the same ratio, this means if for example the pretension in warp is 1 kN/m, the pretension in weft would be 45 kN/m.

All generated saddle shapes looked very flat and completely different to the architect’s idea. Together with him, we defined a solution with additional valley cables, to get a comparable overall shape, but this solution ended up with a very sharp geometry for an even stress distribution, so we increased the pretension in weft to keep the faces slightly curved.

Figure 11: Shape development
Figure 12: prestress in warp direction

Figure 13: prestress in weft direction

Figure 14: result of the structure at night
4 NUVOLA, CENTRO CONGRESSI, ROME

The project Centro Congressi will become Rome’s new congress centre. The 200 m long and 75 m high cube is partially sitting below floor level.

Almost floating, fixed with only few points to the ground a cloud, in Italian Nuvola, is dominating the inner space of this huge glazed cube. The Nuvola is a 126 m long and 65 m wide organic shaped object. Inside is a cafe, foyers, conference rooms and an auditorium for 2000 spectators.
The supporting steel structure was developed by cutting parallel planes through the outer surface of the Nuvola. Different possibilities how to cut these planes were checked. The steel consultant decided for orthogonal planes through the surface.

This steel layout simplified the steel production, but at the same time it created many problems for the membrane cladding. Our task is to cover the heavy steel structure with light white silicone coated glass fibre fabric. In curved areas the steel sections were typically not in the perfect location so that we got sharp edges predominating the appearance of the Nuvola.

The initial geometry was smooth and mainly with synclastic curvature. In-between the main sections the membrane formed saddles depending on the curvature of the axes. With the modification of the prestress ratio, only a slight reduction of the saddles was possible.
5 CONCLUSION

Software tools available on the market offer new possibilities to designers in the development of free form shapes. This leads to a wide range of interesting structures and enriches the today architecture.

In a close collaboration between designer and engineer feasible membrane shapes can be generated fully satisfying the architectural intend. Essential in this process is the understanding of each other. Sometimes it is necessary to turn the wheel back in the design process, as if the involvement has been from the beginning.
IMPLEMENTATION OF A SIMPLE WRINKLING MODEL INTO ARGYRIS’ MEMBRANE FINITE ELEMENT

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Key words: nonlinear analysis, membrane structures, Argyris’ membrane element, membrane wrinkling and slackening.

Summary. This paper presents the implementation of a simple wrinkling/slackening model into the classical Argyris membrane element, comparing the solution performance by Newton’s iterations, using either the tangent stiffness matrix (numerically evaluated through a finite-difference approximation), or a secant stiffness matrix (obtained through the modification of the elasticity matrix, according to a projection technique which decompose deformations into elastic and wrinkle components).

1 INTRODUCTION

There may be cases in which wrinkles on a membrane must be represented with accuracy. In these cases, a shell-like formulation is generally required, and numerical solutions require heavy computation. In the case of architectonical membranes, however, wrinkling is to be avoided, at least on the initial configuration.

To guarantee a taut surface, shape finding processes seek configurations where the minimum membrane’s principal stresses are positive everywhere. In past times, this requirement was also extended to the membrane under design loads, and the onset of wrinkling was considered as a type of structural failure. Thus, in order to avoid wrinkling, high initial stresses were usually prescribed at the initial membrane configuration. Nevertheless, in practical architectonical applications, there is not an unconditional reason for the membrane to be free of wrinkles, or even slack zones, in extreme load cases, nor it is required to know the precise pattern of wrinkling, when it occurs. What does is necessary is a method capable to determine the correct load transfer mechanisms, which are distorted if the adopted finite elements do not avoid spurious compression states. Besides, if wrinkling and slackening are allowed in some conditions, the resulting anchorage loads are reduced, because smaller initial membrane stresses are then required, and also because larger membrane displacements contribute to a more favorable load distribution.
In this paper, we present the implementation of a simple wrinkling/slackening model for the classical Argyris membrane element, already available in the SATS program\textsuperscript{1,2}, and compare the performance of the tangent stiffness matrix (consistent with the proposed wrinkling/slackening model, but numerically evaluated through a finite-difference approximation), to the performance of a secant stiffness matrix (obtained when the classical linear elasticity matrix is replaced by the modified elasticity matrix derived by Akita \textit{et al.}\textsuperscript{3}, using a projection technique to decompose deformations into elastic and wrinkle components).

2 NONLINEAR EQUILIBRIUM. VECTOR NOTATION

Upon discretization, the problem of \textit{equilibrium of a membrane structure} can be expressed as finding a displacement vector $\mathbf{u}^*$ such that

$$
\mathbf{g}(\mathbf{u}^*) = \mathbf{p}(\mathbf{u}^*) - \mathbf{f}(\mathbf{u}^*) = \mathbf{0}
$$

where $\mathbf{u} = \mathbf{x} - \mathbf{x}_r$ is the \textit{global displacement vector} with respect to a reference configuration $\mathbf{x}_r$, $\mathbf{g}(\mathbf{u})$ is a \textit{residual force vector}, $\mathbf{p}(\mathbf{u})$ is the \textit{internal load vector} and $\mathbf{f}(\mathbf{u})$ is the \textit{external load vector}, all of order $n_{\text{dof}}$, the number of degrees of freedom of the system.

These functions can be computed in sub-domains $\Omega^e$, or elements (Figure 1(a)), as a function of the \textit{element displacement vectors} $\mathbf{u}^e = \left[ \mathbf{u}^e_\alpha \right]$, $\alpha = 1, \ldots, n^e_\alpha$, $e = 1, \ldots, n^e$, where $n^e$ is the number of elements and $n^e_\alpha$ is the number of nodes defining the $e^{th}$ element. Defined onto every element, there exist an \textit{element residual force vector} $\mathbf{g}^e = \left[ \mathbf{g}^e_\alpha \right]$, where $\mathbf{g}^e_\alpha = \mathbf{g}^e \left( \mathbf{u}^* \right)_\alpha$ is the contribution of the $e^{th}$ element to the residual vector, evaluated at its $\alpha^{th}$ node.

The equilibrium problem (1) can be solved –within a vicinity of a solution $\mathbf{u}^*$ – iterating Newton’s recurrence formula,

$$
\mathbf{u}_{k+1} = \mathbf{u}_k - \left( \frac{\partial \mathbf{g}}{\partial \mathbf{u}} \right)^{-1} \mathbf{g}(\mathbf{u}_k) = \mathbf{u}_k - (\mathbf{K}_k)^{-1} \mathbf{g}(\mathbf{u}_k)
$$

where we define the \textit{tangent stiffness matrix}

$$
\mathbf{K} = \frac{\partial \mathbf{g}}{\partial \mathbf{u}} = \left[ \frac{\partial g_i}{\partial u_j} \right], \quad i, j = 1, \ldots, n_{\text{dof}}
$$

Recurrence (2) may converge to a solution even when the consistent linearization of (1) is replaced by some approximation (i.e., a \textit{secant stiffness matrix}), usually at the price of reduced convergence rate.
The element displacement vectors $\mathbf{u}^e$ can be extracted from the global one according to $\mathbf{u}^e = \mathbf{A}^e \mathbf{u}$, where $\mathbf{A}^e$ is a Boolean incidence matrix for that element. Likewise, the global residual force vector and the global tangent stiffness matrix can be assembled according to

$$
\mathbf{g} = \sum_{e=1}^{n_e} \mathbf{A}^e \mathbf{g}^e \quad \text{and} \quad \mathbf{k}^e = \frac{\partial \mathbf{g}^e}{\partial \mathbf{u}^e} = \begin{bmatrix} \frac{\partial g_i^e}{\partial u_j^e} \end{bmatrix}, \quad i, j = 1, \ldots, n_{dof}^e \quad (4), \quad (5)
$$

where $n_{dof}^e$ denotes the number of degrees of freedom of the $e^{th}$ element.

### 3 INTERNAL LOAD VECTOR FOR A MEMBRANE ELEMENT

Figure 1 shows the Argyris’ natural triangular membrane finite element, defined in an initial configuration $\Omega^0$, in which it is already under an initial stress field. A reference configuration $\Omega'$ usually considers stress-free conditions. For small strains, we assume $\Omega' \equiv \Omega^0$. The element’s current configuration is denoted by $\Omega^c$. Element nodes and edges are numbered anticlockwise, with edges facing nodes of same number. Nodal coordinates are referred to a global Cartesian system, and a local coordinate system, indicated by an upper hat, is adapted to every element configuration, such that the $\hat{x}$ axis is always aligned with edge 3, oriented from node 1 to node 2, whilst the $\hat{z}$ axis is normal to the element plane.

![Fig. 1](image_url)

Fig. 1: (a) a domain $\Omega$, discretized into elements $\Omega^e$; (b) a triangular element in three different configurations; (c) position vector $\mathbf{x}_p$ and displacement vector $\mathbf{u}_p$; (d) internal angles $\alpha, \beta, \gamma$ and unit vectors $\mathbf{v}_i$ along the edges; (e) internal nodal forces $\mathbf{p}_i$ decomposed into natural forces $N_i \mathbf{v}_i$. 
Fig. 1(c) displays the current global coordinates of $P \in \Omega$. Nodal coordinates are given by $x_i = x_i^0 + u_i$, $i = 1, 2, 3$, where $u_i$ are the element’s nodal displacements. Fig. 1(d) displays the lengths of element edges, given by $\ell_i = \|l_i\| = \|x_k - x_j\|$, with indexes $i, j, k = 1, 2, 3$ in cyclic permutation. Unit vectors parallel to the element edges are denoted by $v_i = l_i / \|l_i\|$. With these definitions, the vector of internal nodal forces can be decomposed into forces parallel to the element edges, according to

$$
p^e = \begin{bmatrix}
N_2v_2 - N_3v_3 \\
N_3v_3 - N_1v_1 \\
N_1v_1 - N_2v_2
\end{bmatrix} = \begin{bmatrix}
0 & v_2 & -v_3 \\
-v_1 & 0 & v_3 \\
v_1 & -v_2 & 0
\end{bmatrix} \begin{bmatrix}
N_1 \\
N_2 \\
N_3
\end{bmatrix} = CN
$$

where $C$ is a geometric operator, which collects the unit vectors parallel to the element edges and $N = \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix}^T$ is the vector of natural forces (see Fig. 1(d)).

We assume that the taut behavior of the element is linear-elastic, thus a linear relationship exists, such that

$$
N = k' a + N_0,
$$

where $a = \begin{bmatrix} \Delta \ell_1 & \Delta \ell_2 & \Delta \ell_3 \end{bmatrix}^T$, is the vector of natural displacements (with $\Delta \ell_i = \ell_i - \ell_i^0$, $i = 1, 2, 3$) and the element natural stiffness is a constant matrix given by

$$
k'_e = V_e L_e^{1/2} D \hat{T}^{-1} L_e^{1/2},
$$

where $V_e$ is the element volume, $L_e = \text{diag}\{\ell'_i\}$, $\hat{T}$ collects the coefficients of Hooke’s law for plane stresses, such that $\hat{\sigma} = \hat{D} \hat{\varepsilon}$ and, finally, $T_e$ is a transformation matrix, relating the linear Green strains $\hat{\varepsilon}$ to the natural strains, i.e., $\varepsilon_n = T_e \hat{\varepsilon}$, highlighting the fact that Argyris’ natural membrane element is akin to a strain gauge rosette. Explicitly we have

$$
T_e = \begin{bmatrix}
\cos^2 \gamma_e & \sin^2 \gamma_e & -\sin \gamma_e \cos \gamma_e \\
\cos^2 \beta_e & \sin^2 \beta_e & -\sin \beta_e \cos \beta_e \\
1 & 0 & 0
\end{bmatrix},
$$

where the internal angles $\beta_e$ and $\gamma_e$ are depicted in Figure 1(d). The upper or lower index ‘$e$’ indicates that computations are performed in the reference configuration, which, for small strains kinematics, can be superimposed to the initial one. A more detailed deduction of $k'_e$ is provided in references $1, 2$. Since $k'_e$ has only six independent components, its storage is usually economic, reducing the number of operations required to calculate the internal loads and tangent stiffness, and thus the overall computing time.

Inserting (7) into (6), the vector of internal forces at each configuration is given by

$$
p^e = C\left(k'_e a + N_0\right).
$$
It is interesting to define also an external wind load vector, according to

$$f_w^e = -\frac{pA}{3}[I_3 \ I_3 \ I_3]^T n^e,$$

where $p$ is a normal wind pressure acting on the element, $A$ is its area and $n^e$ its normal unit vector, in the current configuration. Thus, the element error vector becomes

$$g^e = p^e - f_w^e$$

Proceeding with derivation of (5), the consistent tangent stiffness matrix of the membrane element is obtained:

$$k^e = Ck^eC^T + \left[\begin{array}{ccc}
\left(M_2 + \frac{N}{\ell_2} M_1 \right) & -\frac{N}{\ell_5} M_3 & -\frac{N}{\ell_2} M_2 \\
-\frac{N}{\ell_5} M_3 & \left(M_1 + \frac{N}{\ell_2} M_3 \right) & -\frac{N}{\ell_5} M_1 \\
-\frac{N}{\ell_2} M_2 & -\frac{N}{\ell_5} M_1 & \left(M_1 + \frac{N}{\ell_2} M_2 \right)
\end{array}\right] + \frac{p}{6} \left[\begin{array}{ccc}
\Lambda_1 & \Lambda_2 & \Lambda_3 \\
\Lambda_1 & \Lambda_2 & \Lambda_3 \\
\Lambda_1 & \Lambda_2 & \Lambda_3
\end{array}\right],$$

where $M_i = I_3 - v_i v_i^T$, $i = 1, 2, 3$, and $\Lambda_i = \text{skew}(I_i)$ are skew-symmetric matrices, whose axial vectors are given by $I_i = \ell_i v_i$, and where each component define, respectively, the constitutive, the geometric and the external components of the element tangent stiffness matrix. The contributions of $g^e$ and $k^e$ to (4) and (5) are given according to $A^e_{i,j} = A^e_{2,j} = A^e_{3,k} = I_3$ and $A^e_{1,m} = A^e_{2,m} = A^e_{3,m} = 0$, $m \in \{1, 2, \ldots, n_i\} \setminus \{i, j, k\}$.

4 A SIMPLE WRINKLING MODEL

Matrix $k^e$ is constant, and provides a fast way to compute the element’s internal loads and tangent stiffness, when the membrane is fully under tension, avoiding explicitly calculation of strains or stresses during solution, as they can be post-processed after equilibrium is achieved.

However, before developing compressive stresses, membranes become wrinkled or slack. Criteria for identification of the status of a membrane element (taut, wrinkled or slack) require consideration of stresses, strains, or both. Therefore, when wrinkling or slackening are possible, equations (10) and (13) have to be replaced for lengthier calculations. For isotropic materials, the principal stress and strain directions are parallel, and stress, strain or mixed wrinkling criteria are equivalent [7]. We choose a pure stress criterion and calculate element stresses directly from natural displacements.

In order to further speed up calculations, we first decompose the element natural stiffness $k^e_n$ according to

$$k^e_n = (V_n \mathcal{L}^n_\tau^T) \left(\mathcal{D}^n \mathcal{L}^n_\nu^T\right) = k^e_n k^e_{\nu},$$

where $k^e_{\nu}$ is the natural stiffness of the element.
where we observe that \( \mathbf{k}_a^r \) and \( \mathbf{k}_d^r = V^{-1}_r \hat{\mathbf{D}}(\mathbf{k}_a^r)^T \) are two symmetric constant matrices that can be conveniently stored during the pre-processing phase. Then stresses in any configuration can be evaluated according to
\[
\hat{\mathbf{\sigma}} = \mathbf{k}_a^r \mathbf{a} + \hat{\mathbf{\sigma}}_0,
\]
and, after calculating the principal stresses \( \sigma_{1,2} \) and the “principal angle” \( \theta \) (between axis \( \hat{x} \) and the \( \sigma_1 \) direction), we determine the element status and eventually modify stresses according to the following criterion:
\[
\begin{align*}
\sigma_2 > 0 & \Rightarrow \text{TAUT} & \Rightarrow \hat{\mathbf{\sigma}} = \hat{\mathbf{\sigma}}_0 \\
\sigma_1 > 0 \land \sigma_2 \leq 0 & \Rightarrow \text{WRINKLED} & \Rightarrow \hat{\mathbf{\sigma}} = \frac{\sigma_1}{2} \begin{bmatrix}
(1+\cos \theta_1) & (1-\cos \theta_1) \\
\sin (2\theta_1)
\end{bmatrix}^T \\
\sigma_1 \leq 0 & \Rightarrow \text{SLACK} & \Rightarrow \hat{\mathbf{\sigma}} = \mathbf{0}
\end{align*}
\]

Thereafter, we replace (10) by (17):
\[
\mathbf{p}^e = \mathbf{Ck}_a^r \hat{\mathbf{\sigma}}.
\]
where column \( \mathbf{k}_j = \frac{\partial \mathbf{g}}{\partial x_j} \) can be interpreted as the directional derivative of \( \mathbf{g} \) with respect to the \( j^{th} \) component of \( \mathbf{x} \). Thereafter, we approximate these \( n_{dof}^e \) directional derivatives \( \mathbf{k}_j^e \) by a central-difference scheme, according to

\[
\mathbf{k}_j^e = \frac{1}{2h} \left[ \mathbf{g}^e \left( \mathbf{x}^e + h \delta_j^e \right) - \mathbf{g} \left( \mathbf{x}^e - h \delta_j^e \right) \right], \quad j = 1, \ldots, n_{dof}^e,
\]

(19)

where \( h \) is a finite scalar parameter and \( \pm h \delta_j^e \) are backward and forward perturbations of the \( j^{th} \) element degree of freedom, such that \( \delta_j^e = [\delta_y] \), \( i = 1, \ldots, n_{dof}^e \), and where \( \delta_y \) is the Kronecker delta. Finally, inserting approximations (19) into (18), we obtain numerical estimates for the tangent stiffness matrices \( \mathbf{k}^e \), which are then assembled into a global a stiffness matrix \( \mathbf{K} \). In references\(^{5,6} \) we have shown that this method provides excellent approximations for the consistent tangent stiffness matrix, as long as \( h \) is small enough. Moreover, since no extra cost is associated to reducing the size of \( h \), it can be taken as a function of the machine precision \( \epsilon \), as small as possible, but without introducing numerical noise. In MATLAB environment, where SATS was implemented, we found that \( h = \frac{4}{\sqrt{\epsilon}} \) is a good compromise estmative.

6 A SECANT STIFFNESS MATRIX

We have also investigated the use of a secant approximation to the stiffness matrix, by which instead of performing the consistent linearization of equation (17), we simply modify the elements’ natural stiffness, equation (14), according to a modified elasticity matrix, borrowed from the paper by Akita et al.\(^3 \). After decomposing the total strains on a membrane element into elastic and wrinkled fractions, according to \( \hat{\mathbf{e}} = \hat{\mathbf{e}}_e + \hat{\mathbf{e}}_w \), these authors finally arrive to

\[
\hat{\mathbf{e}}_w = \frac{s_1s_2^T}{s_2^T D s_2} = Q\hat{\mathbf{e}},
\]

(20)

where \( Q \) is a projection matrix, that extracts the wrinkled portion of the deformation from the total one. Vectors \( s_1 \) and \( s_2 \) are such that \( \mathbf{e} = \mathbf{e}_1 \cdot s_1 + \mathbf{e}_2 \cdot s_2 \), with \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) being the element’s principal strains (never actually calculated, in our method). Explicitly,

\[
\begin{align*}
\mathbf{s}_1 &= \frac{1}{2} \begin{bmatrix} (1 + \cos 2\theta_1) & (1 - \cos 2\theta_1) & 2 \sin 2\theta_1 \end{bmatrix}^T \\
\mathbf{s}_2 &= \frac{1}{2} \begin{bmatrix} (1 - \cos 2\theta_1) & (1 + \cos 2\theta_1) & -2 \sin 2\theta_1 \end{bmatrix}^T
\end{align*}
\]

(21)

Now since \( \hat{\mathbf{e}}_w \) does not rise stresses, \( \mathbf{\tilde{\sigma}} = \mathbf{\tilde{D}} \hat{\mathbf{e}}_e = \mathbf{\tilde{D}} (\mathbf{\mathbf{e}} - \mathbf{\mathbf{e}}_w) = \mathbf{\tilde{D}} (1 - \mathbf{Q}) \mathbf{e} = \mathbf{\tilde{D}} \hat{\mathbf{e}}_w \), where

\[
\mathbf{\tilde{D}} = (1 - \mathbf{Q}) \mathbf{\tilde{D}}
\]

(22)

is the modified elasticity matrix we seek. Inserting (22) into (14) and that into (13), we finally obtain a modified, secant stiffness matrix.
\[ \hat{\mathbf{K}}^e = \hat{\mathbf{K}}_c + \mathbf{k}_g + \mathbf{k}_{\text{ext}}, \tag{23} \]
in which only the constitutive part \( \hat{\mathbf{K}}_c \) is approximate. In principle, this non-consistent stiffness matrix may slow down convergence rates, but its use in reference\(^7\) allowed easy solution, as shown in the following benchmark.

7 A FIRST BENCHMARK – THE ‘MEMORIAL DOS POVOS’

In references\(^4,7\) we presented results obtained by the SATS program\(^2\) for the membrane roof of the “Memorial dos Povos de Belém do Pará”, shown Figure 2, under a uniform upward wind load acting over the whole membrane surface, considering both fully-adherent and frictionless sliding border cables. A discretization much coarser than the one used in actual design was adopted, to ease the visualization of results. In the present paper, we consider only results obtained for the fully-adherent model, which can be compared also with results given by the Ansys FEM code (a sliding cable is not directly available in Ansys, thus the sliding condition was not analyzed by that program).

![Figure 2: The membrane roof of the “Memorial dos Povos de Belém do Pará”](image)

<table>
<thead>
<tr>
<th>Program</th>
<th>SATS</th>
<th>Ansys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>Newton (tangent ( \mathbf{K} ))</td>
<td>Newton (secant ( \mathbf{K} ))</td>
</tr>
<tr>
<td>Maximum displacement [m]</td>
<td>0.40403</td>
<td>0.40403</td>
</tr>
<tr>
<td>Maximum ( \sigma_1 ) [MPa]</td>
<td>13.60979</td>
<td>13.60974</td>
</tr>
<tr>
<td>Minimum ( \sigma_1 ) [MPa]</td>
<td>5.47444</td>
<td>5.47442</td>
</tr>
<tr>
<td>Maximum ( \sigma_2 ) [MPa]</td>
<td>5.37598</td>
<td>5.37598</td>
</tr>
<tr>
<td>Minimum ( \sigma_2 ) [MPa]</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Time to solution [s]</td>
<td>4.586</td>
<td>4.353</td>
</tr>
<tr>
<td>Time per iteration [s]</td>
<td>0.9172</td>
<td>0.6218</td>
</tr>
</tbody>
</table>

We also compare results obtained by SATS, through Newton’s iterations, using both the tangent or the secant stiffness matrices (\( \mathbf{k}' \) or \( \hat{\mathbf{k}}' \)), with those obtained through dynamic relaxation\(^4\), a method which only requires the definition of the modified error vector \( \hat{\mathbf{g}} \), yielding an independent checking for results, besides Ansys model.
Figure 3: Results by SATS (both Newton’s iterations dynamic relaxation) compared to results by Ansys; Top to bottom: displacement norms; $\sigma_1$ on elements; $\sigma_2$ on elements, wrinkled elements shown in grey.

Converged results obtained considering these four different methods were all in good agreement, as can be seen in Table 1, which compares some selected results obtained with SATS and Ansys models. In the Ansys model, the wrinkled elements detected in SATS presented very low but still positive 2nd principal stresses, possibly due to the post-processing extrapolations adopted by that program. Figure 3 compares displacement’s norms and principal stresses on elements, obtained using SATS (differences of results through different methods are visually imperceptible, so figures are not repeated) and Ansys. In the plotting generated by SATS, principal directions are shown with lines of length proportional to the
stress magnitude. The wrinkled elements coincide for any alternative solution, and are displayed in grey, on the $\sigma_2$ plotting.

As expected, the use of the secant stiffness matrix $\hat{K}$ requires more Newton iterations for convergence, but each iteration is about 30% faster than the number-crunching finite difference procedure used to numerically calculate the tangent stiffness $K$. We also remark that, being a research program, SATS is implemented in MATLAB interpreted environment, and a lot of redundant calculations is performed, for the sake of readability, so the program is by no means optimized for speed, being much slower than Ansys, a compiled program.

8 AN INSUFFLATE DOME REINFORCED BY CABLES

Although results presented in the previous benchmark suggest that good results for problems of membrane wrinkling may be obtained using either the secant stiffness matrices $\hat{K}$ or the tangent stiffness matrices $K$, in fact only a few elements of the considered membrane did present a wrinkled status. Besides, models with a small number of DOFs can be deceiving, in non-linear analysis.

In order to assess the performance of the alternative methods proposed above, in a large model, where wrinkling is more widespread, we consider as second application the large cable-reinforced, pneumatic envelope shown in Figure 4. It was designed to cover the site of a new nuclear power plant, during the process of ground preparation, remaining on site for only 6 months. The structure had a roughly rectangular plant, 110m x 86m, 30 meters high, and was anchored to a perimeter concrete wall. The membrane was reinforced by seven cables laid over the membrane, transversally constrained by fabric straps, but otherwise capable of sliding. A relatively small internal pressure $p_0 = 100N/m^2$ was specified, and due to the short lifetime, a reduced basic wind pressure $q = 245N/m^2$ was estimated. Figure 4(b) shows the membrane equilibrium configuration under internal pressure, adopted as initial configuration for wind analyses. In the present paper, we restrict our interest to comparing the convergence rates obtained considering $\hat{K}$ or $K$, and only for the case of the envelope constrained by adherent cables, under a transversal wind load, for which the wind pressure coefficients are given in Figure 4(c). We direct the reader to reference 8 for a detailed description of this system, as well as an explanation on the influence of cable sliding on its behavior.

![Figure 4](image-url)

Figure 4 – (a) A large pneumatic dome reinforced by sliding cables; (b) equilibrium geometry under internal pressure; (c) wind pressure coefficients.
Table 2 shows some selected results for two load cases: “internal pressure” and “internal pressure + transversal wind”, both with secant and tangent stiffness matrices. Figure 5 shows displacement norms, $\sigma_1$ and $\sigma_2$ stress fields for these two load cases. Differences between the alternative secant or tangent stiffness matrices are visually imperceptible, so results are not repeated. From top to bottom, Fig. 5 shows displacement norms, $\sigma_1$ and $\sigma_2$ stress fields. A large wrinkled zone is observed in the case of transversal wind loads (right column), at the wind side.
Table 2. Comparison of results for the pneumatic envelope, for two load cases ($\|\mathbf{g}\|/\|\mathbf{g}_0\| \leq 10^{-4}$)

<table>
<thead>
<tr>
<th>Load case</th>
<th>&quot;Internal pressure&quot;</th>
<th>&quot;Internal pressure + transversal wind&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness matrix</td>
<td>Secant</td>
<td>Tangent</td>
</tr>
<tr>
<td>Maximum displacement [m]</td>
<td>0.11894</td>
<td>0.11894</td>
</tr>
<tr>
<td>Maximum $\sigma_1$ [MPa]</td>
<td>5.46716</td>
<td>5.46718</td>
</tr>
<tr>
<td>Minimum $\sigma_1$ [MPa]</td>
<td>0.90308</td>
<td>0.90308</td>
</tr>
<tr>
<td>Maximum $\sigma_2$ [MPa]</td>
<td>1.43734</td>
<td>1.43733</td>
</tr>
<tr>
<td>Minimum $\sigma_2$ [MPa]</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Total time [s]</td>
<td>143.072</td>
<td>129.074</td>
</tr>
<tr>
<td>Time per iteration [s]</td>
<td>15.90</td>
<td>21.51</td>
</tr>
</tbody>
</table>

9 PROVISIONAL CONCLUSIONS

It was seen that both secant and tangent stiffness matrices are able to cope with the problem of membrane wrinkling, under the simplified assumptions described above. When wrinkling is not widespread along the membrane, both secant and tangent matrices provide good convergence rates. On the other hand, when wrinkling is widespread, the secant stiffness renders convergence more difficult. Iteration cost also becomes larger for the secant stiffness, whilst it is practically invariant when the tangent stiffness is evaluated by finite differences. Therefore, although implementation of the secant matrix requires little extra computation when a linear-elastic material is already available, the finite difference approximation provides a systematic, straightforward way to keep quadratic convergence, requiring the sole definition of force vectors. At this point of research, this last is our preferable method.

REFERENCES

The eXtended Updated Reference Strategy for the form finding of tensile structures

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Key words: Tensile structures, form finding, updated reference strategy,

Summary. In this paper, the eXtended Updated Reference Strategy is presented. Starting from the established Updated Reference Strategy all related issues, which are involved for this methodology, are identified. It will be shown that the eXtended Updated Reference Strategy is able to solve the “correct” form finding problem in one non-linear iteration step. By applying the eXtended Updated Reference Strategy to well-known form finding problems the difference in convergence in comparison to established methods like the force density method or the Updated Reference Strategy is discussed.

1 INTRODUCTION

Tensile Structures are lightweight structures, which combine an optimal stress state of the material with an impressive language of shapes [1]. The shape of tensile structures is defined by the equilibrium of surface stress and cable edge forces in tension. Throughout the whole design process of tensile structures the variation of prestress constitutes the main shaping parameter. Due to this direct interaction of shape and prestress, the shape of a tensile structure cannot be set like for conventional structures (e.g. concrete bridges, wooden slaps, steel frames, etc.). The step of form finding is always necessary in order to find the final shape.

The first solutions for the task of form finding were made by using soap film and hanging models. From this approach some of the most challenging tensile structures were developed. Certainly, Frei Otto is one of the most important pioneers using physical models to solve the problem of form finding [2]. Nowadays the effort in research is mainly focused on the development of appropriate numerical methods for the form finding of tensile structures. This evolution is in large paths based on the huge impact of the introduction of Finite Element Methods (FEM) in engineering and the constantly growing computation capacities are the basis of this development. The starting point for the development of numerical methods for form finding of tensile structures is the work of Klaus Linkwitz with the well-known Force Density method (FD) [3].

In the following sections, starting from the correct continuum mechanical description of the form finding problem, the numerical issues in the solution process will be discussed. A method, which was introduced by [4] in order to solve the form finding of tensile structures,
will be presented. Starting from this an extension will be derived which improves the convergence behaviour as well as the usability.

2 FORMFINDING OF TENSILE STRUCTURES

From a mathematical point of view the form finding of tensile structure is closely related to the well-known task of the determination of minimal surfaces. The connection between the pure geometrical and the mechanical model is the overall prestress in the surface, as minimal surfaces are characterized by an isotropic stress distribution. For centuries mathematicians have investigated research in the solution of minimal surfaces for different cases of boundary conditions [5]. Certainly, the experimental work of Joseph Plateau in the 19th Century was one of the most important contributions to this research.

From a mechanical point of view the form finding of tensile structures is the task to find the shape of equilibrium w.r.t. a given surface stress state $\sigma$ and natural (in terms of edge forces) or geometrical (e.g. clamped edges) boundary conditions. Additional loading, as e.g. internal pressure (cushions), has to be considered, too. Considering the non-linear kinematics of large deflections the equilibrium condition in the deformed, actual configuration is defined by the principle of virtual work. See Eq. (1):

$$\delta w = t \int_\alpha \sigma : \delta e da - \int_\alpha p \delta u da = 0$$  \hspace{1cm} (1)

The total virtual work $\delta w$ consists of the internal work given by the Cauchy stresses $\sigma$, the virtual Euler-Almansi strains $\delta e$, the thickness of the membrane $t$ which is assumed to be thin and constant throughout the form finding process and the external work given by the external loading $p$ and the virtual displacements $\delta u$. Due the formulation of the equilibrium in the current configuration, the integration is carried out over the current domain $da$. In the following the external loading will be neglected. The discussion of the influence of the external load onto the governing equations is presented in [4] and holds for all of the derived equations in the later sections.

The equilibrium condition in Eq. (1) w.r.t the current configuration can also be transferred into the reference configuration by applying Nanson’s relation which is given in Eq. (2).

$$a = \int_\alpha da = \int_A \det F dA$$  \hspace{1cm} (2)

In Eq. (2) $\det F$ represents the determinant of the deformation gradient $F$ which connects the reference configuration to the current configuration. Inserting Eq. (2) in Eq. (1) leads to the equilibrium condition w.r.t. the reference configuration in Eq. (3).

$$\delta w = t \int_A \det F \sigma : \delta e dA = 0$$  \hspace{1cm} (3)

By applying the relation between the virtual Euler-Almansi strains $\delta e$ and the Green-Langrange strains $\delta E$ which is given in Eq. (4), the equilibrium can be written by values
which are all defined w.r.t. the reference configuration.

\[ \delta e = F^{-T} \delta \varepsilon F^{-1} \] (4)

Finally, after some rearrangements the equilibrium condition can be written as in Eq. (5).

\[ \delta \nu = t \int_{A} \det F (\sigma F^{-T} \cdot \delta \varepsilon F) dA = 0 \] (5)

For the special case of minimal surfaces the prestress state can be expressed as a constant value \( s \) and the Identity Tensor \( I \) since it represents an isotropic stress distribution in the surface. See Eq. (6):

\[ \sigma = s I \] (6)

With Eq. (6) the equilibrium condition can be reformulated from Eq. (5) to the expression which is given in Eq. (7).

\[ \delta \nu = s t \int_{A} \det F \cdot \delta \sigma dA = 0 \] (7)

The derived equation up to this point is totally derived from continuum mechanics. In the following it will be shown that Eq. (7) describes a minimal surface in a mathematically correct way. From a mathematical point of view minimal surfaces are defined as surfaces of minimal area content between given boundaries. The minimum of area content can be derived by the vanishing variation \( \delta a \) of the area content \( a \). See Eq. (8):

\[ \int_{A} a \delta a = 0 \] (8)

Again, using Nanson’s relation (c.f. Eq. (2)) the variation of the area content can be formulated as given in Eq. (9).

\[ \int_{A} \delta a dA = 0 \] (9)

Herein, the variation of the determinant of the deformation gradient \( \det F \) has to be formulated. This variation can be derived as shown in Eq. (10).

\[ \delta (\det F) = \det F \cdot \delta F \] (10)

Inserting Eq. (10) in Eq. (9) the variation of the area content can be formulated as given in Eq. (11).

\[ \int_{A} \det F \cdot \delta F dA = 0 \] (11)

Obviously, by applying an isotropic stress field to a tensile structure the equation which is derived from continuum mechanics is identical to the equation that is derived from a mathematical point of view. Both approaches are able to describe the task of finding a minimal surface.
In order to solve the problem stated in Eq. (1), standard numerical methods (e.g. FEM) can be used. In this context a discretization of the governing equation has to be done, in order to reduce the number of unknowns to a finite number. Furthermore, a geometrical nonlinear analysis is necessary due to the fact that the given problem includes large displacements.

Trying to solve the given problem from Eq. (1) it turns out that the system matrix to evaluate the unknown discretization parameters is singular. The reason for this deficiency originates from the inverse character of the given problem where, stresses in the deformed configuration are given without considering material properties. This inverse character can be understood in comparison to standard structural analysis, where based on a defined reference configuration the deformation w.r.t. a certain load situation is computed. Therefor the stresses can be evaluated from displacements by applying the material law. In contrast to that, form finding already knows the stress and tries to determine the deformed geometry. Due to the prescribed stresses this can be done without defining any material property (see Fig. 1).

On important feature is that surface stresses and strains are not related. As a consequence it turns out, that the position of the nodes on the surface cannot be evaluated uniquely, since it is possible to describe the same surface with differently shaped finite elements: The nodes can float freely on the surface. Hence, the fact, that the same surface can be described by an infinite number of discretizations leads to the singular system matrix (see Fig. 2).
To eliminate this singularity various methods have been developed in the past, like e.g. the dynamic relaxation [6], [7] or the force density [3], [8]. All of them try to stabilize the singular system matrix by different kinds of approaches. In the following, a further, most general method is presented which is consistently derived from continuum mechanics.

3 UPDATED REFERENCE STRATEGY (URS)

The updated reference strategy (URS) uses general mathematical methods to stabilize the singular problem given in the previous section [4], [9]-[10]. The idea is to modify the original, singular problem by a related one which fades out as we approach the solution. Therefore Eq. (1) will be expanded by an additional term which describes an alternative formulation of the internal virtual work. See Eq. (12):

\[ \delta w_{\text{URS}} = \lambda \int_{a} \sigma : \delta \varepsilon da + (1 - \lambda) \int_{a} S : \delta \varepsilon da - \int_{a} \mathbf{p} \delta \varepsilon da = 0 \]  

(12)

The first part of Eq. (12) is the original problem from Eq. (1). The second part represents the stabilization in terms of an added similar problem. The last part again represents the external virtual work. The stabilization term formulates the equilibrium condition w.r.t. the reference configuration where the true surface Cauchy stresses \( \sigma \) are replaced by \( S \), the 2nd Piola-Kirchhoff stresses. As they are artificially related to tangential deformation of the mesh a formulation using \( S \) does not suffer from singularity but the solution deviates from the intended one. On the other hand, if there is no deformation \( S \) and \( \sigma \) are identical. Obviously, the homotopy factor \( \lambda \) controls the solvability of the whole problem. For the choice \( \lambda = 1 \) only the original problem will be considered and for \( \lambda = 0 \) the pure stabilization term is solved. It is guaranteed that the system of equations is solvable as long as \( \lambda \) is small enough to stabilize the whole problem.

The biggest advantage of the URS is that the stabilization term becomes more and more alike the original problem as the reference configuration gets closer to the final shape. Hence, a further improvement of the method can be done, by using the solution of Eq. (12) with any arbitrary choice of \( \lambda \) as a new improved reference configuration for the next approximation. This means by solving Eq. (12) the reference configuration will be iteratively updated towards the optimal solution. The principal sequence of a form finding by applying the URS is shown in Figure 3.
Starting with an almost arbitrary reference configuration the 2\textsuperscript{nd} Piola-Kirchhoff stresses $S$ are assumed to be equal to the Cauchy stresses $\sigma$. Herein, the chosen reference configuration has to fulfill the boundary conditions. After solving the governing equation from Eq. (12) the resulting displacements are added to the reference configuration and set this state (= actual configuration) as the new reference configuration. The method will be repeated until the occurring displacements will converge to be small enough.

As the method is totally dependent on the choice of $\lambda$, the speed of convergence and the solvability is directly connected to this choice. In the following a new extension to the URS is presented which cancels out the drawback of choosing a value for $\lambda$ in each form finding step by maximum possible convergence speed.

**4 EXTENDED UPDATED REFERENCE STRATEGY (X-URS)**

The idea of the \textit{eXtended Updated Reference Strategy} is to modify the principle of virtual work in a way, that still the original problem is solved the singularity in the tangential direction are neglected. In the following, the terms which cause the singularity in the governing equation will be referred to as singular terms. To identify the singular terms the principle of virtual work which is given in Eq. (12) has to be stated in the linearized form. In Eq. (13) the residual form is shown. Herein the virtual work is linearized w.r.t. the virtual displacements $\delta b$, which are points in the direction of the discretization parameters. For the sake of simplicity the external load is neglected in Eq. (13). The discussion of the influence of the external load onto the governing equations is presented in [4].
\[
\frac{\partial w_{	ext{USI}}}{\partial b_r} \partial b_r = R_r \partial b_r = \left\{ \lambda \tau \int_{a} \sigma : \frac{\partial \epsilon}{\partial b_r} \, da + \left(1 - \lambda \right) \int_{A} S : \frac{\partial E}{\partial b_r} \, dA \right\} \partial b_r = 0
\] (13)

Again the residual forces for the respective parts of the original problem and the stabilization can be identified. \(R_{r,\sigma}\) represents the residual force of the original problem in the direction of the discretization parameter \(b_r\). The residual force of the stabilization term is given in \(R_{r,S}\).

By investigating the forces w.r.t. to their influence on the singularity of the final stiffness matrix in the numerical solution process it can be identified that the singularity originates from the original term (which was already discussed in section 2). More precisely the singular term is related to the derivative of the residual force which points in the direction tangential to the surface. Figure 4 illustrates the different parts of the residual forces: \(R^n\) acts along the surface normal \(n\) and \(R^t\) along the tangential direction \(t\).

Figure 4: Separation of the residual force

To perform the separation of the forces in normal and tangential direction basic mathematic definitions can be used. Eq. (14) and Eq. (15) show the residual force in normal and tangential direction.

\[
R^n = (n \otimes n)R
\] (14)

\[
R^t = R - R^n = (I - n \otimes n)R
\] (15)

The summation of the normal and tangential part of the residual force leads again to Eq. (13), but now the possibility to identify the singular terms is given. In Eq. (16) the original and the stabilization problem are shown by separating them to the normal and tangent direction.
In Eq. (16) the singular term is fully related to the tangential residual force. The stabilization \( R^s \) ensures the solvability in the tangential direction. Due that it would be sufficient if the stabilization just affects the tangential residual force of the original problem. From Eq. (16) it can be seen that the stabilization also affects the residual force in normal direction. The splitted residual form know offers the opportunity to just take into account the terms of the residual force which are needed in order to solve the form finding problem. Obviously, the residual force of the original problem in the normal direction \( R^n \) is needed to find the shape of equilibrium of the tensile structure. To stabilize the form finding problem the residual force of the stabilization term in tangential direction \( R^s \) is needed, too. The other two terms (\( R^t \) and \( R^n \)) can be neglected as they are not of importance for the description of the form finding problem. Due to the absence of a singular term in the governing equation the homotopy blending is not necessary anymore and the given non-linear problem can be solved directly. Finally the governing equation of the extended Updated Reference Strategy can be given in Eq. (17).

\[
R = \lambda \left( R^n + R^t \right) + (1 - \lambda) \left( R^s + R^n \right) = 0
\]  

(16)

As the governing equation of the X-URS solves the original problem without any compromises in the normal direction of the surface, the solution after the first form finding step is identical to the analytical one. Additionally, the solution in normal direction might also be influenced by the deformation in the tangential direction (e.g. edge cables).

In Figure 5 the method and the convergence behavior is discussed for the Schwarz minimal surface. The surface is discretized with 4 finite elements, which results in 3 global degrees of freedom at the middle node. The reference configuration satisfies the boundary condition w.r.t. the straight edges. Obviously, the middle node just has to move in the vertical (here z-) direction. Due to that in the following the residual force \( R_z \) will be investigated.
It is obvious that the final position of the middle node has to be at the half of the height of the surface (here 5). From a mechanical point of view, a “correct” residual equation should have a zero value of the residual forces for a displacement of 5 in the vertical direction. In the following the residual forces for the force density method (FD), for the URS and the X-URS are compared in the first form finding step. In Figure 6 the residual forces for the named methods are plotted. In case of the URS two different solutions are plotted for different choices of the homotopy factor $\lambda$.

Figure 5: Schwarz minimal surface reference configuration; Top view (left); Iso view (right)

Figure 6: Residual forces for the Schwarz minimal surface for different form finding methods. Convergence is archived for displacement of 5 (red line).
From Figure 6 it can be seen that only the X-URS converges to the final solution within the first form finding step. The force density method shows the worst convergence within the first form finding step, while the URS convergences to the final solution with increasing homotopy factors $\lambda$. It has to be stated at this point that the non-linearity increases from force density to X-URS. When applying the force density method to a form finding problem in each form finding step just a linear system has to be solved. This advantage has the price of an increasing amount of form finding steps. In contrast the X-URS leads to a non-linear problem in each form finding step but with a decreased number (in the best case just one) of form finding steps. In the following, examples for the successful application of the X-URS are presented.

5 EXAMPLES

In this section two examples are shown. For both examples the comparison to the force density and URS are investigated.

5.1 Catenoid

The first example is the well-known Catenoid minimal surface. The observed displacement is at the half of the height (point M) of the height of the surface. The reference configuration was a cylinder. Due to that the analytical solution based on a catenary curve can be determined (see Fig. 7)

![Catenoid](image1)

Figure 7: Catenoid (top left) [11]; FEM discretization (top right); Convergence graphs for FD, URS and X-URS for the displacement at the half of the height (distance b) (below)
5.2 Four Point Tent

The second example shows the well-known four point tent. Starting from an arbitrary reference configuration which satisfies the boundary conditions (in terms of high and low points) the final shape of the surface is determined. Again, the comparison to the force density method as well as to the URS is plotted (see Fig. 8).

Figure 8: Initial reference configuration (top left); Converged state (top right); Convergence graphs for FD, URS and X-URS for the vertical displacement in the center of the surface (below)
6 CONCLUDING REMARKS

In the latter sections a new method for form finding of tensile structures has been presented. The \textit{eXtended Updated Reference Strategy} describes the form finding problem in a non-linear residual equation. When solving this equation the final equilibrium shape is achieved. The additional introduction of pseudo time step iteration (form finding steps) is not necessary. The capacity of the method is illustrated on two well-known examples (Catenoid and Four Point Tent). For both of them, the convergence behavior of the respective displacements demonstrates the advantage of the method w.r.t. to nowadays established methods. The final shapes are achieved for both cases within the first form finding step. The improvement of the tangential stabilization w.r.t. the influence on the overall solution should be the topic of further discussions on the method.

REFERENCES

USING NURBS AS RESPONSE SURFACE FOR MEMBRANE MATERIAL BEHAVIOR

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Key words: inflated membranes, membrane materials, finite element method, NURBS surfaces

Abstract. This work proposes a NURBS for the determination of a smooth response surface relating biaxial strains and stresses from discrete test data. These NURBS surfaces are based on two axes of strain and one axis of stress. The constitutive material tensor is calculated with the derivatives of the NURBS surfaces and curves. The motivation of the proposed work came from the use of new materials in membrane structures that requires complex material models to describe the complex material behavior. A method for the establishment of a matrix of material coefficients from these surfaces is developed aiming its application in finite element models. The response surface stress-strain relation and the material matrix derived are compared to classical hyperelastic and Mooney-Rivlin material models. The response surface approach using NURBS allows for an easy implementation in an existent FE code, requiring few changes. A similar application is found in the work of Bridgens and Gosling [1]. This approach provides a direct correlation between stresses and strains in the wide range of possible stress paths the material is subject to. Curve fitting based on least squares approximation is employed to generate NURBS surfaces for the experimental data. The advantage of this material model is that a smooth stress-strain response surface can be obtained directly from the experimental results. On the other hand, in order to generate good NURBS surfaces the experimental data should provide an adequate point distribution. This could require a large range of experimental data. We conclude that this material model is a good alternative to conventional material models for complex material behavior.
1 INTRODUCTION

Non Uniform Rational Basis Splines (NURBS) is a mathematical representation of a geometry in 3D used for curves and surfaces. This representation is widely used in Computer Aided Design (CAD) to create and modify designs offering smooth surfaces. Due to the success of the use of NURBS in CAD, it has been suggested in other applications. An example of this is the isogeometric analysis introduced by Hughes et al. [4], to solve problems governed by partial differential equations such as structures and fluids. Another application of NURBS in numerical analysis is the NURBS-enhanced finite element method (NEFEM). Sevilla et al. [8] reports that the NEFEM uses NURBS to accurately describe the boundary of the computational domain. The NURBS application proposed in this work aims the determination of a smooth response surface relating biaxial strains and stresses. These NURBS surfaces are based on two axes of strain and one axis of stress. The constitutive material tensor is calculated with the derivatives from the NURBS surfaces and curves.

A similar application is found in the work of Bridgens and Gosling [1]. In Bridgens and Gosling [1] Bezier functions, B-spline and NURBS are used to represent the bi-axial behavior of coated woven fabrics. The validity of the approach is assessed through an extensive fabrics testing program. This approach provides a direct correlation between stresses and strains in the wide range of possible stress paths the material is subject to. As pointed out in Bridgens and Gosling [1] this representation has the additional ability to represent surfaces with rapid changes in gradients and discontinuities in the data. Also, the plane stress constraint, frequently used by the analysis of films and membrane structures is not explicitly imposed. Curve fitting based on least squares approximation is employed to generate NURBS surfaces for the experimental data. The response surface methodology based on NURBS is tested on classical hyperelastic and Mooney-Rivlin constitutive models. A set of results of an aluminum testing program were used to illustrate the response surface construction procedure from test results. Aiming the application of this methodology together with a finite element non-linear analysis program for the investigation of global structure behavior the derivation from the NURBS surface of a constitutive matrix is developed. Most general purpose FE programs provide user access to add new functionalities such as user constitutive models. The response surface approach using NURBS presented in this work allows for an easy implementation in such programs.

2 NONUNIFORM RATIONAL B-SPLINE CURVES AND SURFACES

The concept of NURBS curve and NURBS surface used in the present study refers to the works of Piegl and Tiller [6] and L. Piegl [5].

The definition of NURBS curve/surface is the rational generalization of the tensor-product nonrational B-spline curve/surface. According to Rogers [7], technically, a NURBS surface is a special case of a general rational B-spline surface that uses a particular form
of knot vector. For a NURBS surface, the knot vector has multiplicity of duplicate knot values equal to the order of the basis function at the ends. The knot vector may or may not have uniform internal knot values.

3 MATERIAL MODEL BASED ON NURBS FOR PRINCIPAL DIRECTIONS (PD–NURBS)

The proposed material model covers isotropic nonlinear materials under plane stress conditions. This model is based on principal directions of stress and strain. Therefore only one surface is required for its definition.

PD–NURBS is valid for isotropic materials because of the use of orthogonal transformation to calculate the response of the stress. According to Gruttmann and Taylor [3], for isotropic material response the contravariant components of the second Piola–Kirchhoff stress tensor are recovered by an orthogonal transformation of the principal stresses.

The second Piola–Kirchhoff stresses and the Green–Lagrange strains in principal directions are given by:

\[
\hat{S} = \begin{bmatrix} S_1 & S_2 & \hat{S}_{12} \end{bmatrix} \tag{1}
\]

\[
\hat{E} = \begin{bmatrix} E_1 & E_2 & \hat{E}_{12} \end{bmatrix} \tag{2}
\]

where \( \hat{S}_{12} = 0 \) and \( \hat{E}_{12} = 0 \).

The constitutive material tensor in general directions is obtained with the rotation matrix calculated as follows:

\[
d\hat{S} = \left[ \begin{array}{ccc} \frac{dS_1}{dE_1} & \frac{dS_1}{dE_2} & \frac{dS_1}{dE_{12}} \\ \frac{dS_2}{dE_1} & \frac{dS_2}{dE_2} & \frac{dS_2}{dE_{12}} \\ \frac{dS_{12}}{dE_1} & \frac{dS_{12}}{dE_2} & \frac{dS_{12}}{dE_{12}} \end{array} \right] = \mathbf{T}^T \cdot \frac{d\hat{S}}{d\hat{E}} \cdot \mathbf{T} \tag{3}
\]

where \( \frac{d\hat{S}}{d\hat{E}} \) is the constitutive material tensor in principal directions

\[
d\hat{S} = \left[ \begin{array}{ccc} \frac{dS_1}{dE_1} & \frac{dS_1}{dE_2} & 0 \\ \frac{dS_2}{dE_1} & \frac{dS_2}{dE_2} & 0 \\ 0 & 0 & \frac{dS_{12}}{dE_{12}} \end{array} \right] \tag{4}
\]

and derivatives of the NURBS surface for \( S_1 \) in directions \( u \) and \( v \) are given by

\[
S_{u_1}^{NURBS}(u, v) = \left[ \begin{array}{c} \frac{dE_1}{du} \\ \frac{dE_2}{du} \\ \frac{dS_1}{du} \end{array} \right] \tag{5}
\]

\[
S_{v_1}^{NURBS}(u, v) = \left[ \begin{array}{c} \frac{dE_1}{dv} \\ \frac{dE_2}{dv} \\ \frac{dS_1}{dv} \end{array} \right] \tag{6}
\]
and analogously for the derivatives of the NURBS surface for \( S_2 \) in directions \( u \) and \( v \).

\[
S_{u_2}^{\text{NURBS}}(u, v) = \begin{bmatrix}
\frac{dE_1}{du} & \frac{dE_2}{du} & \frac{dS_2}{du}
\end{bmatrix}
\]

\[
S_{v_2}^{\text{NURBS}}(u, v) = \begin{bmatrix}
\frac{dE_1}{dv} & \frac{dE_2}{dv} & \frac{dS_2}{dv}
\end{bmatrix}
\]

(7)

(8)

and the rotation matrix \( T \) is given by:

\[
T = \begin{bmatrix}
\cos^2 \phi & \sin^2 \phi & \cos \phi \sin \phi \\
\sin^2 \phi & \cos^2 \phi & -\cos \phi \sin \phi \\
-2 \cos \phi \sin \phi & 2 \cos \phi \sin \phi & \cos^2 \phi - \sin^2 \phi
\end{bmatrix}
\]

(9)

The constitutive material tensor in principal directions is computed with the NURBS surface derivatives:

\[
\begin{bmatrix}
\frac{dS_1}{dE_1} \\
\frac{dS_1}{dE_2} \\
\frac{dS_2}{dE_1} \\
\frac{dS_2}{dE_2}
\end{bmatrix} = \left( \begin{bmatrix}
\frac{dE_1}{du} & \frac{dE_2}{du} & \frac{dE_1}{dv} & \frac{dE_2}{dv}
\end{bmatrix} \right)^{-T} \cdot \begin{bmatrix}
\frac{dS_1}{du} \\
\frac{dS_1}{dv} \\
\frac{dS_2}{du} \\
\frac{dS_2}{dv}
\end{bmatrix}
\]

(10)

\[
\begin{bmatrix}
\frac{dS_1}{dE_1} \\
\frac{dS_1}{dE_2} \\
\frac{dS_2}{dE_1} \\
\frac{dS_2}{dE_2}
\end{bmatrix} = \left( \begin{bmatrix}
\frac{dE_1}{du} & \frac{dE_2}{du} & \frac{dE_1}{dv} & \frac{dE_2}{dv}
\end{bmatrix} \right)^{-T} \cdot \begin{bmatrix}
\frac{dS_1}{du} \\
\frac{dS_1}{dv} \\
\frac{dS_2}{du} \\
\frac{dS_2}{dv}
\end{bmatrix}
\]

(11)

The algorithm of the material model based on NURBS for principal directions is presented in the following box:

1. Update the strain tensor.
\[
\mathbf{E}_{n+1} = \mathbf{E}_n + \nabla^S u
\]

2. Calculate the strains in principal directions
\[
\tilde{\mathbf{E}}_{n+1} = T^T \mathbf{E}_{n+1}
\]

3. Calculate the local parameter \( u \) and \( v \) from the strains.

4. Obtain the stress values \( S_1(u, v), S_2(u, v) \).

5. Calculate the derivatives \( \frac{dS_1}{dE_1}, \frac{dS_1}{dE_2}, \frac{dS_2}{dE_1}, \frac{dS_2}{dE_2}, \) and \( \frac{dS_{12}}{2dE_{12}} \) (equations 10, 11 and ??).

6. Constitutive material tensor is obtained:
\[
\frac{d\mathbf{S}}{d\mathbf{E}} = T^T \cdot \begin{bmatrix}
\frac{dS_1}{dE_1} & \frac{dS_1}{dE_2} & 0 \\
\frac{dS_2}{dE_1} & \frac{dS_2}{dE_2} & 0 \\
0 & 0 & \frac{dS_{12}}{2dE_{12}}
\end{bmatrix} \cdot T
\]

7. Calculate the stress tensor.
\[
\mathbf{S} = T^T \cdot \dot{\mathbf{S}}
\]
4 VALIDATION EXAMPLES

The PD–NURBS material model is applied to examples with different material responses to validate the proposed material model. Attention is given to materials with large strains.

Data fitting based on least-squares approximation is used to generate NURBS surfaces for the experimental data. For more details see the works of Piegl and Tiller[6] and L. Piegl [5]. An alternative approach for the generation of NURBS surfaces is the use of a CAD software.

4.1 Hyperelasticity – Mooney-Rivlin

This example consists of the stretching of a square sheet with a circular hole. This example is found in Gruttmann and Taylor [3] and in Souza Neto et al. [9]. The length of the sheet is 20m, the radius of the circle is 3m and the thickness is 1m. Due to problem symmetry, one quarter of the sheet was analyzed and the mesh with 200 linear quadrilateral membrane elements is presented in figure 1(a). The material used is on of the Mooney-Rivlin type with the constant values of $C_1 = 25MPa$ and $C_2 = 7MPa$. Thus the Ogden material constants are $\mu_1 = 50MPa$, $\mu_2 = -14MPa$ and $\alpha_1 = 2$, $\alpha_2 = -2$. The analysis was performed under load control conditions in three steps.

![Figure 1](image-url)  
*Figure 1*: Square sheet with a circular hole (a) undeformed sheet mesh with applied load (b) displacement result in y direction with deformed sheet in a scale of 1:1.
The results obtained are compared with the nonlinear material model based on NURBS surfaces. Figure 3 shows the NURBS surfaces used in these examples. This surfaces are composed by a net of control points 120(u) x 120(v) and degree 3 ($p = 3$ and $q = 3$).

4.1.1 Results

Figure 2 show the load-displacement curves, of three points on the mesh (A, B and C highlighted in figure 1), for the work of Gruttmann and Taylor [3] and the results obtained with the proposed material model based on NURBS. The results show good accuracy.

4.2 ETFE-Foil modeled with PD-NURBS

This example shows the application of PD-NURBS to model a material making use of the available experimental data. The experimental results used to generate the NURBS surfaces are those of the biaxially loaded ETFE–foil under two loading programs ratios of applied force: 1:1 and 2:1 presented in the work of Galliot and Luchsinger [2]. The available experimental data is not enough to generate good NURBS surfaces. In order to obtain a point cloud data necessary for the generation of the NURBS surface, data points based on the von Mises elastoplastic material formulation will be used. Figure 4 shows the experimental data points represented by the filled circles and the artificial ones by hollow squares. In this figure the gap between the points of the experimental test is observed. With this data points, NURBS surfaces in principal directions for stress and strain are generated and figure 6 shows the NURBS surface in conjunction with the experimental data points.

There is a dependence of the material model formulation with the size of the NURBS surfaces, in other words, input strains outside the NURBS surface, do not generate output.
stress results. In these regions artificial data is used to supply the stresses and strains information.

In figure 6 it is observed that the experimental data points are on the NURBS surfaces. The test is carried out for two load ratios 1:1 and 2:1. The mesh used is a rectangular membrane presented in figure 5. This mesh has 441 nodes and 400 quadrilateral linear elements. In figure 5 the boundary conditions and the applied loads for this model are presented. These examples are symmetric, therefore one quarter of the problem is modeled.

The analysis is carried out with the arclength control method and an equivalent nodal force is applied on the edges.
Figure 4: NURBS surface with experimental data

Figure 5: Mesh, geometry and boundary conditions for the biaxial test

4.2.1 Results

For both load ratios, the results are compared with the experimental results of Galliot and Luchsinger [2]. Table 1 shows the relative error of the numerical model with PD–NURBS material for stress and strain results.

Table 1: Relative error of biaxial test for the PD–NURBS material

<table>
<thead>
<tr>
<th>Error (%)</th>
<th>Biaxial 1:1</th>
<th>Biaxial 1:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain</td>
<td>Stress</td>
<td>Strain</td>
</tr>
<tr>
<td>direction 2</td>
<td>direction 1</td>
<td>direction 1</td>
</tr>
<tr>
<td>0.42</td>
<td>1.99</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Table 1 shows that the error with the PD-NURBS material for the biaxial test for load ratios of 1:1 and 2:1 is small compared to the experimental results.

5 CONCLUSIONS

The present work presents a material model, which use NURBS surfaces as response surfaces for material behavior. The material behavior is defined with NURBS surfaces with stresses and strains in principal directions. These NURBS surfaces are generated with the results from biaxial tests. The advantage of this material model is that from results of experimental tests, a material model can describe the material behavior. On the
other hand, the experimental data should provide a such point distribution as to generate good NURBS surfaces. This point distribution could result in a necessity for a large range of experimental data.

The results obtained for the perforated square membrane with Mooney–Rivlin material model are compared with the results from literature. The results obtained are in complete accuracy.

Numerical analysis with the finite element method using the PD–NURBS material model are applied to model the ETFE material. The error obtained is small and the results can be improved with the optimization of the NURBS surface.

With respect to computational time for the analysis no significant difference between the PD–NURBS material and conventional material was observed.

We conclude that this material model is a good alternative to conventional material models.

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Textile and Film Based Building Envelopes – Lightweight and Adaptive

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Key words: Building envelope, multilayer textile construction, acoustic insulation, spatial acoustics, urban acoustics, adaptive acoustic system.

Summary. This paper presents recent advances in the field of multilayer textile cladding systems with a focus on the latest findings. Primary topics are the special characteristics of textile materials in building envelopes in relation to thermal insulation, vapour issues and changing weather conditions as well as the acoustic evaluation of such ultralight systems and the ambitious demands of acoustic insulation and spatial acoustics.

1 ADVANCED TEXTILE MATERIALS, AUTOMATED PRODUCTION PROCESSES AND MULTIDIMENSIONAL REQUIREMENTS

The development of production and manufacturing processes for textiles and a variety of established technical products have allowed the extension of the functionality of textile materials and the expansion of market shares over the past three decades, especially in the sectors of automotive technology, aeronautics and astronautics, sports and leisure, protection and security, as well as the medical field, communication and information technology.1,2 Fibres, their semi-finished and their final products are omnipresent in our durable goods and commodities. Breathability, non-combustibility, energy absorption and storage capacity are just a few of the possible characteristics of textiles. Such properties have been achieved in recent years through the development of combined materials, coatings, embeddings, and the advancement of process technologies for fabrication, joining and processing textile materials. Gries and Klopp summarised comprehensively the development of processes and catalogued the possibilities and applications from a textile technological perspective.1,2 Industry and science work on textile materials and membranes using different methods, approaches and goals. Cell biologists, for example, try to promote fundamental investigations by exploring the behaviour of semipermeable membranes. The material sciences enhance the structural performance of carbon fibres or nanotubes. The fashion industry, on the other hand, works on a user-oriented implementation of comfortable, heat-regulating clothes.2,3

The knowledge gain associated with research in textiles usually takes place in different disciplines and can be separated into clusters such as materials, processes and applications. The findings diffuse very slowly between the disciplines and inspiration for product design is often limited to sketches and preliminary studies. A comprehensive transfer of knowledge and an exchange of expert opinions in an interdisciplinary design process is, however, essential.
for a successful system and product development. Maeda focuses on complex interactions and simple-appearing solutions. Ropohl describes this kind of process in his introduction to transdisciplinary thinking and discusses the benefits based on a general systems theory. Such an interdisciplinary, iterative process seems to get more important if the setting of requirements and criteria gets complex and if subjective estimation influences the design process. Those ‘soft’ factors for the design process are e.g. aesthetics, haptics, or sensation.

Promising technologies that will gain importance for new functional applications in architecture can be seen in material science, communication technology or mechanical engineering. The use of these new materials and manufacturing techniques offer the opportunity to revitalize or replace existing solutions that are exposed to increasing criticism due to resource and energy consumption. The level of difficulty increases for the development and implementation of those technologies if the respective demands and operating conditions extend. Geotextiles or fabrics for textile-reinforced concrete can be easily developed with simple focus on material parameters. They can be produced, characterized, and finally established more easily and faster. On the other, the building envelope is likely the one of the most ambitious and complex components in the construction industry. It combines numerous requirements as an essential separation, functional and design element.

2 CLADDING CONCEPTS AND DESIGN PROCESS

For many years there have been several efforts in research and practice to develop the building envelope as an ultralight, adaptable system through the use of different textile materials. A summary of results of recent research projects at the Institute for Lightweight Structures and Conceptual Design (ILEK) and description of practical examples of multilayer textile building envelopes is given by Haase et al.

Building envelopes – the sum of the exterior walls and the roof – must achieve various functional qualities. Environmental protection, security, sound and thermal insulation, aesthetics as well as static properties and low maintenance requirements throughout the life of a building are essential demands to be fulfilled simultaneously. The influence of façade construction on increasing interior comfort criteria and correspondingly the well-being, health, and performance of occupants extends the affected requirements. In addition, functions important during production, transport and the separation of the components at the end of the lifecycle are to be considered. Existing solutions in the construction industry typically respond to this diverse range of requirements with complex components and assemblies using a differentiated construction typology.

The concept of an adaptive multilayer textile building envelope as a façade solution can consistently implement the reduction of resource consumption, the interchangeability of all components of the system, and the application-specific adaptation to changing environmental conditions and user requirements. So far, the basic principles related to construction, thermal conditions and vapour transmission behaviour have been investigated and are integrated in the design process. Corresponding prototypes focusing on air regulation, moisture transfer and heat balance as developed over the last 5 years were characterized and verified using conceptual prototypes, simulations and experimental measurements.
The design of the conceptual prototypes mentioned above is based on the approach of modular façade systems well-known in the construction industry. Modular façade systems are one of the advanced solutions to fully meet the complex demands of modern buildings and cladding systems.6,10 They can be designed with very little variability from element to element in semi-industrial processes, equipped with different technologies, and transported and installed on site in a routine procedures.6 Within the design process, single components of the modular façade system, their functionality and their arrangement can be supplemented, reduced or altered according to the varying requirements. This characteristic applies to the design process of adaptive multilayer textile building envelopes as well. Different aspects, such as requirements for a structural support system, the implementation and conception of individual layers and the manufacturing processes, joints and connection capabilities are developed in parallel processes and merged at their points of interface. As a result, pultruded fibreglass reinforced profiles for multifunctional purposes were realized in addition to the prototypical textile layering. Figure 1 shows the geometry of the substructure and a fully equipped façade element.

![Figure 1: Pultruded Profiles and Equipped Module of a Multilayer Textile Façade Element (ILEK / Photo: G. Metzger)](image)

Within the work on structural-physical and material-specific principles, a significant result is that modularity is consistently complemented by the strategy to use multiple textile layers. The different requirements of a façade system can be fulfilled in multiple ways exploiting the material characteristics and the system design of the framing in combination with the textile surface elements. The main task of the design is ultimately to incorporate the potential materials in the necessary position within the system in order to achieve the maximum impact on relevant physical, structural and aesthetical demands as well as the requirements related to building science as they were identified in the design process. Thus, the central challenge in the development of textile building envelopes consists in resolving the conflict between many competing demands. These demands inevitably lead to opposing objectives and interactions between different system elements. Balancing these interactions and solving the conflicts is only possible in a continuous, iterative design process, which leads to an improved façade system.
3 THERMAL INSULATION, VAPOUR AND CHANGING WEATHER CONDITIONS

Thermal insulation and a controlled vapour transport concerning in changing weather conditions represent two of the important requirements of modern façade systems. Therefore, the relevant aspects were investigated in previous research projects at the ILEK.¹

The reduction of transmissive heat loss during winter is one prime aspect of textile façade systems. During summer, heat generation is an issue of consideration as to avoid cooling loads and reduce incident solar radiation while maintaining adequate daylight lighting conditions.¹ The reduced mass of textile building envelopes leads to major challenges with respect to climate control. In conventional wall and roof construction, heat transmission occurs mainly through heat conduction and convection.

In contrast, heat transfer within textile layers is governed by radiation exchange. Furthermore, the low thermal mass of textile systems leads to a more dynamic behaviour when changes in environmental conditions occur. This dynamic behaviour influences the vapour balance as well. Condensation and moisture formation within insulating layers are often described as causes of damage in buildings that use multilayer textile building envelopes.¹⁰ At night, exchange of radiation with the clear, cold firmament can reduce the surface temperature of membrane layers below the ambient air temperature. If the air directly at the membrane surface drops beneath the vapour saturation temperature, condensation will form (see Figure 2).¹

![Figure 2: Principle of Night Cooling and Condensate Formation (ILEK)](image)

The vapour diffusion resistance of the materials used must be considered in order to prevent the risk of moisture formation in the insulation. Three options to provide a reduction in the risk of condensation are available: the controlled ventilation of the insulation and interspaces to remove the vapour, the finishing of the outside layer with a low-e-coating to decrease heat losses, and the selective heating of single membrane layers to increase the surface temperature and avoid condensation.

As result of the investigations, the following criteria for the arrangement of the textile layers in a sensible system configuration are relevant: the material properties (thermal
conductivity, radiation behaviour), the thickness of the insulation, the distance between the layers to provide ventilation and avoid condensation, and the reduction of the vapour diffusion resistance from the inner to the outer layers.¹

As a proof of concept of the functional integration in a multilayer textile element, different materials and technologies were integrated in prototypical designs and were investigated conceptually. Figure 4 shows two of the various alternatives. Concept 1 provides a sufficient level of insulation using phase-change material (PCM) as a thermal buffer and aerogel granulate as translucent high-performance heat insulating layer. Adjustable vents are integrated to manage vapour transport and to equip the façade with a controllable ventilation system. Concept 2 uses common opaque, non-woven fabric insulation and compensates the resulting lower thermal insulation through a twin-collector. This collector combines a flexible photovoltaic layer with a solar thermal collector. The thermal radiation of the collector can be used to heat up the surfaces of the interior layers in order to avoid condensation.
4 NOISE INSULATION, SPATIAL AND URBAN ACOUSTICS

While the research described above focuses on structural profiles, thermal insulation and vapour management, the evaluation of the acoustic behaviour of materials and systems is an issue that has evolved as another important aspect of ultra-light textile façade systems. The aural perception of spaces increasingly affects the cognition through noise as one of the greatest environmental impact factors. Excessive background noise and acoustic deficiencies often underline a poor audibility and lead to significant loss of spatial quality.  

Designing spaces without considering spatial acoustics is as meaningless as focusing on noise insulation and spatial acoustics as an independent, physical topic. Blésser and Salter describe that the combination of design and acoustical performance needs to be discussed with an understanding of the underlying physics as well as the necessities of the application.11 Being aware of visually non-perceptible influences will gain importance together with the increasing demand for sustainable, environmentally and health friendly buildings for the future. The relationships between material, construction, space and the resulting constructive and acoustic consequences will become of greater importance for designers and planning professionals in order to build high-quality spaces.12

Noise protection in an urban environment is achieved today mainly by reducing the noise of the cause, by establishing sound protection walls that shield the noise mainly due to reflection, and finally by integrating noise reducing glazing into existing buildings or by appropriate orientation of the rooms in new buildings. Usually, adequate quality for the occupants can be achieved only through the combination of schemes in all three categories. Even if the reduction of the actual sound source is the most effective method for reducing noise, it is not sufficient to significantly reduce the overall noise level especially in the urban environment with increasing sound-emitting infrastructure.12,13 For an effective improvement of the urban quality in the context of increasing population density and noise-emitting infrastructure, reducing the sound level by means of noise-absorbing façade solutions is a potential addition. Emissions would thus no longer be redirected into the urban space through reflection and sound levels could be reduced by the surfaces. The comparison of absorbing and fully reflective sound-insulating walls in Figure 5 is described by Möser. It shows the effect in reduced noise levels and lower diffraction effects.13

![Figure 5: Sound Pressure Level Distribution in Reflective and Absorbing Walls (Möser: p. 357)](image-url)
In addition to new buildings, existing façades offer a great potential for improving the urban acoustics in connection with the pending modernisation of thermal insulations of buildings. However, façades must fulfil the functional, energetic, environmental, structural, aesthetic and social needs in comprehensive urban and architectural concepts. The application of textile materials with their inherent aesthetic qualities promises a restrained, effective and aesthetically pleasing solution for retrofitting façades and reducing noise levels in streets and at noisy infrastructure.

The optimisation of spatial acoustics is primarily determined by the adjustment of the reverberation time. This necessary adjustment becomes increasingly difficult in flexible used spaces and for short-term building conversions. As a result, those spaces increasingly use universal reference systems with electro-acoustic support. These methods, however, lack behind the distinctive acoustic potential of the utilized space. Actively changing acoustically effective systems that can be adjusted specifically depending on the usage, are a different but expensive method. Such systems require a professional service and are used occasionally in high-quality multi-purpose halls.

The functional and modular concept of adaptive multilayer textile building envelopes offers the possibility to use the façade system as an actively changing acoustic conditioning element for interiors. The façade system must be developed further, focusing on cost effectiveness, material-saving, and independent controllability. Thus, spatial acoustics can be adapted automatically to different usage scenarios of rooms to provide an environment that allows, for example, concentrated working or clear conversation.

From prototypical specimens for acoustically effective layering and the experimental examinations of potential textile materials, a solution according to the sheet, Helmholtz, and wideband resonators used commonly in spatial acoustics (see Figure 7) is identified. Core components are two or three vibrating sheets with higher area-specific mass, as well as porous, absorbing, and light interlayers. The basic design of a sheet or Helmholtz resonator allows the variation of the heavy layers by means of textile materials, additional ballast layers, inlays or of the material of the sound-absorbing layer. A perforated layer for the interior acoustics can be attached to expand the system to a wide-band resonator.

With regard to an active variable setup, the system offers the possibility of varying the amount of area-related mass, the distances and initial tension of the layers.
5 SYSTEM SYNTHESIS

After studying the basic principles described above, the findings were combined into a prototypical system design, facilitated within an interdisciplinary process combining workshops and iterative development phases. The development of a passive system was focused on first in order to expand upon the configurations of existing membrane structures with minimal interventions. This enables a comparatively easy conversion of existing production and manufacturing processes. A dual-layer, non-woven system evolved from the conceptual phase, the study of different prototypes and an assessment of the feasibility. This system was developed to enhance the existing thermal insulation layer in order to increase the efficiency of the acoustic insulation.

A multilayered system was considered to be advantageous for the purpose of a sound-absorbing layer setup. One layer functions as heavy sheet with increased area-specific weight. From a manufacturing perspective, a weight of 1.5 to 2.0 kg / m² was considered feasible. The porous filling is used between the heavy sheets to reduce the dynamic stiffness and to prevent standing waves and interference. This porous filling is also produced as a non-woven fabric and combined into one element with the heavy fabric by needling. Figure 8 outlines the relevant elements of the compound material.

The compound material was then used for a first prototypical setup. It was integrated twice and positioned as mirrored layers to meet the configuration of a sheet absorber. The different layers and the acoustic behaviour is described in the diagram of Figure 9.
To demonstrate the adaptability of the reflective and absorptive characteristics of multilayer textile building envelopes, the design of the existing system was extended to an acoustically active system. A stitched tube system with a fluid filling was added to the system in front of the heavy fabric in order to vary the area-specific mass. The variability of the amount of the fluid and the filling pressure allows to influence the vibration behaviour of the layer and to increase the reflective characteristic in relation to the urban space or to the interior. Figure 10 shows the extension and the acoustic behaviour of the acoustic active-acting systems.

System designs developed with this adaptive principle can reduce the sound energy in an urban environment (layer positioned to the interior is filled and ballasted). (see Figure 11) It can also be configured inversely to mainly reflect the incoming sound field (layer positioned to the outside is filled and ballasted). In conjunction, the system includes the possibility to
vary the spatial acoustics of the interior space through the reflection and vibration behaviour of the heavy layers.

As part of the system development, the concepts and prototypes are described above are currently being studied in more detail. The investigation of the thermal and vapour behaviour of the systems is being conducted using the simulation tool described by Klaus et al.\textsuperscript{14}

In the context of the ongoing research, the acoustic performance of the prototypes is quantified by the system specific sound reduction index. The determination of this index has been conducted at the Fraunhofer Institute for Building Physics in Stuttgart (see Figure 12). In Figure 13, the results are compared with commonly used textile systems and with a solid wall. The calculated weighted difference level of the passive-acting system (type A) is $D_W = 30$ dB, the active-acting system (type B) reaches $D_W = 34$ dB. In comparison to that, the dual layer PVC/Polyester Membrane described by Maysenhölder reaches a difference level of $D_W = 17$ dB. The acoustic performance of the system integrating a non-woven insulation is significantly better than the one of single or double layer membranes (type D and E). The addition of the fluid filled tube system with an area-specific mass of 17.75 kg/m\textsuperscript{2} (type B) causes a further increase of the sound insulation performance, in particular in the frequency range above 500 Hz. This effect demonstrates the possibility to vary the acoustic behaviour of such systems on demand.

![Figure 11: System Behaviour with Switchable, Ballastable Layers (ILEK)](image-url)
6 CONCLUSION

The studies and experiments performed at the ILEK present the potential of multilayered textile building envelopes as an important high-quality solution for new façade constructions. The concurrent work on the relevant subject areas through systematic system development, the incorporation of a large number of textile layers or functional materials in appropriate structural frames and experimental investigations are part of a necessary discourse to develop the fundamentals for a meaningful choice of material and for an effective synthesis. The findings are used to improve the design of the textile façade solution in the manner of thermal and acoustic insulation, spatial acoustic and the interior quality.

The studies have shown that the thermal and vapour issues can be solved with rational material selection and integrated active elements. An effective solution to solve the acoustic requirements of ultra lightweight envelope systems is represented by the broadband absorber used in spatial acoustics.

Although the results are based on initial conceptual ideas, they nonetheless point in an important direction for a consistent development of multilayer textile building envelopes addressing changing requirements and conditions in a single system. The diverse requirements can feasibly be addressed only through a consistent system development of
ultra-light systems. This must be driven by an interdisciplinary and iterative design process taking all potentials associated with the system synthesis into account. These potentials are exploited only through the appropriate use of material properties, relevant physical and functional features, structural efficiency, and by integrating consideration of production processes and facilities into the design process.\textsuperscript{1,8,9}

The resulting conceptual designs and the findings with respect to construction principles and functionality represent a contribution to establish recyclable textile and film-based multilayer configurations as future ultra-light, highly efficient building envelopes. The importance to focus on such new kinds of façade solutions is obvious by the necessity to redevelop processes, technologies and products for the building sector in the current context of limited resources and urban development.\textsuperscript{1}

REFERENCES

THERMOTECTHICAL OPTIMIZATION OF MEMBRANE CLAMP
PROFILES REGARDING STATICAL AND ASSEMBLY
ENGINEERING ASPECTS

STRUCTURAL MEMBRANES 2013

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Key words: clamp profile, thermal bridges, heat protection, risk of mold formation and water condensation, thermal transmittance, minimum surface temperature.

Summary: The thermal optimization of a two-dimensional membrane model is presented, with regard to typical construction features of a clamp profile. Based on a reference profile, measures of building physics like the use of insulation layers but also geometrical variations of the profile have been investigated in detail to reduce the effect of thermal bridges and heat losses. As a result the inside minimum surface temperature can be raised in order to reduce the risk of water condensation. This helps to avoid possible damages to the supporting structures.

The results were used to develop a thermally optimized membrane profile, which meets both assembly engineering and statical requirements. Key features of these profiles are a significant reduction of the thermal transmittance and a complete avoidance of water condensation on the inside surfaces.

1 INTRODUCTION

Recent research reports¹, ², ³ show that the requirements of heat protection and protection against moisture cannot be totally fulfilled by conventional membrane constructions. The investigated multilayer foil constructions are able to ensure the minimum thermal protection, but there is a significant loss of energy at the clamp profiles as a result of the geometrical and material specific constraints and boundary conditions.

This paper presents the development of optimized membrane profiles regarding assembly engineering and statical requirements. Different variables for the thermally optimized clamp profile are presented and evaluated in serveral geometrical and building physics variation steps for the chosen reference profile. Thereby the International Standards for window
construction are used to determine the characteristic thermal values since there is presently no standard for membrane constructions.

2 REQUIRED CHARACTERISTIC VALUES OF BUILDING PHYSICS

2.1 Characteristic thermal values

The thermal transmittance (U-value) is a significant variable of the thermal insulation of building structures. The U-value of windows, doors and shutters is calculated using DIN EN ISO 10077-2 and defined as:

\[ U_f = \frac{(L'_f - U_p \cdot b_p)}{b_f} \text{ [W/(m² K)]} \]  (1)

2.1 Characteristic hygrothermal values

The thermal bridges in clamp profiles are causing both high heat flows and low surface temperatures. The knowledge of these low surface temperatures is necessary to evaluate the risk of water condensation and mold formation.

The temperature factor f (f-value) is defined as the difference between the interior surface temperature of a component and the exterior air temperature, related to the difference of interior and exterior air temperature:

\[ f_{\text{Rsi}} = \frac{(\theta_{\text{ini}} - \theta_{\text{e}})}{(\theta_{\text{i}} - \theta_{\text{e}})} [-] \]  (4)

A temperature field of the building structure, including an isothermal curve and a color scale, is used to show the specific temperature conditions of a profile. Red represents the maximum and purple the minimum temperature.

The following simulation is based on the characteristic boundary conditions (cf. table 1) according DIN 4108-2. To avoid water condensation on the inner surface of the construction the 9.3°C-isothermal curve should be located in the interior profile. This curve is illustrated as a bold black line (cf. figure 1).

<table>
<thead>
<tr>
<th>Area</th>
<th>Temperature</th>
<th>Relative Humidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indoor climate</td>
<td>20 °C</td>
<td>50%</td>
</tr>
<tr>
<td>Outdoor climate</td>
<td>-5 °C</td>
<td>-</td>
</tr>
</tbody>
</table>

Under these boundary conditions there is a high risk of mold formation when the surface temperature is lower than 12.6°C. The 12.6°C-isothermal curve is shown as a red line (cf. figure 1). In table 2 the minimum surface temperatures and temperature factors for the risk of
mold formation and water condensation are described.

Table 2: Minimum surface temperatures $\theta_{si}$ and temperature factors $f_{Rsi}$ for the risk of mold formation and water condensation

<table>
<thead>
<tr>
<th>effect</th>
<th>$\theta_{si, min}$</th>
<th>$f_{Rsi}$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mold formation</td>
<td>12.6 °C</td>
<td>0.70</td>
</tr>
<tr>
<td>water condensation</td>
<td>9.3 °C</td>
<td>0.57</td>
</tr>
</tbody>
</table>

3 THERMAL ANALYSIS OF THE REFERENCE PROFILE

Two types of the chosen reference profile are analysed, with and without screw joints to the supporting structure. There are three different types of supporting structures which are included in the simulation process according [5]. This paper’s focus is the clamp profile without a fixing in between the supporting structure. The other types of supporting structure, the clamp profile with a steel support and the clamp profile with a continuous supporting structure on a wooden cross-section are not explained further. All measurement data is reported in millimetre.

Figure 1 shows the sectional drawing and the temperature field of the reference profile with (upper picture) and without screw joints (lower picture). The temperature field for the profile with screw joints is indicating a minimum surface temperature $\theta_{si, min}$ of 7.5°C. Water condensation can occur on the inside surface. The $f$-value is 0.50.

Figure 1: Sectional drawing and temperature field of the reference profile with (upper picture) and without (lower picture) screw joints
Compared to the usual thermal transmittance for window frames, a $U_f$-value of 9.2 W/(m$^2$ K) for the profile with screw joints is high and thus representing a large thermal bridge. Without screw joints the values aren’t significantly better; however water condensation occurs. The $U_f$-value for the entire profile is 7.7 W/(m$^2$ K).

4 OPTIMIZATION PROCESS

In the following chapter the most efficient strategies for the optimization of the chosen reference profile are presented. Detailed analyses [5] show the following results:

By filling polyurethane foam (thermal conductivity 0.030 W/(m K)) in the profile cavity, there is no significant improvement of the $U_f$-values and the minimum surface temperature. This may be due to the fact that the biggest heat loss is taking place over the screw joints of the central aluminum fin which is continuing to the supporting structure. A probable assumption is that insulating the cavity has no influence on the heat flow through screw joints and the central fin. Further analyses are confirming this assumption.

The lateral insulation of the interior profile surface (thickness = 2 cm, thermal conductivity 0.030 W/(m K)) does also not result in a reduced risk of water condensation. The application of a lateral insulation leads only to an improved $U_f$-value (4.7 W/(m K)), whereas the surface temperature is falling below the temperature of the reference profile. This difference can cause an increased risk of water condensation.

The warm air of the room cannot contribute to the warming up of the construction from inside as the contact surface to the good heat-conducting aluminum is reduced.

4.1 Exterior insulation of the profile cover plate

In the following this optimization strategy is titled insulation cover plate. An insulation layer (thickness = 1 cm) is placed at the rear side of the cover plate (thermal conductivity = 0.035 W/(m K)). This strategy results in the temperature fields shown in figure 2.

The minimum temperature of the profile with screw joints is 9.0°C and thus 1.5 °C higher than the temperature of the reference profile. The f-value is 0.56. Without screw joints the minimum surface temperature adds up to 13.0°C. Regarding the given boundary conditions in table 1 there would occur no water condensation.

The $U_f$-value of the profile with screw joints decreases in this case to 9.0 W/(m$^2$ K) compared to the reference profile with 9.2 W/(m$^2$ K). This results in a $U_f$-value of 3.4 W/(m$^2$ K) for the entire profile. Although the screw head has only a small contact to the outside, the thermal conduction is relatively high because of the high thermal conductivity of steel. The central aluminum fin increases the heat flow to the outside of the profile. This effect was also observed for the filled cavity profiles. Due to the lower thermal transmittance of the profile without supporting structure, the $U_f$-value is about 60 % lower than the one of the profile with screw joints. The screw joints of the cover plate and the central aluminum fin have the highest influence on the heat flows to the outside. These results have to be considered in the further optimization process.
4.2 Separated foil layers with additional aluminum clamps

In the following this optimization strategy is called *foil spacers*. In this simulation two additional aluminum profiles are installed for the foil layers. Their geometrical structure is based on the reference profile. This measure results in a cumulative construction height of 75 mm. In figure 3 the corresponding temperature field for this profile is visualized.

This case is a first step to investigate the optimization potential for separated foil layers. The possibility of a practical application regarding the air tightness of pneumatic membrane constructions will be investigated in a following chapter.

The minimum surface temperature with screw joints is 8.8 °C and therefore 1.3°C higher than for the reference profile. The f-value is 0.55. The $U_f$-value for the profile with screw joints has increased only slightly to 9.7 W/(m² K), compared to the reference profile (9.2 W/m² K). The $U_f$-value for the entire profile-system is 7.8 W/(m² K). The characteristic values show that the separation of the foil layers does not significantly improve the minimum surface temperature, but the curve progression of the isothermal curves is more homogeneous. Hence, the additional aluminum profiles do not improve the thermal insulation because of the high thermal conductivity of the central fins and their side walls.
4.3 Plastics for the profile construction

In the following text this optimization strategy is titled plastic profile. Unplasticized polyvinyl chloride (PVC) is a common material for window constructions. Especially the low thermal conductivity (0.17 W/(m K)) leads to reduced heat losses.

As shown in the previous chapter, a particularly high heat loss is caused by aluminum profiles. Due to the fact that plastic material has a low strength, a reinforcement section is necessary (e.g. out of aluminum). The position and thickness of the reinforcement section as well as the number and order of cavities in the profile are essential for the thermal transmittance\(^9,10\). This optimization strategy results in the temperature fields shown in figure 4.
The minimum surface temperature in the case of screw joints is 9.8°C. Hence, there is no more risk of water condensation. The $f$-value is 0.59.

In comparison to the reference profile ($U_f = 9.2 \text{ W/(m}^2 \text{ K})$), the $U_f$-value for the profile with screw joints is reduced to 3.9 W/(m$^2$ K) and the total profile has a $U$-value of 3.4 W/(m$^2$ K). By constructing an external insulation made from plastic material, the cover plate screw joint is insulated and decoupled. Better results are obtained. The implementation of a plastic frame with aluminum reinforcement can lead to a reduction of the thermal transmittance for the entire profile of 55%. Moreover it can be seen in figure 4 that the 9.3°C-isothermal curve is close to the edge of the profile. Only in the transition area between foil and clamp profile the curve extend into the interior side, which would enhance the formation of water condensation.

5 DEVELOPMENT OF A THERMALLY OPTIMIZED CLAMP PROFILE

5.1 Most promising optimization method and its practical implementation

Regarding the reduction of the thermal transmittance and the increase of the minimum surface temperature the installation of an exterior insulation layer offers the biggest room for
improvement. This improvement can be achieved by insulating the cover plate with a 1.0 cm thick layer or insulation a PVC-cover plate, which includes cavities.

An insulation layer beneath the cover plate is of no practical use since the foil layers move as a result of the interacting forces.

The clamp profile cover plate offers a consistent contact pressure on the membrane foil layers; similarly the EPDM-layers seal the profile against water and other environmental impacts. Therefore a PVC-cover plate, including cavities, will be chosen.

Although the separation of foil layers with an aluminum construction leads to a higher thermal transmittance, the highest minimum surface temperatures can be reached with the above mentioned strategy. Using this method, the 9.3°C-isothermal curve can be shifted to the inside of the profile. Thereby the risk of water condensation can be minimized, as function of the above mentioned boundary conditions (cf. table 1). Therefore a separation of foil layers will be considered for the development of a thermally optimized clamp profile.

5.2 Installation of foil spacers and insulation of the cover plate

In the following chapter this optimization strategy is titled foil spacers and insulation cover plate.

The analyses in the previous chapter have shown that the separation of foil layers leads to a higher minimum surface temperature and thus a reduced risk for the occurrence of water condensation. The installation of a cover plate profile made from PVC with aluminum reinforcements is resulting in a reduction of the U_f-value and an improvement of the minimum surface temperature.

A profile is developed to combine these two optimization strategies. In order to avoid a too large profile width or height, two clamp devices for each profile side are designed. Due to the fact, that a three-ply foil construction is used, there is for example the possibility to weld and clamp two foils separately from the third one.

Furthermore the use of two clamp devices makes it possible to assemble different types of foils, for instance a foil implementing a sun protection without the necessity for a common cord edge.

This can be beneficial regarding production as well as assembly and maintenance.
The spacer can be made out of composite material. On the one hand the exterior layer has to be elastic and caulking to equalize foil movements and to guarantee an air tight pneumatic foil construction. On the other hand two important requirements for the interior construction are thermal insulation as well as load reduction. The use of foamed EPDM and reinforcements as for example glass fiber reinforced plastics can help to accomplish the required characteristics.

To guarantee an air tight pneumatic foil construction, butt joints of the profile sections (in longitudinal direction) have to be avoided. Therefore, a tongue-and groove-joint could be used. As the required spacer should be further analyzed regarding its practical usefulness, there is no specification of a construction material in the following chapter.

The figures 5 and 6 show the sectional drawing, the temperature field and the building physics specific values of the optimized clamp profile with foil spacers and insulation cover plate with screw joints.

In this considered case the minimum temperature amounts to 13.9 °C and results in an f-value of 0.76 (with screw joints). Compared to the reference profile the minimum surface temperature decreases by 6.4°C. The course of the 9.3°C – isothermal curve is close to the edge of the profile cross-section, but remains in the profile. Therefore there is no risk of water condensation. The minimum surface temperature is located at the bottom inside of the clamp profile.

The $U_f$-value of the total profile drops from 7.7 to 3.0 W/ (m² K) compared to the reference profile. This means an improvement of 61%.
The installation of foil spacers in combination with the insulation of the cover plate results in a considerable improvement of the thermal transmittance. The risk of mold formation and water condensation for standard boundary conditions is eliminated. Due to the fact that the weak points in the foil connecting area are resolved, the minimum surface temperature is not located in the transition of the foil connecting area any more.

12 CONCLUSION

To determine the likelihood of water condensation in membrane clamp profiles, it is necessary to consider the thermal transmittance as well as the minimum surface temperature. A thermal bridge analysis of the chosen reference profile is indicating the biggest heat losses in the profile cross-section. It is shown that the transition area from the clamp profile to the foil layer construction is a weak point as well as the central profile fin. Changes in building physics parameters as well as geometrical parameters lead to improved characteristic values. A reduction of the thermal transmittance is not coercively resulting in higher minimum surface temperature inside the profile and vice versa.

A combination of the most efficient strategies leads to a thermally optimized membrane clamp profile. The profile with separated foil layers and a decoupled profile cover plate shows the best results.

With regard to the thermal characteristic values, there is a large improvement, which leads to the avoidance of water condensation and a significantly reduced thermal transmittance.

As shown it is possible to combine different optimization strategies in order to provide a thermally optimized clamp profile with regard to statical and assembly engineering aspects.
Based on the results of this paper future analyses can be conducted and tested in context of practical applicability.

REFERENCES


ADVANTAGES OF LIGHTWEIGHT TENSIONED COATED FABRICS AND FOILS FAÇADES FOR THE BUILDING SECTOR

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Summary. This paper reviews the advantages of lightweight tensioned coated fabrics and foils applied to the existing building sector in order to improve the insulating/shading performance of the external building envelope. The paper describes the main systems developed in the last few years in this field such as pneumatic façades, double curved membrane façades and flat tensioned façades (prefabricated textile panels or tensioned on-site). In addition, the paper analyses the range of foils, coated fabrics and open mesh coated fabrics commonly in use, such as ETFE, Polyester/PVC, Fibreglass/Silicone, Fibreglass/PTFE and ETFE foils.

1 INTRODUCTION

Recent energy-saving European policies in the building sector have led to a rapid increase in the demand for insulating and shading products. It has been estimated that the market for refurbishment of existing European buildings is currently 545 billion Euros per year which represents 30% of the total building sector output. The improvement of the external building envelope represents a crucial aspect in order to achieve the expected thermal performance required by the new targets for the building sector⁴.

2 THE TRADITIONAL EXTERNAL VERTICAL CLADDING BUILDING SYSTEMS

Currently, the external vertical cladding market is characterised by several systems based on a wide range of cladding panels made of bricks, ceramics, natural and reconstituted stone, high-pressure laminates, composite materials, aluminium, zinc, steel, fibre-cement board, or durable exterior woods. Due to their reduced material efficiency, these systems show several critical aspects in terms of environmental assessment.

The considerable weight of the cladding panels, which for terracotta claddings and
sunscreens can reach 50kg per square meter, requires adequate supporting frames and anchoring systems which can be considerably expensive with not-insignificant environmental loads. In addition, the additional loads could compromise the structural performance of the existing buildings with unpredictable consequences in case of earthquakes.

3 LIGHTWEIGHT TENSIONED MEMBRANE FAÇADES

Lightweight façades realised through the use of tensioned coated fabrics and foils can be classified in the dry assembled building systems with a high degree of prefabrication. The elements are generally prepared at workshops, partially pre-assembled, numbered and transported to the construction site, where they are put together to form units for erection in a further preassembly process. Lightweight tensioned membrane façades are characterised by several distinguishing aspects such as the intrinsic lightweight and efficient use of the materials, reduced environmental impacts (especially when recyclable materials are used), cost-effective alternatives for flat or complex 3D surfaces, good safety behavior in case of fires or earthquakes and a level of insulation (light, heat and noise) which can be progressively improved with specific surface treatments (Low-E coatings, reflective patterns, micro-perforated surfaces etc.) or additional insulating layers and materials.

Figure 1: Detail of the textile façade designed by M. Majowiecki for the two-floor exhibition hall at the Bologna exhibition Centre, 1999.

Their use in the building industry has considerably increased over the last ten years following the first successful examples of enclosed fabric envelope such as the Schlumberger
Cambridge Research Centre in 1984\(^4\) and one of the first uses of fabric exclusively for façades, the two-floor exhibition hall at the Bologna exhibition Centre in 1999\(^3\), Figure 1.

There are three main systems for tensioned membrane façades currently on the market: pneumatic façades, double curved membrane façades and flat tensioned façades.

### 3.1 Pneumatic façades

Pneumatic façades use air under pressure to achieve the required load bearing capacity. In addition, the pressurised air chamber considerably increases the insulating performance, which can progressively be improved with multiple layers. Compared with double- or triple-glazed façades, the thermal performance is considerably compromised by the dimensions of the air chambers, which allow a not-insignificant movement of the enclosed air with consequent energy losses due to convection. The cushion are generally obtained by welding together two or more layers and then fixing them to a rigid boundary through keder rail profiles. The efficiency of the system, in terms of overall weight per square metre, is strictly related to the ratio between the area of each cushion and its perimeter: the bigger the cushion, the more efficient the system. The main limit to the dimensions of the cushion is its thickness, which is directly related to the curvature of the membrane and its stress state under the same pressure. To overcome this limit, the shape of the cushion is generally a rectangle in which primarily the short direction influences the tension in the material. A similar effect can be achieved using supporting cables or frames arranged in a wide supporting net which reduces the tension in the membrane, Figure 2. Fritted patterns can block out solar radiation, thereby cutting down on cooling costs during the summer. They are commonly printed on the external layer in order to maximize the effect. Recent projects such as the Dolce Vita shopping complex, in which includes north-light-like roof cushions and the Duales System Pavilion at Expo 2000 Hannover have shown the potential of static and dynamic patterns printed on multiple layers.

![Figure 2: The pneumatic envelope of the Finmeccanica pavilion designed for the Farnborough Airshow, 2006.](image-url)
3.2 Double curved membrane façades

Double curved membrane façades achieve the required load bearing capacity through the double curvature of the surface and the pretension introduced into the membrane. The level of prestress depends on the material and geometry chosen with the stress state required inversely proportional to the level of curvature of the surface. The shapes in use reproduce the most common geometries for tensile roofs such as high point structures (Royal Artillery Barracks, London), ridge and valley structures (Unilever building, Hamburg, Figure 3) awnings (King Fahd National Library) and arch structure (Basketball Arena, London).

According to the performances required, which can vary from the simple shading of the façade to a complete water and air tight insulated envelope, the membrane material can be chosen within a wide range of products which cover all the main types of fabrics currently in use in architectural structures.

![Figure 3: The double curved membrane façades developed for the Unilever building in Hamburg.](image)

3.3 Flat tensioned façades

The structural performances of flat membrane façades rely on the pretension introduced into the membrane and its ability in recovering from temporary deflections due to external loads such as the wind pressure/suction. Despite the intrinsic low load bearing capacity due to the absence of double curvature, this type of façade is extremely attractive to the market due to the reduced thickness of the systems (which can be less than 5cm if no insulation and ventilation is required) and its similarities with the more traditional external cladding systems.

There are two main building principles currently in use for flat tensioned façades: in the first case the façade is realised connecting the membrane to a set of supporting profiles which have been previously fixed to the façade; in the second case the fabric and the rigid boundary frame are preassembled into modular panels in the workshop and then fixed to the façade.

Like for pneumatic cushions, the efficiency of the system is strictly related to the incidence of the supporting metal profiles. Unfortunately, due to the industrial costs of a new extrusion
matrix, these systems are generally based on standard profiles mainly developed in order to optimise the manufacturing and mounting process (corners, windows, intermediate support of the membrane etc.) with no attention to the environmental impacts due to specific technical choices (i.e. modular panels require twice the boundary profiles used for an equivalent façade assembled on site).

A key aspect of this type of façade is the details developed in order to accommodate the applied loads through a temporary deflection of the membrane. These details are covered by several patents such as the Texo® system (based on an aluminum modular frame and a silicon elastomer which acts as a spring), the Profil Tension System (based on aluminum profiles combined with plastic clamping profiles derived from the advertising sector) and the Facid system (based on a metallic keder clamped on the fabric and tensioned into the ribbed aluminum supporting profile). Self-tensioning systems based on the thermo-shrinking of the membrane are currently under development.

Figure 4: The modular façade for the Expo Shangai 2010 based on the Texo® system.

4 COATED FABRICS AND FOILS

Tensioned membrane façades are based on the use of flexible and thin materials in the form of coated and uncoated woven fabrics or polymer foils. As in the biological acceptation of the term, tensioned membrane façades have to satisfy a wide range of requirements which go beyond the mechanical properties and have to consider safety aspects, the influence on the environmental aspects of the enclosed space and the durability of the whole system under the expected weathering conditions. The most relevant engineering textiles for this field are mainly based on polyester, PTFE and glass fibres coated with PVC, PTFE, THV or silicone.

In case of façades designed for solar protection, the same fibres and coatings can be used to obtain open mesh coated fabrics which allow the modulation of the amount of radiation reflected, absorbed and transmitted through different weaving patterns and coatings.

Transparent envelopes can be easily obtained through the use of clear or fritted foils such
as PVC, PE, THV and ETFE.

4.1 PVC-coated polyester Fabric

Polyester-PVC is one of the most used textile membrane in the building industry due to the good compromise of price and performance. The five types of polyester woven fabrics cover a wide range of tensile strength suitable for all the main structural applications. In addition, thanks to a relatively good flex cracking resistance, this type of fabric is successfully used also for deployable structures. The main limitations of PVC-coated polyester fabrics are related to the light transmittance, the resistance to soiling and the long-term stability (which, however, can be considerably increased through top coats mad from fluoride lacquer).

Its successful use for tensioned façades is well-documented by a wide range of projects all over the world where cost-effective solutions and short to medium service life are required, such as most of the recent projects for the Olympic Games in London. Furthermore, its use for temporary pavilions is widely supported by the possibility of recycling the coated fabric, thereby reducing its environmental impacts.

4.2 THV-coated polyester Fabric

The use of THV coatings as replacement for PVC is quite recent in the building industry with little data and examples regarding their use. Compared with PVC coatings, THV offers a better behaviour in terms of weathering resistance, self-cleaning properties, light transmittance and UV resistance with the advantage of a similar manufacturing process and equipment.

4.3 PTFE-coated glass-fibre fabric and mesh

Considered one of the most durable membrane materials, glass-fibre fabric coated with PTFE is the most recommended material for permanent projects with an expected service life over 25 years. The material, characterised by a good light transmittance, combines the advantages of the PTFE coating, which provides excellent long term stability and resistance to soiling, with the mechanical resistance of the glass fibres. However, the relatively high cost of the material, especially compared with Polyester-PVC, combined with the additional manufacturing and installation costs due to its low flex cracking resistance, has reduced its use for temporary and low budget projects and for highly double curved surfaces.

PTFE-coated glass-fibre fabrics have been used for several high quality tensioned façades such as the Berlin Brandenburg Airport and the Burj Al Arab Hotel in Dubai.

4.4 Silicone-coated glass-fibre fabric

The disadvantage of PTFE-coated fabrics, which are susceptible to wrinkling, is overcome by using clear or opaque silicone treated with additives. Silicone shows an excellent light transmittance, flex cracking performance and resistance against chemical attack and UV radiation, and it does not become brittle. One disadvantage is that its surface charges up statically and attracts dirt. In addition, the high cost of the raw material and the relatively complex and expensive manufacturing process (the material has to be vulcanised or glued)
reduces its use in architecture.

Due to the price and the performance, Silicone-coated glass-fibre fabrics are mainly used for permanent applications such as the Zenith concert hall in Strasbourg.

### 4.5 Coated and uncoated PTFE fabric

The main properties of PTFE fabrics are their extremely high flex cracking resistance, light transmission, long-term stability and resistance to soiling, which make it the most recommended material for convertible structures, especially when uncoated and with no budget restraints. Its relatively infrequent use for façades is mainly due to the high price of the raw material. However, the potentialities of PTFE fabrics are clearly expressed by recent project such as the façades of the clusTEX research pavilion in Milan and the NRW Bank travelling exhibition ship.

### 4.6 ETFE foil

ETFE is one of the most stable chemical compounds and its films are largely employed in the building industries due to the very good long-term stability, resistance to soiling and high light transmittance. The mechanical strength is relatively good, especially considering that the material is not reinforced by a woven support, and makes ETFE foils suitable for load bearing envelopes characterised by small spans or supported by cables. The best known ETFE façade is the Beijing Aquatics Center, however, the material has been recently used for single layer projects such as the Unilever building in Hamburg.

### 4.7 PVC foil

PVC foils are characterised by an extremely poor mechanical resistance, long-term stability and resistance to soiling. The optical properties, which deteriorate soon despite the initial transparent and clear aspect, are inferior if compared with ETFE, especially considering specific wavelengths. However, the flexibility of the material and the extremely low cost, make PVC foils a valid alternative for indoor or temporary applications. One of the most relevant projects is the façade for the Finmeccanica Pavilion designed for the Farnborough International Air Show, see Figure 2.

### 4.8 PE foil

PE foils do not present any relevant property for the building sector except for the extremely low price, which compensates for its very poor UV and soiling resistance. For this reason its use is mainly confined to greenhouses and the agricultural field with no relevant permanent projects in architecture. However the film has been successfully used for temporary pavilions and installations such as the Mobile Action Space in Berlin.

### 4.10 THV foil

THV foils offer a good flex cracking resistance and long-term stability comparable with those provided by ETFE foils. However its optical properties and resistance to soiling are considerably lower than ETFE. Although it can be easily welded with high frequency welding
machines, its use in architecture is quite sporadic due to the lower mechanical and tearing resistance which reduce its use over medium and large spans.

5 ADVANTAGES

5.1 Technical

Like for other uses of membranes in architecture, the advantages in the use of tensioned fabrics and foils for external claddings are related to the efficient use of the materials with no concerns for bending and buckling with the only exception being the rigid boundary profiles, when used. In addition, the use of a membrane envelope intensifies the level of industrialization of the construction, reducing the number of processing phases and workers involved in the building on-site, and increases the proportion of factory-built components which are only assembled on-site. This has several advantages in terms of efficiency (1 to 10 m² of material processed for each hour of work, according to the material and the type of joints chosen) and quality of the final product.

![Figure 5: Installation of the Torium Shopping Center pneumatic ETFE façade](image)

The manufacturing of the fabric panels is executed in safe and clean spaces by highly specialised workers and specific equipment such as computerised cutting tables and welding machines. The manufactured membrane can be easily packed, stored and shipped due to the reduced weight and volumes of the fabric once folded. Thanks to the relatively low
transportation costs, the packed façade can be delivered all over the world using trucks, railways, waterways of even aeroplanes when required, virtually with no limits in terms of logistics.

Once on site, the assembly process is relatively fast and efficient due to the high level of accuracy of manufacture, the adjustable boundary details and the reduced weights and volumes to be handled, which require less (and smaller) lifting equipment. In addition, there are no limitations in terms of combination with others building materials such as steel, aluminum, wood, reinforced concrete, composites etc.

Finally, tensioned membrane façades can provide maintenance benefits using self-cleaning materials and top coatings, and other intrinsic advantages in terms of inspection and substitution of the components.

The result is an overall cost-effective product able to compete with the main traditional solutions and materials currently in use for planar and complex three-dimensional façades.

5.2 Environmental

The intrinsic efficiency and lightness have a direct influence on the environmental impacts due to the reduced amount of natural resources, energy and waste produced compared with traditional cladding system. However, the membrane’s low mass per area is not directly synonymous with reduced environmental impacts due to the substances and energy consumptions often required during the manufacturing of the main architectural coated fabrics and foils. For this reason, several studies and certifications \(^9,10,11\) have recently investigated the impact of tensioned membrane structures according to a life cycle approach which assess the environmental impacts associated with all the stages of a product's life from-cradle-to-grave\(^12\).

![Figure 6: Comparison of Primary-Energy-Calculation for a Glass roof compared with an ETFE Cushion-Roof for a project in Munich\(^16\)](image-url)
Another insidious aspect which has to be considered is the energy consumption during the operation of the buildings, which, for pneumatic structures, is inevitable, associated with the energy consumption required to maintain the adequate air pressure\textsuperscript{13}, although potentially this can be partially provided by energy produced from photovoltaics.

Despite uncertain aspects, several recent works \textsuperscript{14,15,16} have shown the high potential of lightweight membrane structures in reducing the environmental loads in the building sector. In addition, further advantages are related to the accurate design of the boundary and supporting profiles which can be reduced due to the bigger spans and the minor permanents loads\textsuperscript{17}.

Thanks to their reusable connections, the components of the façade can be easily disassembled and recycled (ETFE foils and PES/PVC fabric are entirely recyclable) or reused (such as the bags produced by the famous brand Freitag from recycled fabrics). Finally, tensioned membrane envelopes can be successfully used in order to improve the thermal performances of existing buildings reducing the heating/cooling costs thanks to better insulation and shading of the external façade. For specific projects, a further advantage could be obtained through the use of high quality self-cleaning surfaces which reduce the use of cleaning products.

5.3 Comfort

One of the main challenges of the building sector in recent years is focused on the reduction of the running costs and the environmental impacts without compromising the levels of comfort. Despite the critical aspects related to the use of lightweight membrane envelopes, mainly due to their intrinsic low thermal inertia and their complex building physic driven by radiation input and losses, several recent projects have shown the advantages of membrane façades in new buildings and retrofittings\textsuperscript{18,19}.

In addition, when the translucency of the façade is not a priority, it has been shown that the thermal properties of tensioned membrane envelopes can be progressively improved with a wide range of additional layers such as the latest translucent aerogel mattress or the more traditional sandwich systems based on fibrous insulating material, such as in the Chatham Maritime Food Court, developed by Architen Landrell\textsuperscript{20}. The use of fibrous insulating material can considerably improve the acoustic insulation of the envelope and it is generally combined with open mesh fabrics or micro-perforated foils in order to increase the acoustic absorption properties of the envelope, thus reducing the reverberation time which generally affects large spaces. Specific research in this field has been carried out in recent years at the Eindhoven University of Technology\textsuperscript{21}.

The use of tensioned façades for retrofitting is generally combined with the need to improve the shading of the external wall and/or protect additional external insulation. The shading performance of coated fabrics and nets has been measured in several combinations\textsuperscript{22} and can be successfully applied to reduce the solar gains during the summer. A correct design of the envelope should find the appropriate compromise between a waterproof and airtight envelope and a good level of natural ventilation in order to reduce potential risks of condensation.

Finally, membrane envelopes offer an excellent way to provide a good level of natural
illumination, which, however, has to be carefully considered in order to provide the correct balance between direct and diffuse light, an adequate color of the light and a good perception of the inside/outside.

5.2 Safety

The high level of prefabrication of tensioned membrane façades can lead to a considerable reduction of the of manufacturing activities carried out on site, which are reduced to the mere assembly of components and the final installation. The reduced weight and volumes managed have positive impacts on the installation process which is generally quicker and based on smaller lifting equipment. But the safety aspects of tensioned membrane envelopes are not solely restricted to the building phase. The work of Kawaguchi has shown the remarkable advantages due to the flexibility and lightweight of membrane ceilings and envelopes in the case of earthquakes. ETFE foils provide similar safety benefits when used as a replacement for (brittle) glass in overhead situations, such as atria. In addition coated fabrics and foils have a good fire behavior and have been successfully approved in several projects. In particular PVC coated polyester fabrics will self-extinguish quickly upon removal of the heat source and the use of specific additives prevents the formation of flaming droplets. On the other hand, glass-based fabrics are classified as non-combustible materials. In case of fire the seams connecting the membrane panels immediately above or adjacent to the area of the fire will open creating an hole in the envelope through which the intense heat and smoke is discharged.

6 CONCLUSIONS

In summary this paper aims to present the recent developments in the design and manufacturing of tensioned coated fabrics and foils façades. The paper describes the main systems and materials currently in use and analyses their main technical, environmental, comfort and safety advantages.

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ARCHITECTURAL DESIGN OF MEMBRANE LIGHTWEIGHT STRUCTURES AND THE CONSIDERATION OF BUILDING COSTS

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ABSTRACT

In the context of membrane architecture it is a necessity that design teams of people with different expertise collaborate in an integrated design process. Knowledge in geometric design, engineering, installation, and costing as well as understanding of their correlation is fundamental for the success of membrane projects.

Especially cost estimation is often neglected in the early design stage. As a consequence decision makers cannot reason over the realization of a project until very late. The result is, time and money are spent without knowing the future of the project.

For their intentions and ideas to be realized, it is crucial for designers to have knowledge about the expected costs and the economic influence of various design parameters already at the very beginning of a project.

A software tool that gives the designer an instant cost estimation of the current design could provide valuable knowledge about how to optimize the costs and bring the original design intention into reality. Surveying and quantifying the parameters within the designer’s sphere of influence was the basis to create an interactive tool integrated into the “Formfinder” design software for membrane lightweight structures.

This paper discusses those parameters the designer can control in this early stage of a project. The results of cost estimations of membrane roofs with different constraints will show their influence on the project costs. Furthermore a short description of our software tool will be given.

Conference TOPIC / CATEGORY
Invited Session 50: “Detailing – Case studies – Installation Process"
1 Introduction

The main difference between designing rigid or form-active structures is the approach to find the geometry required for a certain purpose. In general one can say: rigid structures are shaped; form-active structures are derived.

Conventional (rigid) structures are given a specific shape or geometry. Spans, dimensions, and materials are chosen to be checked mathematically in relation to given load cases. If the proves fail, either the geometry, sections, or materials have to be changed. This iterative process continues until an acceptable result in ULS (= Ultimate Limit State) and SLS (= Serviceability Limit State) is achieved.

The form-finding of tensile surface structures on the other hand starts with the definition of the layout of an area to be covered as well as the arrangement of fixation points and boundary conditions (such as stiff or elastic edges, or stress ratios between warp and weft). Within these the membrane is supposed to find equilibrium. One layout of boundaries allows only for one mathematical solution, meaning one particular membrane shape. Every alteration of the arrangement results in a new variation of geometry. [2]

Therefore designers should be aware of the potential they have in hand to control the (commercial) success of a membrane project. Right after the project idea the first steps are the most important and have a major influence on the following design process. In the beginning the architect has to deal with a lot of uncertainties. At first the knowledge and the degree of detailing are very little and the risk for the project not to get realized is very high. The more time passes the more knowledge is gained and the risk of failure reduces.

2 Study

To visualize the relation between optimization of geometry and the resulting costs, design studies have been performed on a four-point sail. Every single variation of the base geometry ran through a complete predesign of all main construction components, such as membrane, edge cables, struts, guying cables, foundations, and anchors. In a next step the individual project costs were estimated and the gained values were visualized in figures for further analyses.

The three independent studies included the following:

1. Variation of the membrane curvature
2. Variation of the edge cables curvature
3. Variation of the inclination of the guying cables

2.1 Geometry

For easier understanding of the effects of altering the geometry the most simple roof type was used – a four-point sail.

![Figure 1. Layout of standard sail](image)

2.2 Study 01 – Variation of membrane curvature

The membrane surface of the base roof is designed with an arch rise of 10%. In this study the curvature of the membrane was altered from 3-20%. As an additional precondition the minimum clearance of the sail at the highest point was set 3m. As a result the height of the corners changes from model to model.
2.3 Study 02 – Variation of edge cable curvature

In this study the effects of different curvatures of the edge cables were analyzed. Starting with the base roof, the considered range of the arch rise is 3-20% of the fixation point distance. The clearance and the lengths of the struts remain the same in all models.

2.4 Study 03 – Variation of inclination of guy cables

On basis of the reactions of the standard roof, the angles of the guy cables against the vertical were altered. The angle between the strut and the guy cables varies from 7.5 – 57.5°.

3 Study 01

To visualize the effects of altering the membrane curvature the following figure gives an overview of the development of the bearing loads (red and green graphs) and the forces in the edge cables (purple graph). To meet the precondition of avoiding ponding, the prestress (blue graph) increases with the diminishing curvature.
As generally known, a reduction of membrane curvature causes bigger internal stresses and reaction forces (Figure 5). Taking the standard sail as a point of reference, the results show, when the curvature increases from 10 to 20 percent ($\Delta = 10\%$), the reaction forces decrease only by 25 percent. Almost the same variation occurs when the curvature is reduced from 10 to 8 percent ($\Delta = 2\%$). At the extreme end of the investigated range (curvature = 3%), the reaction forces are 175 percent higher than at the base geometry.

As expected, the less the membrane is curved the higher the total costs grow. The unit costs change almost in parallel to the project costs.

4 Study 02

In Study 02 the altered design parameter is the edge cable curvature. Figure 7 summarizes the development of the bearing loads (red and green graphs) and the edge cable forces (purple graph). In contrary to Study 01, here the prestress (blue graph) rises with the increasing curvature.
Throughout Study 02 the location of the fixation points remains identical to the base geometry. The most obvious effect of reducing the edge cable curvature is a bigger membrane surface. Furthermore the edge cables become stressed more and consequently the reaction forces increase. Having the same membrane curvature in all models, bigger arch rises at the borders reduce the membrane stresses. As a result the sail requires higher pretensioning the more the edges are curved.

The variation of the edge cable geometry has a major impact on the development of the covered and the membrane surface area. While the progression is almost linear over the whole range, the membrane surface of the biggest sail measures 2.3 times the smallest.

Looking at the data of the structural calculations (Figure 7) the progression of the forces eases at a curvature of about 12 percent and does not vary much until 20 percent. Although the total costs grow from high going to lower edge curvature (caused by higher reactions forces), the development of the unit costs is misleading (Figure 8). Dividing the costs (having a polynomial progression) by the surface area (almost linear development) results in a U-shaped graph. First the unit costs follow the total costs but at 12 percent curvature the chart comes to a turning point. From this point on the unit costs grow again. The reason is the membrane surface decreasing faster than the costs.

Figure 7. Study 02 – Overview structural calculations

Figure 8. Study 02 – Cost calculation
In reality this situation is more of a theoretical kind. The roof geometries of the last models (with strong curved edges) do not seem reasonable from an architectural, practical, and economical point of view. In reality strong curved edge cables will be used for architectural reasons or if local stress reduction is required.
5 Study 03

The following diagram indicates the progression of the strut forces (blue graph) respectively the forces in the guy cables (red graph) and as a result the required size of concrete foundations (green graph).

![Diagram showing strut and guy cable forces and concrete foundation size](image)

**Figure 9.** Study 03 – Overview structural calculations

As only the strut and guy cables were considered in this study, Figure 9 only shows the progression of the reaction forces and the corresponding size of the block foundations. The strut is constantly inclined at 12.5°. Therefore the angle between the strut and the guy cables varies from 7.5° to 57.5°. As expected, the smaller the angle, the higher become the reaction forces of guys and struts. In the range from 57.5° to 37.5° the forces develop almost linear. While the investigated inclinations augment in steps of 5°, in this range the internal forces grow by about 10 percent from model to model. From 37.5° up to 7.5° the augmentation is more exponential. The size of the block foundations evolves accordingly.

The effect of a variation of the enclosed angle between guy cables and struts is best seen when related to the costs (Figure 10). The visualization shows a development according to the progression of the forces. The single points on the left give the derived values for a support solution with restraint columns. It is interesting that the costs of sails with very narrow strut-cable-arrangements top the ones of restraint column solution. In this case

![Diagram showing cost calculation](image)

**Figure 10.** Study 03 – Cost calculation
architectural reasons may still be controlling. At an enclosed angle of 7.5° the required steel profile is a CHS 168.3/6 (buckling stresses authoritative) while the restraint column has to be a CHS 406.4/12 (bending stresses authoritative). As explained in the presumptions the more disadvantageous high point (h = 4m) was dimensioned and taken for all fixation points. The conclusion is that restraint columns are only reasonable up to a certain column length.

The costs certainly mirror the progression of the forces.

6 Perspective / Outlook

The idea for the topic of this study is based on the fact that there are plenty of different cost estimation (software) tools for conventional rigid structures on the market. All of them needed a certain degree of detailing for more or less reasonable results. Yet, there is no such tool for form-active structures. Some design guides give advice regarding geometry and detailing based on research and on practical experience. Only the relation to the costs is not discussed in any of those.

As mentioned in the introduction it is valuable to know about the efficiency of a structure (regarding the load transfer). However in reality the costs are the most authoritative design parameter. Very often architecture is driven by the available budget. Maximization of profits usually is the main aim. Concerning textile architecture aiming for a cheap solution may not be an issue. Deciding for a membrane roof already implies, an architectural solution is desired for the intended purpose, meaning tensile structures are usually not the cheapest one. Anyhow, regarding sustainable design a meaningful utilization of resources is to aspire at all times.

This study should lay the basis for a series of investigations and studies on design parameters and how the optimum costs can be achieved by considerate amendments. The intention is to extend this research to other types of geometries as well as to other stages of the project. This thesis even leaves space for further investigations on the four-point sail. Additional fixation points, rigid edges, use of other materials and products, or special design elements (like loops) are additional design parameters and their effects can be interesting for everyone involved in membrane design.

The idea is to collect data and to publish the results in a real design guide. In schools design is often seen as a combination of appearance and function of a product. Reality shows that the commercial aspects, costs and return on investment, are at least as important. This design guide should focus on all of these aspects. The wheel will not be invented anew but established knowledge will be extended by and associated with new aspects in design.

Furthermore the data gained should be used in a software tool that gives designers an instant cost estimation of the current design. An interactive tool integrated into a design-software for form-active structures, such as the Formfinder, would allow for a significant simplification of the calculation process. Reduced complexity increases the understanding of the cost-driving parameters. A first version of this software tool which is not released yet visualizes these cost factors and can support designers to keep their own design by providing a set of positive arguments to convince decision makers of the design intention and the design concept.

We believe, comprehensive research on this topic can make a major difference. If more and more people involved in membrane design understand the complex relation between design and costs, the number of realized projects will increase significantly. Eventually this could encourage the whole industry and make membrane architecture more common.

References


BATENSO NEW TEXTILE FAÇADE FRAMING SYSTEM: DEVELOPMENT, CFD WIND ANALYSIS OPTIMIZATION AND APPLICATION TO THE “OASIS” HOTEL AESTHETICAL REFURBISHING AND CLIMATIC IMPROVEMENT

STRUCTURAL MEMBRANES 2013

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Key words: Textile Façade, Framing System, BATenso, Solar Protection, Wind, CFD.

Summary. This paper describes the use of BATenso Textile Façade framing system and its application to the Hotel Oasis refurbishing, for aesthetical and climatic improvement.

1 INTRODUCTION

The “Oasis” Hotel was built on the Canary island of Lanzarote during 1990-1991 in an eclectic style, which mixed references to Islamic architecture built with traditional and sumptuous materials considered a symbol of luxury, with building systems that were considered "high tech" at the time (a large curtain wall façade and pyramidal skylights over the main access to the lobby).

Figure 1: Hotel Oasis façade, before and after the textile façade installation.

This curtain wall, besides representing an energy problem because it did not have any sun protection system, is crowned with pyramidal skylights that once met the aforementioned iconic role, but after more than twenty years, it was necessary to be redesigned since this icon image had not resisted with enough dignity over time. For this purpose, BATenso Textile Façade framing system was used in the refurbishing project.
2 THE SITE: EARTH, WIND AND FIRE

Canary Islands’ climate is subtropical, due to the geographical location some 4º from the tropic of cancer. Temperatures between the seasons may vary only some 6º C, and the monthly islands average temperatures are from 18º to 24º C. Nonetheless, each one of the islands has specific conditions and climate variations.

Lanzarote is one of the seven Canary Islands, placed in the Atlantic Ocean at 1.800Km South West from the Spanish coasts and only 95Km West to the African coast and the Western Sahara and the desert Morocco’s coast. Its climate is also subtropical regarding the temperatures, but dry and subdesertic referring the rainfall.

Earth is mainly black lava and basaltic stone, due to the volcanic origin of the islands, Wind is constant with a 2012 average of 22Km/h and a maximum of 85Km/h in June, and volcanic Fire is still present in the island with several active fumaroles and geysers.

A remarkable phenomenon in close relation with the strong wind which whips constantly the island is the so called “calima” or “sirocco” haze. Main wind direction is usually from northeast, but when it blows from southeast (sirocco wind), Sahara Desert dust in suspension flies to the Canary Islands raising temperatures and covering all with an orange colour dust.

Figure 2: Lanzarote and the Canary islands under calima or sirocco haze phenomenon. Strong wind throws streamers of orange dust from the African desert coast. Nasa Terra/MODIS.

It will be shown later how this haze phenomenon was considered in the fabric type selection, its composition, finishing and colours.
Wind was not considered as a “static” load applied on a stiff building, following the current regulations (CTE, Código Técnico de la Edificación in Spain), but as a dynamic load applied on a deformable shape, which generates different shapes depending on time and therefore, a certain amount of wind energy is dissipated in textile + frame + structure deformation.

Figure 3: Whilst a flag flutters in a turbulent flow (with high oscillation frequency and fast deformations), a membrane structure gallops in it (low oscillation frequency, slow shape variations).

By means of CFDtex calculation method developed by the author in his Ph.D.¹, consisting on several iterations between CFD (Computer Fluid Dynamics) and tensile structures software, a significant wind load reduction was achieved, as shown on Table 1.

Figure 4: example of one of the frames (red) CFDtex wind analysis and deformed shape after several iterations.
Table 1: CFDtex results: CTE (Spanish Code regulations) vs. CFDtex.

<table>
<thead>
<tr>
<th></th>
<th>CTE (SPA code)</th>
<th>CFDtex method</th>
<th>LOAD reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iteration 1</td>
<td>1.29</td>
<td>1.19</td>
<td>-92%</td>
</tr>
<tr>
<td>Iteration 2</td>
<td>1.29</td>
<td>1.04</td>
<td>-81%</td>
</tr>
<tr>
<td>Iteration 3</td>
<td>1.29</td>
<td>1.02</td>
<td>-79%</td>
</tr>
<tr>
<td>Iteration 4</td>
<td>1.29</td>
<td>IRRELEVANT</td>
<td>IRRELEVANT</td>
</tr>
</tbody>
</table>

Modelling the complete façade alike, the load reduction is higher regarding CTE values (for a stiff and continuous façade), since wind flows through the frames’ joints (50mm vertical joints and 100mm horizontal ones) reducing the final wind load.

Figure 5: BATenso tests: the system shows higher tensile strength than any kind of membrane.

In spite of these reductions, since wind load was still the main challenge of this façade due to its location, two BATenso Textile Façade framing system characteristics made it specially suitable for this project: its high prestress level applied in workshop, and the possibility to restress the fabric on site after some time.
3 THE HOTEL AND THE REFURBISHING PROJECT

As part of the renovation project, it was raised the partial occultation of the main façade, the pyramidal skylights over the lobby and the panoramic elevators’ cores inside it.

The solution should allow nuanced transparency from inside, as it was not seeking to generate an opaque screen but a filter to keep the view of the Lanzarote landscape that existed from the hotel atrium. In panoramic lifts, it was searched precisely not to lose its panoramic nature, keeping the views from them, but hiding from outside all the machinery and enclosure walls cladding in “old fashioned” marble.

Regarding the façade role, it had to be a solution not only to renew the look, but also to improve the energy efficiency performance, using a product that would combine characteristics adapted to this target, with a range of enough colors in order to achieve an aesthetical solution. To fulfill all these premises, it was selected the recyclable Polyester/PVC mesh Stamisol FT 381 by Serge Ferrari, following the architects color design: purple red color in the elevator cores and the combination of three different colors (changing grey, blond ash and grey dust) on the exterior façade, not only regarding aesthetical criteria but also as the best colors in terms of aging and suitable for the haze “sirocco” phenomenon described above.

Fabric longevity is guaranteed due to the special construction of the mesh, which is prestressed during its manufacture and allows high prestress levels which last after many wind exposure cycles, and a thicker coating of the yarns than similar meshes which is dirt-and-dust protected, making easier its cleaning, and suitable for marine environment.

Regarding the climatic improvement due to solar protection, each color has different values: the average percentages are 35% Rs (Solar Reflection), 30% Ts (Solar Transmission) and 40% As (Solar Absortion). So a final value of shading coefficient “g” total exterior = 0.33 was achieved, whilst former curtain wall façade “g” was over 0.65.

Previous thermal simulations estimated a power consumption reduction for cooling of around 42% along the year.
The main project credits were:
Property: INVERSIONES INMOBILIARIAS OASIS.
Property Manager: SUMASA, Miquel Fibla
Project: INTEGRAL S.A.
Architect: Jordi Seguró Capa
Technical architect and Project Manager: Ángeles Atoche Peña
Collaborators (design): AdM arquitectes
General Contractor: HORMICONSA CANARIAS S.A.: Mario García, Juan J. Fernández

Textile façade engineering: BATSPAIN: Javier Tejera, Marian Marco
Textile façade manufacture and installation: BATSPAIN
Fabric supplier: SERGE FERRARI

4 ADAPTING THE FAÇADE TO THE EXISTING BUILDING

For adaptation to the existing building, both in the core lifts as in the outer façade it was designed and built a steel structure which generated the new envelope, with different nuances since the exterior had to bear the Lanzarote’s wind loads.

4.1 Façade, secondary structure

Thus this structure was designed Vierendeel type, wide 60cm from the glass façade, maintenance catwalks with tramex floor were designed, solution which while generating a light and highly resistant structure against wind, enabled an easy installation of the frames from inside the structure without using lifts or cherry-pickers, and allowing maintenance and cleaning of the textile façade and glass curtain wall.

Figure 7: BATenso Textile façade framing system applied on 3D model

Former curtain wall façade did not have any external structure, so all the wind loads were
transferred from the frames to the new steel structure and from it to selected points in the façade, which had a steel sheet cladding and a steel structure behind them. Due to the new façade design and volume, in several points there were no even curtain glass façade behind the new textile one, so it was necessary to transfer loads to other planes of the glass one, as shown below.

Steel structure was built on site and textile façade cladding was installed immediately after each structure module was finished.
Special BATenso hanging clamps were designed specifically to enable three-dimensional adjustment and regulation of the frames in order to adapt them to the structure with tolerances, avoiding errors during manufacture or installation since the frames were manufactured on theoretical measures and prior to the structure construction.

![Figure 9: BATenso hanging clamps detail](image)

Complete installation plans with simple colours codes were drawn in order to fit each frame with its own hanging clamps, making an easier installation.

![Figure 10: Façade installation plan. Each frame had specific colour and sizes codes, and hanging clamps type](image)
4.2 Elevators façade, secondary structure

In panoramic elevators, the textile façade had only an aesthetical role, since it was not exposed to wind loads, so it was feasible to design a lighter structure: simple trusses were hanged from each slab, in order to bear the façade frames + structure self weigh loads.

For this purpose it was crucial the selection of a textile façade system which allows big frames sizes with a relatively light weigh in comparison with traditional façades, the same way that textile roofs must have structural ratios much lower than conventional ones[^3]. In elevators’ façade, the Textile Façade + steel structure weigh was less than 10Kg/m².

[^3]: For more information, refer to the original source or further research on the topic.
5 BATENSO TEXTILE FAÇADE FRAMING SYSTEM

As it was exposed above, regarding the textile façade system to be used, the wind area of Lanzarote, with strong winds whipping constantly the island, and the need to send by sea in containers precast complete façades finished, decided the use of the new textile façade framing system BATenso by BATSPAIN, consisting on precast aluminum frames in a transportable size with special features:

Table 2: BATenso Textile Façade framing system, main characteristics.

<table>
<thead>
<tr>
<th>BATenso TEXTILE FAÇADE, CHARACTERISTICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 It is a framing system, precast in workshop. Big and light frames.</td>
</tr>
<tr>
<td>2 It allows a high prestress of the membrane.</td>
</tr>
<tr>
<td>3 It is possible to restress on site after some months or years.</td>
</tr>
<tr>
<td>4 It is possible to install PES/PVC, GLASS/PTFE, GLASS/Si, PTFE, ePTFE.</td>
</tr>
<tr>
<td>5 It fits membranes and films, not only mesh.</td>
</tr>
<tr>
<td>6 It adapts to temperature changes without wrinkles.</td>
</tr>
<tr>
<td>7 It avoids dirt to come between membrane and frame.</td>
</tr>
<tr>
<td>8 It allows anticlastic geometries on façades.</td>
</tr>
<tr>
<td>9 It does not need any machine to be assembled-prestressed, just hand tools.</td>
</tr>
<tr>
<td>10 It is economically competitive.</td>
</tr>
</tbody>
</table>

In the “Oasis” project a total amount of 182 frames were installed, which were transported to the site properly protected and ordered by colours and sizes according to the installation plan, in order to minimize movement in situ that could damage them due to its big size.

Figure 13: BATenso framing system process, from workshop to installation
6 CONCLUSIONS

- Textile Façades fit perfectly with the special requirements of refurbishing projects in which lightness, economy, solar protection and aesthetical renovation are imperative.

- BATenso Textile Façade framing system is suitable for refurbishing projects as the exposed above, in which its main characteristics (precast framing system, lightness, high wind resistance, etc.), makes it perfect for this purpose.

- Special features such as BATenso high prestress levels, or its on-site retightening ability, fits perfectly in projects with wind exposed facades.

- Analyzing membrane structures wind loads with the combination of CFD and Tensile Structures software, significant reductions can be achieved, optimizing the result.

- All pieces in a textile façade should have adjustment ability and respect tolerances, in order to avoid errors and to make an easier installation.

REFERENCES


CASE STUDY TERRA ALTA SKY LOUNGE MEMBRANE STRUCTURE IN EL SALVADOR CENTRAL AMERICA

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webpage: http://www.construguate.com/
Colegio de Arquitectos de Guatemala (Guatemalan Architects Association)
webpage http://colarq.com/

Key Words: Central America, Tensile Membranes, Guatemala, Development, History

Summary: This document is about a project developed with unique circumstances around it. Such as altitude, wind current and the adaptation to an existent and occupied building which was not prepared for a tensile structure from its original layout. It also includes examples of other projects developed by our firm throughout Central America that have become unique mainly due to its function, design and use of color.

INTRODUCTION

Tensile membrane structures are often built on the ground or within solid surroundings that can bring enough counterbalances to the tensioned system through foundations, lateral anchors or connections to other building systems.

However, when a membrane structure must be installed 17 stories above ground, the membrane is already cut and welded ready for installation and there is not enough resistance on the flagstone in which the tensile structure is going to be installed the project itself is in danger of not becoming a reality at all. The membrane structure built at the sky lounge of the Terra Alta building in El Salvador, Central America, was nothing short of a real challenge.

Besides the challenge itself, or the elements considered normal in any other project, there were many other factors that needed to be taking into account:

- From the contractor’s perspective: the project’s profit margin, analysis of the flagstone with the building’s structural designer and contractor, architectural form and new elements, and finally an agreed general layout between the parties involved

- From the client’s perspective: the designated budget, delays, and structural interventions to its property and tenant control since the apartment complex was at the time already fully operational.
1. REQUIREMENTS FOR THE DESIGN

The flagstone on the 17th floor was designed as a final terrace, not to be occupied nor used except for the sole purpose of maintenance. However, due to the increasing need of social areas in projects of such nature, the 17th floor was then re-designed to hold a social lounge for the building's tenants.

It included a wooden deck to be constructed above steel frames with rubber bases and what it's known as a pergola. A pergola is an arbor, constructed of vertical posts or pillars that usually support cross-beams and a sturdy open lattice, often upon which woody vines are trained. It served as a sort of protection for the open terrace.

The membrane proposed was serge ferrari pre contraint 902
The design lacked originality and the element of surprise, astonishment, beauty and rhythm that only a tensile structure can provide.

The building as a residential development exploited one of its main features, its view. To the south, it shows an open view of the clear San Salvador’s sky, a lot closer it shows the magnificent view of El Salvador’s main elite golf course, Cuscatlán golf club.
2. MEMBRANE ORIGINAL DESIGN

When tensile architecture alternative became into play as a serious alternative, it was clear that the design had to be developed around that single element, the view. The cover was then, conceived as a 100 square meters four point’s ridges and valleys membrane, three of which were masts, and a fourth point anchored to a concrete column. Of the three masts, one was higher.

Another key element that had to be complied with was the geometry of the membrane due to the location of supports within the flagstone, which ultimately was the main challenge in this project.

3. STRUCTURAL DESIGN AND CONSIDERATIONS

The structural conditions of the building were not appropriate for the implementation of a tensile membrane. The challenge was to figure out how to accomplish the right design and the right function. The flagstone was too weak for the load required, mainly because:

- Height of the masts
- Momentum at the base
- Concrete section of the flagstone
- Site’s strong wind currents

Since 15 cm of the top of the flagstone was just a filling. **Step 1** of the solution conceived was to build a double “foundation” to increase the structural section of the base with new and more resistance concrete.

**Step 2** was to place one inch thick steel plates above and under the flagstone and dew structural concrete filling with structural 1 meter long steel bolts through the whole section. However, the geometry of the membrane caused the bases to coincide with the structural concrete beams of the building, making it impossible to perforate.

Therefore, the concrete beams needed to be avoided so additional 1 inch steel plates were placed.

**Step 3** After the proper placement of the steel
plates, steel joist like elements were design in the lower part beside each of the two main masts to counter balance the great momentum at the base caused by the height and weight of the structure itself and the wind load on the membrane system.

The joists designed needed to take into consideration not only solving the structural problem itself but also the architectural criteria in not creating an element that could obstruct the view. The height of the wooden deck contributed to this factor.
Step 4 There wasn’t enough horizontal space for the steel cables that help counter rest the membrane load, similar steps as the reinforcement of the flagstone were taken.

4. EXECUTION

Once the design and general layout for the project was fully developed, a detailed execution program was set into motion, considering a key factor, the building was fully occupied and the social area, on which the membrane was to be erected, was also fully operational, making the window schedule narrower
A. The poor concrete filing was removed, and the perforations for the later placement of the steel bolts were made. Note that the perforations are not completely aligned due to the asymmetrical steel reinforcement of the flagstone.

B. Before the new concrete was poured we used SIKADUR 32 as an adherent bridge for the older and newer concrete. The diameter of each hole was rehearsed and pvc tubes were put into place, assuring the smooth later placement of the 1m long steel bolts.

C. After the concrete had dried, a stencil was drawn for the exact dimensions and perforations of the steel plates. The steel plates where then cut and perforated, and later where put in their right place above and beneath the flagstone. The steel bolts where
also placed and sealed to avoid corrosion and water penetration

D. The joist elements where welded to the two main masts. And additional bracings where also placed at the base of each mast
5. PROJECT FINALIZED
Central Courtyard Cover

Departmental Educational Institution Enrique Pardo Parra

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Key words: Courtyard, cover, forming process, tensile structure, membrane, assembly.

Summary. The following proposal for a “tensile structure” is originated by the need to generate a better use of the central courtyard of the school Enrique Pardo Parra, by using a textile cover which protects the 700 students from the sun and rain in recreational and cultural events. Project development must consider a number of demands given by the existing building and the climatic conditions of the place. The end result is a symmetric membrane, inspired by the logo of the character "Batman" with independent masts arranged symmetrically ground articulated with different lengths corresponding to the proposed architecture and geometry of the membrane. This paper presents the design experience starting from the initial approach to the constructive concept, analyzing the different aspects of design, installation steps, and final construction, using resources such as models, digital models, functional prototypes, which complemented the development of construction details, the pattern and assembly strategy for the structure.

1 INTRODUCTION

Departmental Educational Institution Enrique Pardo Parra, located in the town of Cota 26 km from Bogotá, Colombia invited in 2012 several proponents for design and construction. AR KLEIN ENGINEERS LTD, is hired by one of the proponents to execute the design of a lightweight cover for your central courtyard with the aim of creating a covered space outside the classroom to protect them from the rain and high temperatures and to allow the realization of cultural and sporting events, regardless of the prevailing weather.

The school has no indoor sports areas, theatres or additional facilities for these activities and could not afford within its budget to build a sports centre or additional building conventional for this type of event. The College has generated a central courtyard cloister spatial configuration, which is used to propose a cover from a tensile structure that would allow height variability, not darken the atmosphere and the approach of few supporting
elements unlike other structural proposals raised.

Once the site of implantation of the deck was defined, there were some school characteristics, design determinants generated by the existing construction and climatic conditions of the place. In this direction one of the first limitations was that the tensile structure proposal was not supposed to be supported to the existing school building, because the building’s maintenance report to date, had found some cases of fissuring in the main structure from reinforced concrete frames covered with masonry, and a worsen condition was not desired. For this reason, we proposed a completely independent structure to the structure of the school, using the outside area to locate the anchor masts and cables tensile structure.

The shape definition should take into account the climatic conditions of the place. Cota's population has long record rainy seasons ranging from three to four months of winter season, with frequent hailstorms. This determines the importance of the geometry of the membrane for proper water drainage and prevents rain water damming on canvas, plus all this cross winds from the middle mountain in great force winds blast.

2 FINDING THE FORM

2.1 Design concept

The architectural image that is achieved starts from a striking idea that was appealing to the students. The membrane shape is inspired by the logo of the character "Batman", getting in the front view image of the superhero symbol. This image was considered close to the emotions of students, differentiating their school, compared to some others in the region with one similar to the very students of this age, something furtive, yet attractive.

The designer wanted to give the impression of movement of its "wings", at a time when the wind passes through the deck. Inside, the intention was to link the classroom spaces, and
show it more like a sports arena, since within the same construction there will be spaces for reading and reflection.

Based on this concept, layouts and models made to scale were performed to see the first approaches to the definition of the form, inclinations and location points of the support structure. It tries to choose the direction of escape of water and the height of the masts central to shaping the final form.

### 2.2 Definition of the form

The nominal area to be covered corresponds to the central square of the campus with dimensions of 32 x 30 meters. Digital modeling is done through Rhino Membrane software design always retrofitted with physical modeling; both were indispensable tools for the conceptual design of the tensile structure.
One of the main challenges in the search process was to define the height of the two central masts that simulated the bat ears. There, the membrane generates a cone shape in each of these high masts. The definition of the shape at these points needed to achieve a proportionate height for geometry and space to cover, where an architectural use was intended for different sports activities and likewise maintain the architectural set. Cones finally lowered enough to have an adequate safety factor in the stresses produced by its own shape and not significantly affect the initial geometry of the membrane.

The result from a top view is almost a square, symmetrical elevation with two high points in the center amounting shaped cones and variable height lows distributed on the perimeter of the membrane, achieving a maximum height of 12 meters and a minimum height of 3 meters.

3 STRUCTURAL BEHAVIOR

Continuing the calculation step, we analyzed the structural behavior of the tensile structure under different external loads software platform MPanel Meliar Design. At this stage it was very important to define the wind profiles and the loads generated pressure and suction. As the deck is immersed in the heart of the school, the wind profiles could be very erratic especially in wind directions SE - NW so there were several models to account for this effect.

The loads generated by rain and hail, and their combinations, showed possible sites where the fabric was a little strained and clothing effect could generate wrinkles.

![Figure 4: Analysis of structural behavior under loads of wind, rain and hail](image)

The work was very thorough and required several geometric adaptations, prioritizing the prestressed cables that stress the structure, on the slopes and at the height structural elements to achieve a satisfactory deformation response. Parabolic profiles were initially used in the fabric, so that the weighing cable that would go on these profiles could have a catenary consistent, not too deep nor too smooth. Stressing a fraction of the load that can endure, you got a very suggestive and "pre-stressed" the whole deck. For wind suction, this weighing cable, would take most stresses produced by the suction, while the others would take the inverted efforts by effect of the pressure, leaving this cable only its initial pretension. Also as mentioned above, the cones’ heights were modified on efforts to try and prevent over-stress to
the fabric and finally achieve a good balanced structure.

Figure 5: Isometric. Main structure, masts and cables

4 PATTERN

The Membrane area of 1100 m². The decision to define how many patterns and dimensions conformed the membrane was taken from the process of assembly and transportation of the structure, based on a model made 1:10 scale, which facilitated the understanding of many construction details and the thinking about the mounting strategy. It was clear that by the dimensions of the pedestrian entrances of the school, it was impossible to use a crane for lifting PH membrane. For this reason, the membrane is divided transversely into two equal parts, to facilitate the lifting of the same structure with a limited workforce and mechanical equipment that could enter the access in question. Once the two sections are reinforced membranes, bind both sides, before the elevation through a rack.

Figure 6: 1:10 scale model. Observation and analysis of pattern for the cones

A total of 64 patterns set with a between width of 1.10 -1.30 meters and 8 meters of lengths longer. Each of the strips of the membrane was sealed according to industry standards with sealing width is 5 cm.
5 STRUCTURE AND MATERIALS

A supporting structure was chosen of steel pipes. The membrane is supported in total by 17 structural steel tubular masts with diameter of 10 - 15 inches with varying lengths depending on the design requirement, having masts to a height of 12m and a minimum height of 3m.

The foundation includes reinforced concrete footings, setting the mast to the shoe, fixed initially proposed (without the possibility of rotation) with steel bolts and nuts, while performing the lift and shape of the supporting structure in cables. And then subsequently to expand the covers bolts are removed some prevent rotation and the support is articulated. The wires are of SAE 1070 steel with 6 strands of 19 wires each RRW accordance with standard 410D in diameters of 3/4 to 1 ½ inch.

The base membrane material is polyester yarn coated with a high strength polymer layer. (PVC) Ferrari. Across the membrane is made of a single type of material and of the same series. The membrane will warp and weft stability, tensile strength (warp / weft) greater than or equal to 420/400 daN/5cm, tear resistance greater than 55/50 daN, with a grip 12 daN / 5 cm, the coating thickness of at least 200 microns and 100% recyclable.

6 MANUFACTURING AND ASSEMBLY

In charge of the company ARTE Y DISEÑO HAWAR S.A. with Quality control from JAM ENGINEERING, SAS has specialized machinery to cut and paste membranes, along with the expertise in mounting this type of structure. The work done to them was constant feedback to devise the best installation strategy, taking into account not only a crane PH
could not access the installation site, if not by the size of the income and the type of floor was difficult access for smaller cranes. Once ready, the foundation and the anchors of the masts, the filament and the support structure tied to the steel cables, so that the steel cable structure is now able to withstand lifting of the cover through the same support structure and additional cables. Assembly is done in the morning when there is less wind has low speed and scheduled holiday season to avoid the presence of the student community in the area.

The membrane reaches the mounting place divided into two equal parts, located in its preliminary position, proceeds to join the two parts of the membrane on the ground through the zip fastening system. Together the two sides begin to raise the membrane from the center through the central mast. Funky first rises and extends the left side of the membrane while the right side of timber such that the structure has no asymmetric loads. It was very important to understand the assembly process, to define the best pattern, and decide correctly in many parts divided the membrane, since parties have split over the membrane, the assembly process had more complications at the time of lifting these sections. This way installation coordination was made easier by dividing the membrane into two sections alone to avoid unions that might make mounting a headache caused by asymmetric deformations.

The membrane is stretched as it rises through the external wires and the positioning of the masts. Tightening the perimeter cables is performed in several stages in all lights. The cable length paraboloid is performed accurately in respect to the geometrical shape of the membrane. The structure reaches its final position after thirteen days of installation, additional cables are secured on a single network, subject to the upper deck of the masts and make sure the bolts and nuts so that the structure is stable and secure. The structure was designed to fail membrane, leaving no trace structure of local or general failure. Finally, we take appropriate security measures to visualize tensioner’s cables that reach the floor, and have additional structures for the management of rain water flow.

The images below illustrate the assembly process more clearly:

Figure 8: Assembly process structure, masts, anchor plates and cables
12 CONCLUSIONS

- The main defining feature of this project is dynamics of supported cables that carry the prestressed cables that stress the structure, ensuring the stability of the structure at the time of additional loads.
- It is a self-supporting structure that does not affect existing school building. Thus takes advantage of the outdoor area to locate the main structure based on poles and tensioning cables.
- The architectural image is achieved satisfactory structural lightness and nice architectural space for institutional use, inspired by the wings of the bat symbol character from "Batman". The proposal meets the objective being a lightweight shell, which darkened the central square and to provide students and teachers a useful area for conducting sports and cultural events, protected from rain and sun.
- During the design and structural calculations were definitive analysis of wind profiles and the upper cone geometry to achieve a balance and structure that was not about struggling, achieving proper slopes without affecting the architectural image.
- The overall design lasted a total of five months; two months organized in architectural design and structural calculation, and three months the manufacturing and assembly process. This structure was a good experience to propose structures for the development of recreational and educational activities taking advantage of open spaces and present in the type of faculty, very common in Latin America, and with
limited budget to build conventional structures with the same result.

Figure 3: Central courtyard cover Departmental Educational Institution Enrique Pardo Parra

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Low-tech or High-tech?
“cut.enoid.tower” - three times two facets of irregularity

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SUMMARY

Do membrane structures belong to low-tech or high-tech products?

The concept of low-tech is mostly understood as the counterpart to high-tech and refers to technology which is developed under the aspects of easy function, easy production, easy service, robustness and easy maintenance. In most cases low-tech-solutions bear a huge amount of intelligence deriving from long-lasting processes of trial and error. Therefore the majority of our structural systems is based on low-tech considerations. Building with textiles is one of them.

This paper looks into the question above by the means of the realized, experimental structure “cut.enoid.tower” a complex system of combined structural principles. The three aspects – Irregularity as a result of combined structural systems, Irregularity in detail as a result of geometrical necessity and Irregularity as a feature of architectural complexity – spanning from the global design approach to detailing, general rules (low-tech) used tools for formfinding (high-tech) and the final result (low-tech) are highlighted.

Keywords: Catenoid, minimal surface, membrane, irregularity, lattice structure, tower, selforganizing forms, architecture and structure, aesthetics,

CONTEXT

The objectives of Structural Membranes 2013 are to collect and disseminate state-of-the-art research and technology for design, analysis, construction and maintenance of textile and inflatable structures. In this context we face a question, which represents a balancing act between structural/formal principles originating in selforganizing processes and developments on the material- and software sector:

Do membrane structures belong to low-tech or high-tech products?

The concept of low-tech is mostly understood as the counterpart to high-tech and refers to technology which is developed under the aspects of easy function, easy production, easy service, robustness and easy maintenance. So low-tech in general does not give information on the grade of intelligence behind a certain technology or product but focuses on the output under above mentioned aspects. In most cases low-tech-solutions bear a huge amount of intelligence deriving from long-lasting processes of trial and error. Regarding architecture and structural engineering the majority of our structural systems is based on low-tech considerations. So low-tech can be described as a design and construction philosophy. Vice versa the newest scientific knowledge is often used for the development of low-tech and therefore complex solutions. Low-tech products are mostly as well characterized by the consideration of aspects like of least material consumption, least (human) energy consumption and waste prevention – aspects of sustainability. Building technologies are mostly connected to a special kind of material, its specific material properties and the way the material is used - the technique. Arches build from stone, hanging bridges build from robes or even from grass like the Qeswachaka hanging bridge in Cuzco, Peru or forming bundles and bending them like the March-Arabs are just a few examples of the utilisation of simple active principles. Building with textiles is one of them.
This paper looks into the question above by the means of the realized, experimental structure “cut.enoid.tower”/\(^{(*)}\) a complex system of combined structural principles. These structural principles, which can be declared as tension- (minimal surface catenoids), compression- (pin-joint-columns) and actively bent (plates) elements, simultaneously represent low-tech solutions each for itself as well as combined system, which is constituted from interacting elements. Especially the procedure of iterative formfinding respectively finding a structural equilibrium that fulfills other functional and architectural needs and its simulation is part of a high-tech approach using sophisticated tools.

The result of this design approach appears as formal output and can be summarized in three aspects bearing both facets low-tech and high-tech. In this regard the “cut.enoid.tower” highlights the three aspects – Irregularity as a result of combined structural systems, Irregularity in detail as a result of geometrical necessity and Irregularity as a feature of architectural complexity – spanning from the global design approach to detailing with a special focus on the general rules (low-tech) used tools for formfinding (high-tech) and the final result (low-tech).

What is understood by irregular?

Regularities – a misunderstood Simplification?

Usually irregularity is described as behavior that breaches the rule, as deviance, deviation - deviate behavior, not characterized by a fixed principle or rate.

As part of the environment we are surrounded by irregularity. This phenomenon can be found in every field, every scale, in various forms, etc. Based on this insight irregularity can be seen as a complex, not on the first hand comprehensible order.

To demonstrate the two different respectively mutual contradictory definitions following question may be asked. What would be the correct attribute of a square? On one hand each change of a side length creates a variation into a rectangle, in other words each rectangle is a deviation of a square. So the square is the regular geometry, the rectangles an irregular one. On the other hand we could argue that there are infinite rectangles. Therefore both, length and width, having the same dimension must be a very special case. From this point of view labeling the square “regular” could be a misunderstood simplification.

In architecture a move from geometrically clear forms towards more and more complex and individual space creating forms can be recognized. This phenomenon is directly correlated with emerging technologies, which provide the possibility of these kinds of design, in terms of architectural and structural design –, allow for better exchange between designer and producer in terms of data transfer as well as for more complex calculation and production of spatial structures.
Today we can also recognize a change in the perception of irregularity for example as asymmetric geometry in our society. Not too long ago seen as imperfection and not following “divine proportions” irregularity is rated customized and individual today.

This development in design also raises the question if irregularity is produced or received by both - architects and engineers, meaning that irregularity on one hand leads to a higher degree of freedom in the creation of spatial structures – producing – and on the other hand irregularity represents the result of functional, structural, etc. parameters – receiving –.

1 Irregularity as a result of combined structural systems

In general “cut.enoid.tower” merges different structural members like tension- (minimal surface catenoids), compression- (pin-joint-columns) and actively bent (plates) elements into an overall system. Analogue and digital simulation of these combined structural systems result in (seemingly) irregularity, which is necessary in order to achieve a state of equilibrium [iii].

1.1 Tension Elements - Minimal Surface Catenoids

Catenoids are the only rotational bodies that can be a minimal surface in soapfilm-analogy at the same time. [01] For the „cut.enoid.tower“ we can recognize a “spinning” (Fig.04, 10, 11) respectively a “branching” (Fig.12,13) version of prestressed, tension-only, minimal surface catenoid [2,3], which are spanning seemingly “freely” shaped cut-outs of the wooden plates. In fact the cut.enoid.tower’s cut outs were generated by the overall size and distance of the wooden plates, the possible “radius” at given distance, which can be equated with a given height, the position and the eventual collision of catenoid and pin-joint columns. So their irregular “free” shape is caused by several reasons and has further impact on the active-bending-behaviour of the plates. Referring to the fundamental rules in the generation of catenoids we know that its maximum height is directly related to its radius. Further on we know that in contrast to shifted boundaries boundary conditions which are deviating from primal forms like circle or square have little influence on the general shape of the catenoid. [01] So the change of boundary geometry has marginal effect on the surface, which shows fluent forms. On the other hand the boundary is the only geometry that can be visually exactly captured.

Fig. 03  Traditional, low-tech “catenoids” for fishing purposes

Fig. 04  Different stages in the design process of the boundary of the “spinning” catenoid
1.2 Irregularly Arranged, Skew-Whiff Hinged Columns

A lattice-like structure, consisting of irregularly arranged, skew-whiff hinged columns, represents the compression-counterpart to the tension-catenoids. Compared to conventional lattice structures, here pin-joint members do not meet in their reference points or axes. When joining two or more flat or curved surfaces by the placement of hinged, linear elements all preliminary physical and digital experiments showed that a minimum number of six skew-whiff hinged columns are necessary to achieve a stable structure. The placement constitutes an arbitrary irregularity, but exactly this irregular arrangement generates a stable structural equilibrium locking all directions in space.

Due to its longitudinal proportion the plates of „cut.enoid.tower“ are connected by more than six hinged columns. In the design phase six hinged columns basically locked the structure. Step by step columns were added but also removed if possible.

1.3 Irregularity – A State of Equilibrium

Generating a distorted, irregular appearance of the wooden plates simultaneously caused and combined with the interaction of skew-whiff hinged columns and different versions of prestressed, tension-only catenoids, which are spanning “freely” shaped cut-outs, a state of equilibrium can be gained. The distortion of the wooden plates can be investigated as a special case of active bending, which is defined as follows:

“Active-bending is applied intentional for the shaping process to achieve a predefined geometry. It is one advantage of actively-bent elements that the same straight/flat elements can be used for different curvatures. Actively-bent elements are defined as elements that are transformed from the stress-free start-geometry to their end-geometry by elastic bending. Possible elements are beams and plates with a usually straight respectively flat start-geometry.” [5]

The unity of all elements and forces can be read as deflections which were simulated, found and analyzed by means of a series of physical and digital models (Fig.07).

A modification of only one element or parameter of the system will cause a variation of the whole tower-structure in order to find a new equilibrium. This procedure was run through in an iterative process involving the size of the wooden plates, the geometrical boundary preconditions for the catenoids, their position and eventual collision with hinged columns and the correlation of tension- and compression-members, which is visualized by the distortion of the plates, as well as functional issues like climbing (Fig.08).

The distorted geometry again represents a new starting position – we face a classical “the chicken or the egg causality dilemma that has to be solved in constant feedback loops.
2 Irregularity in detail as a result of geometrical necessity

Details of column-heads or the manufacturing of the nets show the simultaneous use of high-tech and low-tech strategies. Low-tech knotting techniques meet high-tech-tools like Grasshopper scripts in order to solve the geometrical positioning of column-heads or the analysis and assessment of the prestressed minimal surface nets. The aspect of irregularity arises as a geometrical phenomenon.

2.1 Irregular Meshes of Minimal Surface Catenoids

As mentioned, catenoids are basically rotational bodies generated by a catenary rotating around a vertical axis. Postulated a simulation / realization as a 3d mesh, this would allow for many identical mesh faces. As soon as the geometry of the boundaries is not identical, respectively different from a primal form and/or arranged in a shifted way each single mesh face turns out to be unique.

The “spinning” (Fig.10,11) and the “branching” (Fig.12,13) catenoid of „cut.enoid.tower“ were executed as cable-net structures with an average mesh-size of 10x10cm. After the digital formfinding using for example Rhino Membrane and Forten all mesh-lines respectively all intersection-points were numbered and their distance calculated. This was done with the help of a grasshopper script which exported all data into excel-sheets. These lists with more than 11300 distances between the 5697 intersection points of the cable net were the basis for the fabrication process – an irreplaceable advantage of high-tech tools. Similarly, over-length for connection details and knots as well as a compensation factor in accordance to the elongation of material behavior under prestress were digitally applied.

In the realization process 1,5km of 4mm high-performance yachting robes (Fig.09) meeting in 5697 knots were tied by students in order to produce the unique shapes of the minimal surface catenoids (Fig.09). In advance several techniques and types of knots were tested according to their friction-
properties (to keep knots in place), material use, degree of difficulty to produce, error-rate in the production process, time consumption for the manufacture and so on. “The Ashley Book of Knots”, which is showing and explaining 3900 time- and field-tested knots, was of inestimable help – a low-tech basis for a high-tech process to achieve a low-tech product in the end (Fig.09).

Fig. 09  1.5km of 4mm high-performance “Liros prestretch” yachting robes meeting in 5697 knots according to techniques from “The Ashley Book of Knots”

Fig. 10  Spatial impression of “spinning”  Fig. 11  Comparison of right-angled and “spinning” mesh-pattern(right)  Fig. 12  Realized “branching” catenoid  Fig. 13  boxer-short-like boundary conditions of “branching” catenoid

2.2 Curved Surfaces Seated Position and Rotation of Head-plates

The wooden plates of „cut.enoid.tower“ are deformed simultaneously actively bent (Fig.08) by tension and compression elements. This deflections cause constantly different distances between the plates. Since the connection between plates and columns is designed as uni-axial hinge the right, solidly on the curved surfaces seated position and rotation of the head-plates of all skew-whiff members as well as the real length of all skew-whiff hinged columns have to be found.

By means of Grasshopper scripts a plain was rotated according to the (structural) system point and angle of the skew-whiff column (Fig.14). Together with the system point the UV values gave information on the right position and rotation of the head-plates (Fig.15).
3 Irregularity as a feature of architectural complexity

The unity and equilibrium of all elements and forces within the structure of „cut.enoid.tower“ can be perceived as irregularity. Similar to self-organizing forms most beholders experience this kind of irregularity as a logical result. In the case of „cut.enoid.tower“ the design was considering architectural, structural and functional issues like climbing, relaxing and enjoying the stunning scenery as well as maximum wind speed of about 140km/h (Fig.16). Deriving from these different demands, which can be specified as parameters, visual, geometrical, functional, structural and architectural unity was generated. This complex unity, consisting of mostly quite simple low-tech components and merged together by high-tech tools, is shown as irregular structure, pattern, arrangement, visual impression, …
CONCLUSION

The question if membrane structures were belonging to low-tech or high-tech products cannot definitely be answered. Focusing on single components of the design, we can assign them to low-tech-issues. Present, mainly digital tools allow for combining several parameters and components. Merging these components into an overall design asks for high-tech approaches, since a simultaneous handling of several parameters is subject of complex processes.

Complex structures of this kind are brought into equilibrium respecting all parameters and physical boundary conditions. Outside interferences or changes of involved elements reactivate the whole formfinding process.

At the moment, structural and architectural irregularities seem to be produced, sometimes implementing random algorithm in order to create irregular structures. This random irregularity is afterwards re-optimized in a structural sense [7,8].

Our experimental model investigations on physical and digital basis again and again have shown that the phenomena of irregularity have two major coalesce aspects:

- Irregularity can significantly increase the freedom in design.
- Irregularity frequently increases, sometimes it is essential for structural efficiency.

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Selected Examples for the Optimization of Cutting Patterns for Textile Membranes

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Key words: Flattening theories, geodesic lines, widths optimization

1 INTRODUCTION

The cutting patterning is an essential part of the engineering process in textile architecture. In the past it was very costly and time-intensive if the waste of material was to be minimized. Nowadays we have fast or even automatic patterning tools where those problems are solved efficiently as we are willing to show in this paper.

In a short introduction we present different flattening theories, the meaning of geodesic lines in this context, the definition of constraints as angles and distances, and the final estimation of the quality of the patterning by numbers. Then the preparation of the cutting patterns for the production is shown briefly as e.g. seam allowances, welding-marks, etc. But patterning is not only a in the responsibility of the membrane-engineer, the architects are using the seam-lines as an important factor in the design of lightweight architecture. We support it with our software by so-called hierarchical cuts, by a fast extension of seam-lines in the neighborhood-membrane-fields (to avoid staggered lines), etc.

Then we put the focus on procedures with respect to selected examples where cutting patterns are calculated and optimized efficiently. The optimization is almost always related to the waste of material. The widths of the patterns should be a maximum with respect to the role-width of the material by considering the quality of the patterns.

Starting from a small four-point-sail we describe precisely the process chain. Then we show the automatic patterning generation of a high point membrane and a big air-hall.

2 FLATTENING THEORIES

Formfinding of membrane structures is usually done by using finite element meshes. The result of the formfinding procedure are 3D coordinates and the mesh consisting of lines, triangles or polylines. In general we have to define seam-lines on the surface in order to receive the widths of all patterns with respect to the role-widths of the chosen material. The
problem of the flattening remains unchanged, if we have to flatten total net-parts or by seam-lines separated sub-surfaces as patterns. The problem can be defined as follows: a surface consisting of points and lines has to be projected into a 2D system. This task can be seen completely independent from the mechanics of the membrane structures, if the so-called compensation is neglected. The compensation is the reduction of the pattern size depending on the material properties of the membrane due to the pre-stress in the 3D-shape. I repeat again: by neglecting the compensation our problem is treated by the map-projection theories. This field is already very old since the surface of the earth (as sphere, ellipsoid or geoid) is shown in 2D maps with all problems that we face also in membrane flattening procedures. The mathematical map-projection theories were developed especially by Carl Friedrich Gauss, who was also a surveyor and responsible for the triangulation network of the Kingdom Hannover (1818-26); he also developed the Adjustment Theory, where the square-sum of residuals of observations are minimized, in order to get the ‘best’ unknowns [1]. We are going to use now both of his concepts for the flattening of patterns; the adjustment theory to minimize the so-called distortions energy Π in the map (2D situation) and the distortion energy itself is a function of differences between values in their 3D- and 2D-positions of the net. Generally the unknowns in our systems are the 2D coordinates of all flattened points, transformation parameters and Lagrange multipliers in case of additional constraints.

Figure 1: 3-D surface with points, links and polylines
Now we start with a simple formulation of the distortion energy $\Pi$, where the energy is only a function of the 2D-coordinates of the flattened net.

$$\Pi(x, y) = \frac{1}{2} v^t P v \Rightarrow \min.$$ (1)

Very generally the distortion energy $\Pi$ - to be minimized – can be expressed as the square-sum of residuals $v$. The residuals are combined with so-called observations $l$ having the weight $p$. All weights are ordered in a diagonal matrix $P$. See also [2] [3] [4].

$$l^* = l + v = f(x, y) \text{ with weight } p$$ (2)

The observation $l$ is corrected by a residual $v$ and the adjusted observation $l^*$ is a function of the 2D coordinates $(x, y)$. We would like to add some examples for those observations in order to clarify the situation. We assume that $l$ is the 3D-distance between two neighbor points A and B. The projected points (in the flattened situation) are $A^*$ and $B^*$. The distance $l^*$ in 2D should be more or less in the same range as $l$. This range ‘more or less’ is called residual $v$ in the adjustment theory. We can write the equation for the 2D distance as follows:

$$l^* = l + v = \sqrt{(x_{A^*} - x_{B^*})^2 + (y_{A^*} - y_{B^*})^2}$$ (3)

To be complete we show 3 more equations, including equation (3) again:

$$l^* = l_{2D} = l_{3D} + v = \text{distance 2D with } p = p_l$$ (4)

$$\alpha^* = \alpha_{2D} = \alpha_{3D} + v = \text{angle 2D with } p = p_\alpha$$ (5)

$$A^* = A_{2D} = A_{3D} + v = \text{area 2D with } p = p_A$$ (6)

On the equations (4-6) we can see that our possibilities are limited as always in map-projection theories, where the maps can be lengths-, angle- or area-preserving. The residuals can be steered a little bit by their weights $p$. In general we can say: the bigger the weight the smaller the residual, but this concept has limitations mainly because of numerical reasons. We do not recommend at all substituting a constraint by an equation (2) and a very big weight. This leads always to numerical problems. In case of real constraints we have to add the constraint equation by using Lagrange multipliers. Now the distortion energy is not only a function of the 2D coordinates, but also from the Lagrange multiplier $h$. Of course we can use any number of constraints without a change of the principle. In order to find the smallest distortion energy we have set the derivations of the energy to the unknowns to 0. We did it in equation (8) for the Lagrange multiplier $h$ in order to show the constraint-equation.
An example for those constraints are lengths or angles between points. When we assume e.g. for an ETFE cushion fixed boundaries we have to make sure that the lengths of the patterns and the lengths of frame - where the foil is inputted - are identical disregarding the compensation and also $90^\circ$ angles may occur, for which we have to use also additional equations with Lagrange multipliers.

\[
\Pi(x, y, h) = \frac{1}{2} v^t P v - h(g(x, y) - g_0) \Rightarrow \min.
\quad (7)
\]

\[
\frac{\partial \Pi}{\partial h} = g(x, y) - g_0 = 0
\quad (8)
\]

Figure 2: Local triangle \( u, v \) coordinates as observations

Now we would like to extend the possibilities of the flattening procedures by very general coordinate-transformations. As we defined at the beginning of our paper the surface to be flattened is described by 3D coordinates of points and an additional net (or a mesh) (see Figure 1). When we assume now that our 3D surface is described by triangles; so we can calculate rectangular \( u, v \) coordinates for all triangles in their 3D position as shown in Figure 2. In order to flatten the surface we write the residuals by assuming observations having the value 0 as follows in (9). The transformation matrix \( T \) is here describing a rectangular transformation with an identical scale in both directions (Helmert-Transformation) [8].

\[
\begin{pmatrix} v_x + 0 \\ v_y + 0 \end{pmatrix} = T \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \begin{pmatrix} x_{2D} \\ y_{2D} \end{pmatrix}
\quad (9)
\]

The transformation matrix \( T \) is here describing a rectangular transformation with an identical scale in both directions (Helmert-Transformation). There are other possibilities for the setting of the transformation matrix \( T \) as e.g. an orthogonal transformation with the scale 1. In this case the transformation parameter \( a \) is substituted by \( \cos(\alpha) \) and \( b \) by \( \sin(\alpha) \). The advantage of the used transformation is the linear equations in (9). By using this formulation we can find easily in one linear step all 2D coordinates. We simply have to fix 2 points in 2D and the 2D coordinates of all points are together with the transformation parameters \( a, b, x_0, y_0 \) for each
triangle the solution of the linear system. The advantage is obvious: a flattened surface is quickly obtained. The disadvantage of the described method is as follows. By the fact that the equations (9) are depending on a coordinate system, we end up with results depending on the chosen 2 fixed points. For a high quality patterning we cannot accept it, and therefore we are going to a free network adjustment. This means we do not use fixed points at all, and therefore we have to add 3 equations, in order to fix the 3 degrees of freedom in the 2D space. Those equations are not simple to use, because it needs Pivot-strategies to find the solution.

\[\sum_{i=1}^{n} x_i = 0 \quad \sum_{i=1}^{n} y_i = 0 \quad \sum_{i=1}^{n} x_i \cdot y_i = 0 \quad (10), (11), (12)\]

Our flattening procedure has to make sure, that the flattened area and the 3-dimensional area are in the same range. Therefore we add nonlinear observations to our system as follows. For all transformations (e.g. 1 system per triangle) one more equation to maintain the scale 1 as good as possible.

\[1 + \nu = \sqrt{a^2 + b^2} \quad \text{with} \quad p = p_s \quad (13)\]

In the Figure 3 we can see a membrane project consisting of 9 parts. All of those parts are patterned individually, here I would like to describe the flattening procedure for the whole part in one piece; the reason is only to get a flat boundary for a good mesh-generation (very similar as we do it for cable-nets!) We see already the flattened boundaries for all parts. We would like to count the unknowns for the patterning of the biggest part (dark red). This part has 1624 points, 1372 polylines (meshes). The unknowns are so 3240 coordinates and 5488 transformation-parameters. All those meshes have at least 4 corner points, for all systems (polylines) we have an equation (13) and at least for 4 points equation (9). Therefore the number of residual rows is at least 9*1372=12348. The number of unknowns is (3240 + 5488) 8728. So we still have a very good redundancy of (12348-8728) 3620. We forgot to mention, that we have 3 more unknowns by the constraints (10-12).
3 GEODESIC LINES

Geodesic lines play an important role in the patterning procedure. Before explaining the reason why we use geodesic lines I would like to give 3 definitions for geodesic lines. A geodesic line is defined as

1. the shortest line between 2 points on a surface.
2. whose bi-normal vector in all points of the line is identical to the normal vector of the surface.
3. a line with a constant force on a surface (without any friction).

The reason for using geodesic lines as seam-lines is very simple and it can be seen directly on point 1 of the 3 definitions. The geodesic lines help to minimize the waste of material. Figure 4 shows the situation. The red strip has to geodesic lines as seam-lines and the violet strip 2 vertical cuts as seams. In the flattening procedure the red strips automatically becomes ‘straight’ and the violet one ‘curved’. The reason is as follows: geodesic lines in the 2D space are straight lines and therefore the strip becomes as straight as possible.
3 SELECTED EXAMPLES

In order to optimize the widths of patterns automatically we need optimization variables. We all know that the widths of our patterns depend on the position of the seam-lines (mainly geodesic lines). In order to achieve appropriate widths for all patterns we have to modify the position of those seam lines until the desired widths of the patterns are reached. Therefore we want to change the position of the geodesic lines depending on one value. We define this value to be a coordinate (for regular meshes), an arc-length for pneumatic systems or an angle for high point membranes [5], [6].

Figure 4: Geodesic lines and vertical cuts

Figure 5: Geodesic line Start and Endpoint on one x-value
Figure 5 shows how the widths are optimized for regular nets. Parallel lines with the desired widths are starting values for an iterative procedure. With those starting lines geodesic lines are produced automatically by a start-and end-point. Then the flattening of all patterns is performed to get real widths for all patterns. With this information new gridlines can be calculated in a way to get widths being closer to the desired width, etc. After some iteration loops all widths are perfect. Let me refer again to Figure 5 to explain it precisely by this example. The distance of the gridlines is 1.80 m in the 1st iteration. We want all patterns to be 1.80 m wide. Now the flattening at the end of the 1st iteration shows us, that that real width of the first pattern (left side) is 2.01 m (0.21 m wider than desired). So we know the gridline have to start in the 2nd iteration with 1.59 m (=1.80 m-0.21 m) in order to get 1.80 m. This procedure converges very fast and after some iterations all widths are exactly 1.80 m except the last one the right hand side.

In Figure 6 we find the same principle with a guideline and automatic produced geodesic perpendicular to the guideline; therefore a widths optimization can be done automatically also for air-halls and cushions.

Also radial patterns can be arranged and optimized automatically by using an angle in order to set a geodesic line. (see Figure 7)
Our system allows to flatten membrane patterns very fast and width-optimized. We also add seam-allowances and welding marks to simplify the production process [7]. Our clients get also information about the quality of the patterns by the simple numbers as length-, angle and area-differences (and the distortion energy itself) see Figure 8. Hierarchical cuts are also possible as we see in Figure 9.

![Figure 8: Radial net with welding marks and quality report](image)

### 4 CONCLUSION

It has been shown that by applying the adjustment theory for the flattening of membrane patterns, in a first step linear and then - after having got approximation values - nonlinear, a powerful system is obtained; its basis are coming from map-projection doctrine.
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STRUCTURAL MEMBRANES IN MEXICO: TWO CASE STUDIES

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Key words: tensile structures, Mexico, lanterns.

Summary. This paper describes two membrane structure projects in Mexico built with similar dimensions and materials but different climatic and structural conditions. The first project is located in Mexico City and called CUT, was built above the roofs of two existing buildings, which serve as counterweights, and has moderate wind speeds and sun exposure. The second project, meanwhile, located in the city of León, Guanajuato and called ENES León, was built at ground level and is exposed to higher wind speeds, temperatures and sun exposure, requiring the construction of a more robust foundation and anchoring systems as well as its own foundation system.

1 INTRODUCTION

Membrane structures in Mexico have rapidly grown in popularity during the first decade of this century. Specific requirements such as protection from the sun or rain, together with their visual qualities, have made this type of structural system a reference point for contemporary architecture in Mexico and a symbol of a reconciliation between architecture and engineering, form and structure, or indeed geometry and mechanics.

This paper presents and describes two projects recently carried out for the Universidad Nacional Autónoma de México (UNAM). The first, called CUT, is located in Mexico City and is intended to generate a space where students of the University Drama Center—in the Cultural Zone of the Ciudad Universitaria campus—can engage in different activities such as rehearsals or stagings, together with a terrace set aside for a café. The second, called ENES León and located in the city of León, Guanajuato, is intended for a cafeteria within a new campus housing the National School for Higher Studies, León. Here, temperatures rise to 40°C in summer. The two projects cover an average area of 344 m².

The description and study of the two constructions follows a clear order based on the range of variables implied by the methodology covered by the design process at the Structures Lab of the UNAM’s Architecture Faculty. This may be summed up as follows: 1. Considerations of architectural and structural design, 2. Formfinding, 3. Structural analysis, 4. Membrane, 5. Load-bearing structure with foundations, hauling system and anchors, 7. Installation. It
concludes with a table of comparative data for the two case studies including covered and developed areas, weights and wind speeds taken into account for the structural analysis.

2 CUT PROJECT – UNIVERSITY DRAMA CENTER

The University Drama Center – CUT is located in the Cultural Zone of the Ciudad Universitaria campus in the UNAM, in Mexico City. This space is dedicated to teaching performing arts and is one of the leading venues of its kind at a national level. In 2011 the Structures Lab of the UNAM’s Architecture Faculty was asked to develop a project for the construction of a tent structure for the center, with the specific requirement of covering an area between two buildings of differing heights to create a space where students are able to engage in a range of activities.

Once these requirements had been analyzed, it was decided to design a roof capable of covering the space between the two buildings and taking advantage of their differing heights in order to create an area above the lower one where a café could potentially be installed, offering excellent views of the south of the Valley of Mexico.

2.1 Considerations of architectural and structural design - CUT

The architectural design was developed by architects Juan Gerardo Oliva Salinas and Marcos Javier Ontiveros Hernández and the structural design by engineer Juan José Ramírez Zamora. The typology of the roof comprises elliptical cones made up of floating posts ending in lanterns, allowing the passage of the sun’s rays in order to create a play of light and shadow inside. It covers a floor area of 417 m² with an actual surface area of 488 m². The total weight is 28.7 kN, resulting in a weight of 58.8 N/m². The shape was worked out using MPanel© software and the static wind analysis was carried out using a specialized computer program for matrix analysis. The load considerations for the static wind analysis are: dead weight, 10.8 N/m²; pre-tensioning, 294.3 N/cm² and a design for wind speeds up to 96 km/h, equivalent to 26.7 m/s. This analysis was carried out in reference to Mexican norms set out in the Federal District Construction Regulations (RCDF) and the Complementary Technical Standards (NTC), as well as the manuals of the Federal Electrical Commission (CFE), the American Concrete Institute (ACI 318-05), the American Institute of Steel Construction (AISC), the American Welding Society (AWS) and support from the Design Guide for Surface Tensile Structures. (Fig. 1)
2.2 Membrane - CUT

The manufacturing of the membrane was carried out using the high-frequency process on a textile composite made of pre-stressed polyester threads covered in PVC and a layer of highly-concentrated Fluotop T-2 PVDF compound on the top surface and of weldable PVDF on the reverse, to provide improved resistance to pollution on the inside of the material, as well as the following characteristics: breaking strength (warp-weave) of 420/400 daN/5cm, tear strength (warp-weave) of 55/50 daN, translucency 8%, solar diffusion 6%, solar absorption 16%, solar reflection index 78%, and acoustic attenuation index 15 dBA. It has a weight of 1050 g/m2, which for the total area of the roof comes to 5.03 kN (513 kg).

2.3 Structure - CUT

The structure comprises 4 main elements: 1. floating posts with lanterns, 2. tensioning framework, 3. corner posts, and 4. anchors to steel and/or concrete structure. It was manufactured using structural steel type ASTM-A36, with a yield strength of 250 MPa, and with a hot-dip electrotyping finish using OC tubular profiles in the following sizes: 4” (114 x 6.02 mm), 3 1/2” (102 x 5.74 mm), 3” (89 x 5.49 mm), and 2 1/2” (73 x 5.16 mm), together with purpose-designed elements made of steel plates in the following thicknesses: 3/8” (9 mm), 1/2” (12.5 mm), and 3/4” (19 mm). The lanterns are ogival and based on an elliptical frame supported by a pair of branching, floating posts built from tubes with variable section sizes and a cover made from sheets of solid polycarbonate (plexiglass). The total weight is 17 kN. (Fig. 2)
2.4 Anchors - CUT

The membrane structure is supported on the two existing buildings, which serve as a counterweight. For this reason different anchoring systems were designed according to the specific site tensile forces (in the case of cables) and compression forces (in the case of articulated posts) would be applied. The following figures illustrate two specific cases of anchoring to the slab and to the metal structure. (Fig. 3)
2.5 Installation process - CUT

The roof was raised using a crane with a beam-cable system connected to the two lanterns. In this way it was possible to raise them to their final position to connect the system of 8 cables and 6 anchoring points that support the floating post, followed by the membrane in the elliptical rings attached to these points, together with the 8 perimeter anchoring points on the buildings. (Fig.4)

Figure 4: Installation of posts and membrane with the help of a crane

3 ENES LEON PROJECT – UNIVERSITY CAFETERIA

The Universidad Nacional Autónoma de México is currently developing a number of campuses in different states around the country. In the city of León, Guanajuato, a major zone of industrial development famous for its footwear industry, work has begun on one such campus with the construction of the National School for Higher Studies, expected to be completed in the next ten years with capacity for 18,000 students.

3.1 Considerations of architectural and structural design - ENES León

The architectural design was developed by architects Eric Valdez Olmedo and Fernanda Gómez Loyo and the structural design by engineer Juan José Ramírez Zamora. Due to the location of the new campus on the outskirts of the city, a need arose to create cafeterias where students, faculty and administrative staff can eat without the need to travel into the city. The dining area covers a circular area measuring 328 m² with space for 108 seats, to include two retail premises and washrooms in a reinforced concrete block. The specific requirement was for the construction of a lightweight roof to protect from rain and the powerful sun that can lead to temperatures of up to 40°C (approximately 104°F) in the summer. It covers a floor area of 272 m² with an actual surface area of 345 m². The total weight is 205.7 kN, resulting in a weight of 588.6 N/m².

The roof is a tensile structure with a radial typology of ridges and valleys and a total of ten
alternating points converging on a central ring; five supported on the block housing the retail premises and five at ground level. The shape was worked out using Easy© software and the static wind analysis was carried out using a matrix analysis program. The load considerations for the static wind analysis are: dead weight, 10.8 N/m²; pre-stressed, 98.1 N/cm² and a design for wind speeds up to 140.1 km/h, equivalent to 38.9 m/s². This analysis was carried out in reference to the standards set out in section 2.1. (Fig. 5)

Figure 5: Exterior and interior views of the dining area

3.2 Membrane - ENES León

The manufacturing of the membrane follows the same characteristics as the previous one (see section 2.2) and has a total weight of 3.6 kN.

3.3 Structure - ENES León

The structure comprises 2 types of posts with a Y-shaped geometry, divided into sections to facilitate transportation: 1. Perimeter posts in the dining area, and 2. Perimeter posts on the reinforced concrete block. The Y1 type posts were fabricated using ASTM-A36 steel, with a yield strength of 250 MPa, using OC tubular profiles in the following sizes 8” (219 x 8.18 mm) and 4” (114 x 6.02 mm). The Y2 type posts used 6” (168 x 7.11 mm) tubular profiles, as well as purpose-designed elements made of steel plates in the following thicknesses: 3/8” (9 mm), 1/2” (12.5 mm), and 3/4” (19 mm). The finish for the structure uses an anticorrosion primer and a polymer varnish-based paint. The total weight is 20.35 N. (Figs. 6 and 7)
3.4 Foundation - ENES León

The foundation for the structure was built using reinforced concrete with a compression strength of 25 MPa and ASTM-A709 reinforcing steel, with a yield strength of 690 MPa. It comprises three types of isolated spread footing foundations located at the level of the dining plaza with a finish grade of -1.20 m: Z-1, Z-2, and Z-3. The first, Z-1 measures 2.5 x 1.8 x 0.20 m with a 0.50 x 0.50 x 1.00 m block and weighs 27.63 kN. Z-2 measures 2.5 x 2.5 x 0.20 m with a 0.50 x 0.50 x 1.00 m block and weighs 36.04 kN. Z-3 measures 1.40 x 1.40 x 0.20 m with a 0.40 x 0.40 x 1.00 m block and weighs 13.26 kN. 11 blocks are located on the building connected to the reinforced concrete structure measuring 0.35 x 0.35 x 0.40 m on average. The foundation as a whole weighs 192.1 kN. (Fig. 8)
3.5 Installation process - ENES León

The installation of the structure and the membrane was carried out with the help of a crane. In the case of the structure auxiliary plates were placed on top of the posts for the hoisting maneuver, placing the element in its final position and carrying out the necessary welding. In the case of the membrane, the hoisting was carried out from its central section with a fastening system adapted to the tensioning ring where the cables of the radial ridges and valleys converge. Once raised into position, it was possible to hold the ten perimeter points and execute the tensile process. (Fig. 9)

This project is part of the research carried out by the Structures Lab of the UNAM’s Architecture Faculty, entitled: Bamboo and flexible membranes–PAPIIT IN-404611, in conjunction with the University College of London (UCL), as a case study for examining the possibility of an alternative, bamboo-based structure.
4 TABLE WITH COMPARATIVE DATA

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<thead>
<tr>
<th>Item</th>
<th>CUT</th>
<th>LEON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Roof surface—projected on the ground</td>
<td>417 m²</td>
<td>272 m²</td>
</tr>
<tr>
<td>2. Surface area in space</td>
<td>488 m²</td>
<td>345 m²</td>
</tr>
<tr>
<td>3. Weight of membrane</td>
<td>5.03 kN</td>
<td>3.55 kN</td>
</tr>
<tr>
<td>4. Weight of loadbearing structure</td>
<td>17.01 kN</td>
<td>20.35 kN</td>
</tr>
<tr>
<td>5. Total weight of structure and membrane</td>
<td>22.04 kN</td>
<td>23.9 kN</td>
</tr>
<tr>
<td>6. Weight of structure and membrane</td>
<td>37.08 N/m²</td>
<td>69.26 N/m²</td>
</tr>
<tr>
<td>7. Weight of structure</td>
<td>34.83 N/m²</td>
<td>58.86 N/m²</td>
</tr>
<tr>
<td>8. Weight of foundation with concrete</td>
<td>10.6 kN</td>
<td>192.1 kN</td>
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<tr>
<td>9. Total weight</td>
<td>28.7 kN</td>
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<tr>
<td>10. Design for wind speeds</td>
<td>26.7 m/s</td>
<td>38.9 m/s</td>
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<tr>
<td>11. Software for formfinding</td>
<td>MPanel&lt;sup&gt;®&lt;/sup&gt;</td>
<td>Easy&lt;sup&gt;®&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

- In both structures there is an average difference of 144 m² between the area covered and the actual surface area of the membrane.
- The manufacturing conditions are the same except for the finish to the structure.
- The location affects the design for wind speeds and therefore the reactions of the ENES León project structure are considerably higher.
- The volume of concrete for the foundation and thus the weight by area unit is considerably higher given that in the case of the CUT, the building functions as a counterweight to the roof, while in the case of the ENES León, it was necessary to build spread footing foundations for the counterweight to the structure. Thus the weight of the foundation is not a relevant aspect for the comparative analysis.
- This work, which reveals the joint efforts between designers, builders, and clients to maintain and improve the quality in the design and construction of tent structures, contributes to the promotion of architectural criticism, the dissemination of knowledge and the state of art in the design and manufacture of membrane structures in Mexico.

ACKNOWLEDGEMENTS

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TENSION ANCHORS FOR STRUCTURAL MEMBRANES

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Key words: soil anchors, foundations in tension, uplift capacity.

Summary. Several types of tensile loaded foundations for structural membranes with
different materials, geometries, manufactures, installations, efficiencies and depths were
studied. As a result, a better understanding of the uplift behaviour and mode of failure have
been acquired. Therefore, the pull-out capacity of anchors can be estimated confidently by
encompassing the complex relationship between various modes of failure, anchor geometry
and soil properties.

1 INTRODUCTION

Structural membranes are highly efficient because, unlike conventional structures, they
rely on shape and pretensioning rather than stiffness and weight. They work in tension and
take advantage of the full strength of the materials. However, their efficiency can be partly
lost by the way in which the tensions are transferred to boundary structures or the surrounding
soil. Self-balancing structures use compression rings, heavy boundaries or stiff beams,
whereas non–self-balancing structures require foundations loaded in tension.

![Tension Anchors for Structural Membranes](image)

Figure 1: An enormous buried concrete block measuring 30 x 13 x 5 m for 5,000 T

Anchoring tension to the soil can be accomplished by relying on the dead weight of the
foundation and through involving the soil reaction. Relying on the dead weight of the
foundation is inefficient because of the amount of material and excavation required, as well as
the amount of backfilling (fig. 1). Lightweight recoverable anchors which mobilize the soil
reaction, as plant roots do, are therefore preferred.
2 ANTECEDENTS

Antecedents of passive anchors can be found in Nature. Roots feed plants and provide uplift resistance against the wind **involving a large volume of soil** (fig. 2). Stakes have been used for anchoring tents like the black tent (fig. 3), the Tabernacle (fig. 4), the military tents (fig. 5) or the circus tent (fig. 6).

![Figure 2: Roots. Figure 3: The black tent. “Enlarge the place of thy tent, and let them stretch forth the curtains of thin habitations: spare not, lengthen thy cords, and strengthen thy stakes ”. (Isaiah 54.2; 701-681 bC). Figure 4: The Tabernacle “All the vessels of the tabernacle in all the service thereof, and all the pins thereof, and all the pins of the court, shall be of brass ”. (Exodus 27.19 ~1.400 bC).](image)

Sea anchors attach ships to the seabed (fig. 7). A high efficiency ratio is needed in order to withstand heavy loads with minimum self-weight. Metal flukes bury themselves in the soft bottom or hook on to rocks. They are connected from a long distance through the chain that attaches it to the mooring vessel. They are recoverable by breaking them out of the bottom by shortening the rope until the vessel is directly above the anchor.

![Figure 8: Anchored mobile home, power pole, antenna and transmission tower.](image)
More recent examples are the anchors for bridges, mobile homes, antennas, poles, transmission towers, pipelines, buried tanks, equipment, runways, agricultural installations and cattle (fig. 8).

3 TYPOLOGY

Soil anchors can be active or passive. Active anchors are those subjected to permanent pre-stressing. They have a cap and a tendon fixed to the ground with mortar. The soil is pre-compacted between the cap and the fixed length. When the external load is applied, the soil decompresses and movements are limited to acceptable levels.

Passive anchors are not subjected to permanent pre-stressing. They act against the soil when loaded. If unloaded, both, the anchor and the soil, remain at rest. They move more than active anchors, but they are simpler and entail fewer problems of relaxation and durability.

The repertoire of passive anchors includes shallow and deep anchors, which are classified according to whether they reach the surface of the ground or remain buried. Shallow anchors include piles, stakes, hooks, grouted bars, sheet piles, diaphragm walls, concrete piers, well foundations and dead man anchors, whereas deep anchors include logs, tubes, steel sections, grillages, plates, arrowheads, expandable plates, screws and ballast foundations.

4 BEHAVIOUR

Uplift is dependent on anchor type (shape, width, depth, weight, inclination, roughness and position), soil conditions (type, density, cohesion, adherence, friction, stiffness, moisture, groundwater table position and the initial stress state), loading mode (rate, repetition, duration, direction and amount) and installation process (static, dynamic, vibrated, driven, bored, grouted, blasted, compacted or drilled).
The anchors mobilize the soil reaction to a varying degree through four mechanisms: the plate effect, friction, earth pressure and self-weight (fig. 9). The plate acts against the soil as a footing turned upside down. It pushes up the material that prevents uplift. The shape of the surface at failure is a function of the location. If shallowly buried, the anchor displaces a volume of soil reaching the surface. By increasing the depth up to the critical depth, the influence of the surface disappears. The strength of the soil determines the volume involved. The plate effect is characteristic of thin anchors such as plates, screws and grillages.

The shaft acts against the soil by friction throughout the full depth. The strength of the soil determines the volume involved. Uplift must overcome the shear resistance of the soil adhered to the lateral surface. The shaft effect is characteristic of thick anchors such as concrete piers, well foundations and piles.

The earth pressure mechanism acts when the load is oblique. The horizontal component mobilizes passive earth pressure forward or backward depending on the depth. Mobilizing earth pressure requires horizontal displacement. The earth pressure increases the lateral friction (shaft effect) and hence, the ultimate pullout resistance.
The self-weight is the minimum uplift resistance. It is the stiffer resistant mechanism, because it moves less, but it is not efficient, because it does not involve the soil: 1 kp of resistance requires 1 kp of material. It is characteristic of ballast anchorages (barrels filled with gravel or sand, sandbags, pre-cast concrete parts, road building slabs or tanks filled with water).

Detailed examination of the available laboratory and field data, completed by the recent full scale tests conducted at the Shelter Research Unit of the Benelux Red Cross Societies, shows the influence of the anchor type, soil conditions, loading mode and installation process.

### 4.1 Anchor type, size, perimeter and depth

Ali (1968) tested piles and plates at different depths (fig. 11). He found that -piles (shaft effect) show a linear increase of the pull-out resistance with depth due to the linear increase of the lateral surface. Conversely, the resistance of the the plates increases more than piles at the beginning. After D/B = 1, the resistance tends to increase at a progressively lower rate and at D/B = 3 it remains constant.

![Figure 11: Piles and plates (Ali, 1968). Figure 12: Plates at different depths (Kwasniewski et al. 1975).](image)

Kwasniewski et al.1975 (fig, 12) showed that the displacing plate causes a progressive rearrangement in the soil structure. It starts at the plate surface and develops gradually into further regions. A kind of bulb forms above the plate and transmits the stresses into the neighbouring regions. The shape and volume of the soil involved change because the failure pattern changes with depth. At lower depths (a), the volume is affected by the proximity of the surface. The bulb increases in volume until the slip surface is formed and the soil wedge is displaced towards the soil surface. The shape of the soil wedge resembles a frustum of cone or a sea shell having walls slightly turned outwards. The resistance drops after reaching the maximum. At intermediate depths (b) the volume involved reaches the surface but the plate is far away enough to maintain the shear strength. The contour of the soil wedge observed in the soil surface disappears gradually. There is a critical depth (D/B = 3) from which the volume of soil involved remains constant. Deep anchor plates fail when subjected to punching shear by the formation of a wedge immediately above the anchor plate. The failure pattern is similar to the pattern formed below deep foundations.
At small spacing (D/B < 3, fig. 13 left), the capacity of the multi-helix screw anchor is equal to the bearing capacity of the top-most helix plus the friction capacity resulting from the shear strength of the soil along a cylinder bounded by the top and bottom helix with a diameter defined by the average of all helix diameters on a multi-helix anchor. At large spacing (D/B ≥ 3, fig. 13 right), the capacity of the multi-helix screw anchor is equal to the sum of the capacities of the individual helix plates. Calculating the unit bearing capacity of the soil and multiplying by the individual helix areas determine the total end-bearing capacity (fig. 14, Bassett, 1977).

The size and depth effects on plates were measured by Das & Seeley (1975, fig. 15). Four model aluminium anchor plates (from 51 x 51 mm to 51 x 255 mm) were tested in silica sand (γ = 1.51 T/m³, θ = 31°). The ultimate breakout load increased with depth and size because of the volume of soil involved.

Kananyan (1966) evidenced the scale effect by computing the contact stress of plates. Unlike the force Q, which increases with an increase in slab diameter, the contact stress σ decreases when the diameter d increases (fig. 16). The trend is attributable to the influence of the ratio between the perimeter and the area in the failure surface of the soil mobilized at uplift. It can be seen, following the lines representing the same area, that by increasing L/B, the perimeter and the perimeter/area ratio also increase. This means more perimeter per unit
of area and hence, more load capacity per in² of area of anchor plate (fig. 17). Example A = area 1 m²; perimeter 4 m; peri/area = 4. Example C = area 2 m²; perimeter 8 m; peri/area = 4. Heikkilä & Lane tested two different plates with the same uplift resistance: example D = area 0,75 m²; perimeter 5,51 m; peri/area = 7,35. Example E = area 1,32 m²; perimeter 4,63 m; peri/area = 3,51. Less surface area is compensated by a greater perimeter (fig. 18).

Position also matters. Kananyan (1966) tested inclined plates under coaxial pull at a constant depth and found that the inclination increases the bearing capacity due to an increase of the surfaces of shear (fig. 19).

The SRU (2013) tested vertical and inclined stakes under coaxial and inclined pull. The best results (+30%) were obtained for vertical stakes submitted to a 45° pull (figure 20). By orientating V-shaped stakes forward, the pull-out resistance is increased by 18% (figure 21).

4.2 Soil conditions

Yokel et al. (1982) compared the load-displacement characteristics of coaxial pullout tests of single helix anchors in silt, sand and clay (figure 22). Anchors placed in silt and clay exhibit considerable ductility, while anchors placed in sand rapidly lose their load capacity after peak resistance was developed. This difference is attributable to the shear-strength characteristics of the different soils. Clay and silt derive most of their strength from cohesion,
which does not substantially decrease with shear strain or minor decrease in depth. Sand derives its strength from frictional shearing resistance, depending on confining pressures which are relieved as shear deformations become large and which disappear as the anchor is withdrawn. The strength of silt is derived from both that of sand (friction) and of clay (cohesion).

Bemben et al. (1973) varied the depth of embedment of a Y plate and found that the pullout resistance increases, not only with depth as was expected, but also with density. This sensitivity to the depth increases with the density of the soil (figure 23). The angle of friction is also favourable (figure 24, Rowe & Davis, 1982). Instead, the increased water content softens the soil (figure 25).

![Figure 25: Relation between shear strength and water content for London Clay (Skempton, 1959). Figure 26: Static load-displacement of a single helix anchor under vertical pull in silt (Yokel et al.1982). Figure 27: Anchor deflection-time plot of long term multi-helix field tests (Clemence, 1982).](image)

4.3 Loading mode

The initial portion of the load-displacement curve of a single vertical helix anchor submitted to vertical pull in silt is rather steep (fig. 26). At a load of about 1.1 kip there is a break (attributed to pre-consolidation of the soil). After 1.1 kip, there is a gradual but not very drastic decrease in stiffness until the anchor yields at a load of approximately 4.8 kip. The anchor subsequently maintains its resistance during an additional 10-inch withdrawal. Resistance is reduced at uplift, when shear is demobilized. (Ductile behaviour is characteristic of soil anchors tested in silt and clay).

The anchor deflection-time plot of long term multi-helix field tests reveals that when the load is sufficiently smaller than the breaking load, progress of displacement nearly stops after a certain time (fig. 27). When the load exceeds a certain limit, displacement increases rapidly after a certain time and causes rupture. The anchor movement ceased after 60 to 100 minutes under each load increment until the failure load was reached. At the failure load, the anchor begins to rapidly pull out.

Yokel et al. (1982) subjected vertical single helix anchors to different load inclinations (fig. 28). As the angle $\alpha_i$ decreases from 90° to 15°, the load-deflection slope (stiffness) of the initial load-displacement curve decreases and the load capacity of the anchor increases due to
the earth-pressure effect. Therefore, inclination increases the displacement and the load capacity of helix anchors. Nevertheless, the performance of inclined anchors submitted to inclined loads is poor (fig. 29).

Figure 28: Vertically installed anchors submitted to different load inclinations. Figure 29: 45° inclined single helix anchors in silty soil submitted to non-coaxial loading (Yokel et al.1982).

Figure 30: Piles in cohesionless soil subject to oblique pull. Figure 31: Piles in cohesionless soil subject to horizontal pull. Figure 32: Compaction is favourable (Heikkilä & Laine, 1964)

Inclination of piles was investigated by Yoshimi (1964). When subjected to oblique (60°) pull (fig. 30), the vertical rough pile position offers significantly greater resistance. For the 10'' smooth pile, the peak resistance is shifted to β = +15°. The data for the 10'' piles show that the effect of pile roughness is significant for β = 0, -15° and -30°, but insignificant for β = 15° and 30°, in which the ultimate load is governed by lateral resistance of soil rather than pull-out resistance. When subjected to horizontal pull (fig. 31), the inclined piles behave similarly for 0 ≤ β ≤ 30, in which the axial load component is zero or downward. The β = -15° inclination gives greater resistance.

4.4 Installation process

Compaction of disturbed soil is favourable as observed by Heikkilä & Laine testing plates under a) 80 cm, b) 40 cm of controlled fill by tamping and c) uncontrolled fill (fig. 32).
Cycles of unloading and reloading a single helix anchor in sand were conducted at 1 kip intervals by Yokel et al 1982 (fig. 33). The reloading curves are steeper then the initial curve, indicating substantial strain-hardening effect, because whenever load is applied, the soil is compacted and its load-displacement characteristics are modified. As soon as the applied load exceeds the pre-load, the load-displacement curve follows the shape of the initial curve which would be obtained with monotonic loading.

Vertical pull-out load versus uplift deflection for a 6 in triangular (arrowhead) anchor indicates the point where the anchor is set under a preload of 3 kip (13 kN). The slope of the load-deflection curve is steeper after a load of 3 kip was applied to the anchor than from the range of loads from zero up to 3 kip (fig. 34).

5 UPLIFT EVALUATION

The uplift resistance can be estimated combining the plate, shaft, earth pressure and self-weight effects. The basic approaches to the plate mechanism are the cone method, the parabolic failure surface and the earth pressure method (fig. 35). For the shaft contribution, five situations are possible according to the initial and final depths in relation to the critical depth (fig. 36) and the Broms approach provides an estimate of the earth pressure effect (fig. 37). When the four mechanisms act in combination, adjustments should be made to take into account the interferences (fig. 38).
There are also empirical methods based on anchor and soil classification (Kovacs, 1975), baseline anchors and adjusting factors (IFAI, 2006) and expressions derived from tests (Shanin & Jaksa, 2003).

6 CONCLUSIONS

- In order to anchor lightweight structures, it is not necessary to use heavy foundations because lightweight recoverable anchors can be used instead. They replace the contribution of the dead weight by involving the soil, whose contributions are lateral earth pressure, friction and weight.
- Several types of anchors are available, which are classified according to whether they reach the surface or remain buried. Differences among them involve material, shape, installation process, efficiency, depth and ultimate uplift capacity, which can be roughly estimated.
- Installation procedures include excavating, vibrating, driving, boring, grouting or drilling.
- More research is needed to analyse and check every aspect that influences the behaviour of each type of anchor. A standard uplift test would help for using and comparing results. An international database of test results would be highly valuable.
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The Structural Behaviour of PTFE/Glass Fabric Structures Integrating Flexible Photovoltaic Modules

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Key words: Flexible Photovoltaics, Membrane Structures, PTFE/Glass Fabrics, Structural Behavior, Form-Finding.

The work presented in this paper has been conducted as part of a finished PhD-thesis by Hend Mohamed Ibrahim entitled “Membrane Integrated Flexible Photovoltaics: Integrating Organic and Thin-Film Solar Modules into ETFE and PTFE/Glass Membrane Structures”.

1. INTRODUCTION

Although the potentials of integrating thin-film Photovoltaic into buildings make it the recommended technology not only for traditional architecture but also for applications where envelopes are characterized by free morphologies such as membrane structures, integrating flexible Photovoltaic technology into architectural fabrics are still facing critical research questions related to the structural behavior of the integrated system under different loading conditions, the feasibility of their application for the wide varieties of membrane forms, the impact of environmental aspects on integrated elements and the know-how of incorporating PV system into the design and engineering phases of membrane projects. The paper investigates the impact of integrating flexible PV modules into PTFE/Glass hypar structures through a multi-layer attachment system developed by an US company. The system is incorporated in the form finding process of four different hypar form-ratios exploring all problematic aspects and then numerically analyzed for determination of structural behaviour under different loading conditions.
2. ATTACHING FLEXIBLE PV TO PTFE/GLASS FABRICS

Developing techniques for integrating flexible Photovoltaic modules into PTFE/Glass fabrics have special requirements related to material and mechanical issues for system flexibility. PTFE, as the basic material for PTFE/glass fabrics, has one of the lowest coefficients of friction against any solid. Whereas PTFE is preferably used where non-stick surfaces against dirt are targeted, it’s also where the challenge exists to attach other materials to PTFE by means of adhesion or welding. The attachment system should also accommodate special techniques for the flexibility to install and de-install modules for maintenance issues when needed without destroying the supporting PTFE/glass fabric. A multi-layer attachment system has been developed by Saint Gobain Performance Plastics that can mount flexible solar modules to PTFE/Glass fabrics. The system composes of a first layer of single coated PTFE/glass which is laminated to a thin layer FEP on the side of PTFE coating which cannot be hot welded. Therefore, the FEP layer with melting point of 280°C is used for the heat sealing with temperatures over 280°C. This temperature would destroy both the butyl adhesive and the polyester of the Velcro hook. For this reason, the single-side coated PTFE/glass strips are welded before a special type of Velcro hook strip is glued onto it. The velcro layer is the key solution to the targeted replaceable use of modules and is finally adhered to the thin-film solar module. Fig.1. indicates the structure of the attachment system.

Fig.1: Layers structuring of the System attaching flexible PV to PTFE/Glass (Cremers, Hightex GmbH/ SolarLoc System by Saint-Gobain PP)
The installation steps are as follow: In the first step, two one-sided laminated PTFE/glass membrane strips are welded to the supporting PTFE/glass with a distance of the module width with the coated side on the membrane surface. In the second step, the Velcro hook strip of the same dimensions as the previously single side coated PTFE/Glass membrane strips are glued onto it with a hotmelt. It should be noted that preheating the raw glass fabric improves the adhesion of the glue. In the third step, the flexible PV module with Velcro loop is unrolled and attached to the structural membrane Velcro hook strips, See Fig. 2,3,4&5. Cables should be put in membrane pockets for protection. The system is developed but further studies on the structural behavior of the membrane surface and the know-how of integrating these systems into the engineering phase are still lacking. And these are the targeted questions for the following part.

3. MECHANICAL PROPERTIES OF THE ATTACHING SYSTEM

As indicated in the Fig.1, the designed system for fixing the solar modules on PTFE/Glass fabrics composes of layer of single coated PTFE/Glass welded to the membrane on the coated side and adhered to Velcro on the other side. The solar module is then glued to the Velcro using adhesives. Understanding the mechanical behaviour of the fixing system is essentially required to analyze the structural performance of the membrane when modules are integrated. As the true behaviour of coated woven fabrics is highly nonlinear and can only be determined...
by extensive biaxial testing, assuming equivalent linear elastic material properties are generally adopted for analysis and design when obtaining the accurate values from bi-axial testing is not provided. The required mechanical values such as the elastic modulus and shear stiffness represent the input data for form-finding and structural analysis phases of the designed prototype in the next research step. For determining the actual mechanical properties for these layers, some bi-axial tests are commonly required. These tests are recommended as a step in the future research. In the case of unavailability of the fabrics mechanical values, these values are commonly assumed in relation to other fabrics values. The materials integrated were selected and fixed in a way that insures the flexibility of layers to absorb forces generated by the membrane. The only stiff layer is the single coated PTFE/Glass which is welded directly to the main membrane surface made of PTFE/Glass. From the mechanical behaviour perspective, the whole fixing system layers can be then considered as one layer of single coated PTFE/Glass, taking into account the self-weight of the layers that’s usually provided in further design steps. Therefore, an assumption is made to consider additional values of a typical PTFE/Glass elastic modulus and shear stiffness for the fixing system. Therefore, where the whole PTFE/Glass membrane surface has the values of eax=2.0 MN/m, eay=1.8 MN/m, eap=1.0 MN/m and G=0.1 MN/m, the surface of welded fixing system has double the previous values leading to elastic modulus of eax =4.0 MN/m, eay=3.6 MN/m, eap=2.0 MN/m and shear stiffness of G=0.2 MN/m. A constant value of 1 KN/m was used for the whole load cases.

4. NUMERICAL DESIGN AND STRUCTURAL ANALYSIS OF A FOUR-POINTS HYPAR PROTOTYPE INTEGRATING FLEXIBLE SOLAR MODULE

4.1. DESCRIPTION OF STUDY

In order to investigate the structural behaviour of PTFE/Glass fabric structure integrating flexible solar module, a 5x5 m 4-points hypar structure is numerically designed and analyzed under different loading conditions. As the study considers the importance of the form-ratio on the structural behaviour of the integrated elements, the analysis considers four different forms of 1:5m, 2:5m, 3:5m & 4:5m height to side length which corresponds to ratios of 0.2, 0.4, 0.6 & 0.8 respectively. The higher the form ratio is the more curved the structure. Based on the fact that the behaviour of membrane structures depends on curvature and geometric aspects rather than span, the analysis can be applicable on the wide range of membrane scales. In order to reduce the number of parameters, a constant value of 1:1 prestress in warp and weft directions has been used for all forms under all load conditions. Generally, an increase in prestress will reduce deflections while increasing stresses and vice-versa. Uniform wind uplift and uniform snow loads are assumed to be (1KN/m²). The Analysis Targets:

1- Investigating the impact of adding flexible solar modules on the form-finding process in relation to different form-ratios.
2- Studying the relation between different form-ratios and their impact on integrating PV modules and the generated stresses and deflection values.
3- Investigating the behavior of the structure under live loads, wind and snow.
The numerical design and analysis is performed using form-finding and structural analysis software programs. The analysis is performed on two stages: the form-finding phase using TLform program, load analysis phase using TLload and WinNetz programs.

4.2. FORM-FINDING PROCESS OF A HYPAR STRUCTURE INTEGRATING FLEXIBLE PV MODULE

The form-finding process starts by defining numerically the geometry system points, lines and regions. The hypar geometry is firstly generated and the flexible solar module fixing system geometry is defined. According to the inhomogeneity of triangulation grid between the defined regions of the membrane and PV surface, the resulted form-finding process showed an instability condition. This has led to redefining the membrane geometry in a grid ratio close to the fixing system dimensions. This problem represents the first consideration that should be taken into account during the form-finding phase which has more significant impact proportionally with the increase of project scale. The form-finding process is then performed for the hypar with the four aforementioned form ratios of 0.2, 0.4, 0.6 and 0.8. The second problem during the form-finding is explored when the form was generated. Because the system lines during the form-finding are generally free positioned in x,y,z coordinates following the geodesic form, this resulted in changing the geometry and dimensions of the pre-defined PV regions which turned from straight rectangular to curved and smaller geometry. A final solution is investigated by approximately defining wider dimensions than the real ones for the PV fixing system, depending on the curvature of structure, that nearly changes to the real dimensions after the form is generated. The impact of the structure curvature on the modified definition of PV region is investigated and described in the following part. Fig.6 is indicating the primary (marked in red) and modified (marked in blue) modules defined regions resulting in nearly the actual dimensions after generating the form. It’s concluded that the more curved the structure is the more significant the geometry change.
4.3. STRUCTURAL ANALYSIS OF A HYPAR STRUCTURE INTEGRATING FLEXIBLE PV

The structural performance of membranes can be evaluated by the two main factors of stresses and deflection. In this analysis, the stresses on warp and weft directions are tested for the four different hypar form-ratios under wind and snow loading. For comparable results, the analysis is performed for both conditions when PV is integrated or not. The results are analyzed and evaluated trying to understand the relation between the aforementioned parameters and the boundary conditions for PV integration regarding the proposed geometries. Fig.9 indicates the stresses analysis of the less curved form ratios (0.2) which showed an increase level of stresses on warp direction of 27% at the edges of fixing system decreasing gradually till reaching 17% increase from 6 to 7KN/M at the center of strips. Regarding the stresses on weft direction under snowload, as indicated in Fig.10, the results showed slight increase of stresses at the strips edges that could be ignored.
4.4. IMPACT OF PV INTEGRATION ON MEMBRANE STRESSES

The performed analysis on the four hypar forms showed that the generated tensile stresses are relatively higher for less curved forms than those generated in more curved ones, ranging from 5.8, 3.9, 3.7 then 2.3 for 1m, 2m, 3m and 4m high hypar respectively, as indicated in Fig.11. Adding a flexible PV module resulted in an increase of stresses that is proportional to the level of curvature. The more curved the structure is the more sensitivity and higher percentage of stresses increase at the surface of membrane where the module will be installed.
The results showed an increase of stresses ranging between 20% and 35% of the original generated stresses before integrating modules. These values should be taken into account during the structural analysis when the integration of flexible PV modules is targeted. All investigated forms exhibited higher stresses levels at the edges of strips which will be analyzed separately in the next parts. The following chart indicates the impact of adding the module on the tensile stresses on warp direction generated for each form under windload. It should be noted that the first three forms are generated from the same geometry definition for the form finding process. Whereas because of geometrical and form instability problems with the fourth form using the same definition, the fourth form with the highest curvature is separately redefined. This is the reason why the analysis showed a significant increase of stresses of about 62% at the form-ratio of (0.6) which could be attributed to some form-finding issues that resulted in higher generated stresses. Therefore, the results of this form have to be excluded from the analysis.

Fig.11: Impact of PV integration on warp stresses (M. Ibrahim H.)

A common observation between all results was analysed regarding the relative increase of warp and weft stresses at the edges of the fixing system strips under different load cases. This can be attributable to the used geometry of the added layer where the forces are moving from single to double layers of PTFE/Glass through sharp and short edge line. Two recommendations are proposed trying to decrease generated stresses at that part. The first solution could be by changing the form of strips ends in a way that allows the distribution of forces along wider edge line more smoothly. Fig.12 indicates the actual and proposed design for the edges. This solution is not numerically investigated in the current research and should be a part of required research in the future.
The second solution for decreasing the stresses generated at the strips edges is reducing the stiffness of the PTFE/Glass layer of the fixing system. The stiffness is reduced 200% to 125% and the same package of structural analysis is performed in order to compare the behavior of the structure using both stiffness values. The results, as indicated in Fig.13, showed that by decreasing the stiffness of the PTFE/Glass, the stresses applied on warp direction under windload dropped from 4.8 to 3.8 KN/M when the module is added. While the stresses decreased from 1 to 0.9 KN/M by welding the layer before adding the PV module under prestress load condition. The results are promising for future research by testing the impact of material mechanical properties on the structural behavior concluding with the optimum material characteristics for PV integration on PTFE/Glass membranes.
5. CONCLUSION

Realizing flexible photovoltaic technology into membrane projects is an integrative process that has to be involved already in the early design stages of form finding and structural analysis. The performed analysis of attaching flexible PV modules to coated fabrics for architectural applications showed promising results in terms of the modules impact on the structural behaviour and the level of generated stresses that has to be calculated for each single project. However, further research is required to investigate the impact of such integration on the module’s performance.

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REFERENCES


DEFORMATION ANALYSIS OF RECTANGULAR COMPOSITE FLEXIBLE MEMBRANE OF THE PHOTOVOLTAIC SPACE SOLAR ARRAY

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Summary. The thin flexible composite membrane stretched on the frame of space solar array should be capable of withstanding the mechanical loadings exerted on the structure during the delivery to orbit and deployment. Nonlinear analysis of the deflections of the orthotropic flexible membrane stretched over the rectangular frame cell and subjected to transverse loading is presented in this paper.

1 INTRODUCTION

Thin film photovoltaic (TFPV) solar arrays offer the potential for providing a higher level of power generation in a lightweight configuration that can be compactly stowed for a space launch [1 - 4]. A typical solar wing design is shown in Fig. 1. Advanced high-modulus, high-strength carbon fibre reinforced polymers (CFRP) are normally implemented in current designs of the frames for solar wings. The thin flexible membrane is stretched on the frame, and then the photovoltaic cells are attached to its surface. The deployable solar arrays are normally stowed folded and could be deployed in various configurations. When stowed, they are usually placed parallel to each other and compactly packaged for launch. During the delivery to orbit, the membranes are subjected to the transverse g-force. The resulting pressure is equal to the product of the weight-per-unit-area of the membrane with the photovoltaic elements attached by the g-force. As a result of this loading the flexible membrane deflects. The excessive deflection could lead to the damage of the photovoltaic cells and/or electrical circuits. For this reason, one of the design requirements is that the deflection of the membrane should be limited to some specified value.

The solution of the problem related to the non-linear deformation of the flexible membrane carrying photovoltaic elements is presented in this paper. The problem is formulated for the orthotropic flexible membrane subjected to the transverse uniform pressure and tensile in-plane forces applied to the edges of the membrane. The membrane deformation is modelled by the system of non-linear differential equations which is solved using Galerkin
method. The similar approach has been applied by Lopatin et al. [5]. An analytical formula for the calculation of the membrane deflection at the central point has been derived. Using this formula, the calculation of the deflections have been performed for orthotropic flexible membranes having different geometry parameters and subjected to different levels of loads. The results have been verified using comparisons with the finite-element solutions.

Figure 1: Spacecraft with solar arrays (Courtesy of ISS-Reshetnev Company).

2 PROBLEM FORMULATION

Consider a flat frame of the solar array shown in Fig. 1 and single out from this frame a typical representative rectangular fragment formed by the four rigid composite ribs. Refer this $a \times b$ cell to the Cartesian coordinated frame $xyz$ as presented in Fig. 2. As shown, the rectangular flexible orthotropic membrane with the photovoltaic plates attached to its surface is stretched with the in-plane forces-per-unit-length $T_x$ and $T_y$ and fixed to the ribs. The membrane is subjected to the transverse $g$-force, $n_z$. It is assumed that this loading can be represented by the transverse pressure:

$$p = B_p n_z g$$  \hspace{1cm} (1)

where $B_p$ is the mass of the unit area of membrane material (including photovoltaic elements attached) and $g = 9.8 \text{ m/s}^2$.

Figure 2: Typical rectangular fragment of the solar array with the stretched membrane and photovoltaic elements attached.
Deformation of the membrane is modelled by the following geometrically non-linear equations including the equations of equilibrium

\[
\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0
\]

(2)

\[
N_x \frac{\partial \omega_x}{\partial x} + N_{xy} \left( \frac{\partial \omega_x}{\partial y} + \frac{\partial \omega_y}{\partial x} \right) + N_y \frac{\partial \omega_y}{\partial y} - p = 0
\]

constitutive equations

\[
N_x = B_{11} \xi_x + B_{12} \xi_y + T_x, \quad N_y = B_{21} \xi_x + B_{22} \xi_y + T_y, \quad N_{xy} = B_{33} \xi_{xy}
\]

(3)

and strain-displacements relationships

\[
\xi_x = \varepsilon_x + \frac{1}{2} \omega_x^2, \quad \xi_y = \varepsilon_y + \frac{1}{2} \omega_y^2, \quad \xi_{xy} = \varepsilon_{xy} + \omega_x \omega_y
\]

\[
\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\]

\[
\omega_x = -\frac{\partial w}{\partial x}, \quad \omega_y = -\frac{\partial w}{\partial y}
\]

(4)

in which \(N_x, N_y, \) and \(N_{xy}\) are the membrane stress resultant; \(\xi_x, \xi_y, \) and \(\xi_{xy}\) components of the finite strain; \(\varepsilon_x, \varepsilon_y, \) and \(\varepsilon_{xy}\) components of the infinitesimal strain; \(u\) and \(v\) in-plane displacements in the \(x\)- and \(y\)-directions, respectively; \(w\) is the transverse deflection; \(\omega_x\) and \(\omega_y\) are the angles of rotations of the lines tangent to the coordinate axes \(x\) and \(y\); \(B_{11}, B_{12}, B_{22},\) and \(B_{33} \) (\(B_{12}=B_{21}\)) are the membrane stiffness coefficients.

Equations Eqs. (2) – (4) are derived based on the geometrically nonlinear equations for the orthotropic plate given in [6] in which the bending stiffness coefficients are neglected. Substituting Eq. (4) into Eq. (3) and subsequently into Eq. (2) yields the following governing system of equations:

\[
B_{11} \frac{\partial^2 u}{\partial x^2} + B_{33} \frac{\partial^2 u}{\partial y^2} + (B_{12} + B_{33}) \frac{\partial^2 v}{\partial x \partial y} + B_{11} \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial y^2} + (B_{12} + B_{33}) \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x \partial y} + B_{33} \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial y^2} = 0
\]
This system provides the governing equations for the pre-stretched membrane under consideration in terms of displacements $u$, $v$, and $w$.

### 2 SOLUTION PROCEDURE

Galerkin procedure is employed for the solution of the system of equations given by Eqs. (5). Taking into account that the displacements of the membrane $u$, $v$, and $w$ are equal to zero at the edges supported by the ribs, and that for the given loading the lines $x = a/2$ and $y = b/2$ are the lines of symmetry, the approximating functions are selected in the following form:

$$
\begin{align*}
U(x, y) &= U \sin \frac{2\pi x}{a}, \\
v(x, y) &= V \sin \frac{2\pi y}{b}, \\
w(x, y) &= W \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}
\end{align*}
\quad (6)
$$

where $U$, $V$, and $W$ are unknown coefficients. Following Galerkin procedure, i.e., substituting Eq. (6) into Eq. (5) the corresponding errors are presented as follows:

$$
R_s(x, y) = -\left( B_{11} \frac{4\pi^2}{a^2} + B_{33} \frac{\pi^2}{b^2} \right) U \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} + \\
+ \left( B_{12} + B_{33} \right) \frac{\pi}{a} \frac{2\pi}{b} V \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} - \\
\frac{\pi}{a} \left( B_{11} \frac{\pi^2}{a^2} + B_{33} \frac{\pi^2}{b^2} \right) \sin^2 \frac{\pi y}{b} - \left( B_{12} + B_{33} \right) \frac{\pi^2}{b^2} \cos^2 \frac{\pi y}{b} \right) W^2 \sin \frac{\pi x}{a} \cos \frac{\pi x}{a} 
$$
\[ R_2(x, y) = \frac{2\pi}{a} \left(-B_{11} \frac{\pi^2}{a^2} \sin^2 \frac{\pi y}{a} - B_{12} \frac{\pi^2}{a^2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} - \right. \\
\left. -\frac{\pi}{b} \left( B_{22} \frac{\pi^2}{b^2} + B_{33} \frac{\pi^2}{a^2} \right) \sin^2 \frac{\pi y}{a} - \left( B_{12} + B_{33} \right) \frac{\pi^2}{a^2} \cos^2 \frac{\pi y}{a} \right) W^2 \sin \frac{\pi y}{b} \cos \frac{\pi y}{b} + \\
+ \frac{2\pi}{b} \left[ -B_{12} \frac{\pi^2}{a^2} \sin^2 \frac{\pi y}{a} \right. \\
\left. + B_{33} \frac{\pi^2}{a^2} \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} \right] \sqrt{W} + \\
\left. \left[ -\frac{1}{2} \frac{\pi^2}{a^2} \left( B_{11} \frac{\pi^2}{a^2} + B_{12} \frac{\pi^2}{b^2} \right) \sin \frac{\pi x}{a} \sin^3 \frac{\pi y}{b} - \right. \\
\left. -\frac{1}{2} \frac{\pi^2}{b^2} \left( B_{12} \frac{\pi^2}{a^2} + B_{22} \frac{\pi^2}{b^2} \right) \sin^3 \frac{\pi x}{a} \cos \frac{\pi y}{b} \sin \frac{\pi y}{b} + \right. \\
\left. + 2B_{33} \frac{\pi^2}{a^2} \frac{\pi^2}{b^2} \cos \frac{\pi x}{a} \sin \frac{\pi x}{a} \cos \frac{\pi y}{b} \sin \frac{\pi y}{b} \right] W^3 - \right. \\
\left. \left( T_x \frac{\pi^2}{a^2} + T_y \frac{\pi^2}{b^2} \right) W \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + p \right) (7) \\

The orthogonality conditions for the errors, Eq. (7) and the basis functions, Eq. (6) have the form

\[ \int_{0}^{a} \int_{0}^{b} R_2(x, y) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} dxdy = 0 \]

\[ \int_{0}^{a} \int_{0}^{b} R_2(x, y) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} dxdy = 0 \] (8)

\[ \int_{0}^{a} \int_{0}^{b} R_2(x, y) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} dxdy = 0 \]

Integrating yields the following system of non-linear algebraic equations:
\[
-a_{11}U - a_{12}V = b_1 W^2 
\]
\[
-a_{21}U - a_{22}V = b_2 W^2 
\]
\[
-c_{11}W + c_{12}UW + c_{21}VW - c_{22}W^3 + p \frac{1536}{\pi^2} = 0
\]

in which
\[
a_{11} = 9(4B_{11} \frac{\pi^2}{a^2} + B_{33} \frac{\pi^2}{b^2}) ,
\]
\[
a_{12} = a_{21} = \frac{64}{ab}(B_{12} + B_{33}) ,
\]
\[
a_{22} = 9(4B_{22} \frac{\pi^2}{b^2} + B_{33} \frac{\pi^2}{a^2})
\]
\[
b_1 = \frac{6}{a} [2B_{11} \frac{\pi^2}{a^2} - (B_{12} - B_{33}) \frac{\pi^2}{b^2}] 
\]
\[
b_2 = \frac{6}{b} [2B_{22} \frac{\pi^2}{b^2} - (B_{12} - B_{33}) \frac{\pi^2}{a^2}]
\]
\[
c_{11} = 96(T_x \frac{\pi^2}{a^2} + T_y \frac{\pi^2}{b^2}) ,
\]
\[
c_{12} = \frac{128}{a} [2B_{11} \frac{\pi^2}{a^2} + (2B_{12} + B_{33}) \frac{\pi^2}{b^2}] 
\]
\[
c_{21} = \frac{128}{b} [2B_{22} \frac{\pi^2}{b^2} + (2B_{12} + B_{33}) \frac{\pi^2}{a^2}]
\]
\[
c_{22} = 3[3B_{11} \frac{\pi^4}{a^4} + 2(3B_{12} - 2B_{33}) \frac{\pi^2}{a^2} \frac{\pi^2}{b^2} + 3B_{22} \frac{\pi^4}{b^4}]
\]

Solving Eqs. (9), for \(U\) and \(V\) the latter can be presented as follows:
\[
U = \frac{-b_1 a_{22} + b_2 a_{12} W^2}{a_{11} a_{22} - a_{12}^2} W^2 
\]
\[
V = \frac{-b_2 a_{11} + b_1 a_{12} W^2}{a_{11} a_{22} - a_{12}^2} W^2 
\]

Substituting these expressions into Eq. (10) yields the following cubic equation:
\[
c_{33} W^3 + c_{11} W - p \frac{1536}{\pi^2} = 0
\]

in which
\[
c_{33} = c_{22} + \frac{b_1 (a_{22} c_{12} - a_{12} c_{21}) + b_2 (a_{11} c_{21} - a_{12} c_{12})}{a_{11} a_{22} - a_{12}^2}
\]

Equation, Eq. (13), is the governing equation for the problem of nonlinear deformation of the orthotropic membrane under consideration subjected to the transverse uniformly distributed pressure and pre-stretching tensile in-plane loads. This equation links the forces \(T_x\) , \(T_y\) , pressure \(p\), membrane dimensions \(a\) and \(b\), membrane stiffness coefficients \(B_{11}, B_{12}, B_{22}, B_{33}\), and deflection at the centre of the membrane \(W\), and it can be transformed into the form
\[
W^3 + SW - R = 0
\]
where
\[
S = \frac{c_{11}}{c_{33}} , \quad R = \frac{1536}{\pi^2 c_{33}}
\]
The discriminant of Eq. (15) is calculated as

$$Q = \left(\frac{S}{3}\right)^3 + \left(\frac{R}{2}\right)^2$$  \hspace{1cm} (17)

In this case, the discriminant is positive so one root of Eq. (15) is real and another two are complex conjugates. According to Cardano’s formula the real root is given by

$$W = \sqrt[3]{\frac{R}{2} + \sqrt{Q}} - \sqrt[3]{\frac{R}{2} - \sqrt{Q}}$$  \hspace{1cm} (18)

Using this equation, the deflection can be found for various combinations of stiffness parameters of the orthotropic membrane subjected to the pressure and in-plane tensile loads.

The equation, Eq. (13) can be used when designing the membrane with the voltaic elements attached. Based on this equation, the membrane parameters and in-plane stretching loads, delivering the specified deflection $W$, can be found for given mass of the unit area of membrane material and $g$-force.

Note that if the pre-tensioning loads $T_x = T_y = 0$, than the coefficient $c_{11} = 0$ and it follows from Eq. (13) that

$$W = 8\sqrt[3]{\frac{3p}{\pi^2 c_{33}}}$$  \hspace{1cm} (19)

For given $W$, the values of $U$ and $V$ are calculated using Eqs. (12) and the displacements at any point of the membrane are determined by Eqs. (6).

Furthermore, substituting Eqs. (6) into Eqs. (4) and the resulting equations into Eqs. (3), the stress resultants can be calculated as follows

$$N_x = T_x + 2\left(B_{11} \frac{\pi a}{b} \cos \frac{2\pi x}{a} \sin \frac{\pi y}{b} + B_{12} \frac{\pi b}{a} \sin \frac{\pi x}{a} \cos \frac{2\pi y}{b}\right) +$$

$$+ \frac{1}{2}\left(B_{11} \cos^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} + B_{12} \frac{\pi^2 b}{a^2} \sin^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b}\right)W^2$$

$$N_y = T_y + 2\left(B_{12} \frac{\pi a}{b} \cos \frac{2\pi x}{a} \sin \frac{\pi y}{b} + B_{22} \frac{\pi b}{a} \sin \frac{\pi x}{a} \cos \frac{2\pi y}{b}\right) +$$

$$+ \frac{1}{2}\left(B_{12} \frac{\pi^2 a}{b} \cos^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} + B_{22} \frac{\pi^2 b}{a^2} \sin^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b}\right)W^2$$

$$N_{xy} = B_{33} \left(\frac{\pi a}{b} \sin \frac{2\pi x}{a} \cos \frac{\pi y}{b} + \frac{\pi b}{a} \sin \frac{\pi x}{a} \cos \frac{2\pi y}{b}\right) +$$

$$+ \frac{1}{4} W^2 \frac{\pi a}{b} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{b}$$  \hspace{1cm} (20)
Obviously, for the orthotropic membrane loaded with a uniform pressure, the largest values of $N_x$ and $N_y$ are reached in the centre of membrane with $N_{xy} = 0$. Thus, taking $x = a/2$ and $b = y/2$ in Eqs. (20), the maximum values of $N_x$ and $N_y$ are

$$N_x = T_x - 2(B_{11} \frac{\pi a}{a} U + B_{12} \frac{\pi b}{b} V) , \quad N_y = T_y - 2(B_{12} \frac{\pi a}{a} U + B_{22} \frac{\pi b}{b} V)$$

(21)

Substituting for the values $U$ and $V$ in this equation their expressions given by Eqs. (12), the stress resultants $N_x$, $N_y$ can be expressed in terms of deflection $W$ as follows:

$$N_x = T_x + 2 \frac{B_{11} \pi (b_1 a_{22} - b_2 a_{12}) + B_{12} \pi (b_2 a_{11} - b_1 a_{12})}{a_{11} a_{22} - a_{12}^2} W^2$$

$$N_y = T_y + 2 \frac{B_{12} \pi (b_1 a_{22} - b_2 a_{12}) + B_{22} \pi (b_2 a_{11} - b_1 a_{12})}{a_{11} a_{22} - a_{12}^2} W^2$$

(22)

It should be noted that the solution of the nonlinear problem under consideration was obtained under assumption that the membranes analysed should not be too long, i.e. their aspect ratio $a/b$ should not be excessively large. If the membrane is too long, the deformed shape would resemble the cylindrical surface and the use of the approximations as per Eqs. (6) could lead to noticeable errors. In practice, the aspect ratio $a/b$ for the cells of solar arrays normally does not exceed 2. So the solution presented in this work can be efficiently applied at the early stages of the design of flexible-membrane stiffened space solar arrays [7].

3 NUMERICAL EXAMPLES

The solution obtained in this work has been applied to the analyses of membranes made from orthotropic material. Consider non-linear deformation of a membrane made of an orthotropic material with the moduli of elasticity $E_x$ and $E_y$, shear modulus $G_{xy}$, and Poisson’s ratios $\nu_{xy}$ and $\nu_{yx}$. The stiffness coefficients of the membrane are given by

$$B_{11} = \bar{E}_x h , \quad B_{12} = B_{21} = \bar{E}_y \nu_{xy} h , \quad B_{22} = \bar{E}_y h , \quad B_{33} = G_{xy} h$$

(23)

where

$$\bar{E}_x = \frac{E_x}{1 - \nu_{xy} \nu_{yx}} , \quad \bar{E}_y = \frac{E_y}{1 - \nu_{xy} \nu_{yx}}$$

(24)

Substituting Eq. (23) into Eq. (11), the latter are transformed into the following form:

$$a_{11} = \frac{\pi^2}{a^2} h \bar{a}_{11} , \quad a_{12} = a_{21} = \frac{h}{a} \bar{a}_{12} , \quad a_{22} = \frac{\pi^2}{a^2} h \bar{a}_{22}$$

$$b_1 = \frac{\pi^2}{a^2} h \bar{b}_1 , \quad b_2 = \frac{\pi^2}{a^2} h \bar{b}_2$$

(25)
\[ c_{11} = T_x \frac{\pi^2}{a^2} \bar{c}_{11}, \quad c_{12} = \frac{\pi^2}{a^2} \bar{h} c_{12}, \quad c_{21} = \frac{\pi^2}{a^2} \bar{h} c_{21}, \quad c_{22} = \frac{\pi^4}{a^4} \bar{h} c_{22} \]

where

\[ \bar{c}_{11} = 9 \left( 4E_x + G_{xy} c^2 \right), \quad \bar{c}_{12} = 64c \left( E_x \nu_{xy} + G_{xy} \right), \quad \bar{c}_{22} = 9 \left( 4E_x c^2 + G_{xy} \right) \]

\[ \bar{B}_1 = 6 \left[ 2E_x - (E_x \nu_{xy} - G_{xy}) c^2 \right], \quad \bar{B}_2 = 6c \left[ 2E_x c^2 - (E_x \nu_{xy} - G_{xy}) \right] \]

\[ \bar{c}_{11} = 96 \left( 1 + \alpha c^2 \right) \]

\[ \bar{c}_{12} = 128 \left[ 2E_x + (2E_x \nu_{xy} + G_{xy}) c^2 \right], \quad \bar{c}_{21} = 128c \left[ 2E_x c^2 + (2E_x \nu_{xy} + G_{xy}) \right] \]

\[ \bar{c}_{22} = 3 \left[ 3E_x + 2(3E_x \nu_{xy} - 2G_{xy}) c^2 + 3E_x c^4 \right] \]

and

\[ c = \frac{a}{b}, \quad \alpha = \frac{T_y}{T_x} \] (27)

are the membrane aspect ratio and the ratio of the in-plane stretching tensile forces, respectively.

The coefficient \( c_{33} \) in Eq. (13) is calculated using Eq. (14) after substitution of coefficients determined by Eq. (25):

\[ c_{33} = \frac{\pi^4}{a^4} \bar{h} c_{33} \] (28)

where

\[ \bar{c}_{33} = \frac{1}{\pi^2} \left[ \bar{B}_1 \left( \frac{\bar{B}_{12} \bar{c}_{12} - \bar{a}_{12} \bar{c}_{21}}{\pi^2} \right) + \bar{B}_2 \left( \frac{\bar{a}_{11} \bar{c}_{21} - \bar{a}_{12} \bar{c}_{12}}{\pi^2} \right) \right] \] (29)

Substituting \( c_{11} \), Eq. (25) and \( c_{33} \), Eq. (28) into Eq. (13) yields the governing equation for the problem under consideration in the following form:

\[ \frac{\pi^4}{a^4} \bar{h} c_{33} W^3 + T_x \frac{\pi^2}{a^2} \bar{c}_{11} W - p \frac{1536}{\pi^2} = 0 \] (30)

Dividing this equation by \( \frac{\pi^4 \bar{h} c_{33}}{a^4} \), the latter can be transformed into the form

\[ W^3 + SW - R = 0 \] (31)

in which

\[ S = \frac{T_x}{h} \frac{\bar{c}_{11}}{\bar{c}_{33}} \frac{a^2}{\pi^2}, \quad R = \frac{1536}{\pi^6} \frac{pa^4}{\bar{c}_{33} h} \] (32)
The solution of Eq. (31) is given by Eqs. (17) and (18). Substituting coefficients defined by Eq. (25) into Eq. (22) and taking into account equations for the stiffness coefficients, Eq.(23), the stress resultants $N_x$ and $N_y$ acting at the centre of membrane are calculated as follows:

$$N_x = T_x + \frac{\pi h}{a^2} F_x W^2, \quad N_y = T_y + \frac{\pi h}{a^2} F_y W^2$$  \hspace{1cm} (33)

where

$$F_x = 2 \frac{E_x (f_1 + v_{xy} f_2)}{a_1 a_{22} - \frac{a_{12}^2}{\pi^2}}, \quad F_y = 2 \frac{E_y (v_{xy} f_1 + f_2)}{a_{11} a_{22} - \frac{a_{12}^2}{\pi^4}}$$  \hspace{1cm} (34)

$$f_1 = \frac{\bar{b}_1 a_{22} - \bar{b}_2 a_{12}^2}{\pi^2}, \quad f_2 = \left(\frac{\bar{b}_2 a_{11} - \bar{b}_1 a_{12}}{\pi^2}\right)$$

Consider analysis of the orthotropic membrane with typical for the solar array dimensions: \(a = 1 \text{ m}, b = 0.8 \text{ m},\) and \(h = 0.5 \text{ mm}\). The membrane is made of a material based on the glass-fibre fabric with the following elastic properties: \(E_x = E_y = 0.8 \text{ GPa}, G_{xy} = 0.15 \text{ GPa},\) \(v_{xy} = v_{yx} = 0.35\) and density \(\rho = 1800 \text{ kg/m}^3\). The mass of the unit area of membrane material with photovoltaic elements attached \(B_{\rho}^{cell} = 1.8 \text{ kg/m}^2\). Assume that \(T_x = T_y = T = 0, 250,\) and \(500 \text{ N/m}\). The resulting pressure exerted on the membrane is determined as follows:

$$p = (\rho h + B_{\rho}^{cell}) n_z g$$  \hspace{1cm} (35)

If \(n_z = 1, 5,\) and \(10\), than the pressure \(p = 26.5, 132.4,\) and \(264.9 \text{ N/m}^2\), respectively. Using Eqs.(17), (18) and (32), the membrane deflection \(W\) have been calculated. The results of calculations for different values of \(n_z\) and \(T\) are presented in Table 1. Analysis of the data shows that the increase in the stretching force \(T\) leads to a reduction of the membrane deflection. This effect is most noticeable for \(n_z = 1\).

**Table 1**: Deflection \(W\) (mm) for different values of \(n_z\) and \(T\).

<table>
<thead>
<tr>
<th>(n_z)</th>
<th>(T) (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>10.92</td>
</tr>
<tr>
<td>10</td>
<td>18.67</td>
</tr>
<tr>
<td>23.53</td>
<td>20.83</td>
</tr>
</tbody>
</table>

The results of the calculations were verified by a finite-element analysis. The non-linear analysis has been performed using the COSMOS/M module NSTAR [8]. The results \(W_{FEM}\) are presented in Table 2 and deformed shape of the membrane is shown in Fig. 3. Comparison of the results presented in Tables 1 and 2 shows that the maximum difference
between \( W \) and \( W_{FEM} \) is -6.75\% for \( n_z = 1 \) and \( T = 0 \) which is acceptable for the approximate analytical solution.

<table>
<thead>
<tr>
<th>( n_z )</th>
<th>( T ) (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10.92</td>
</tr>
<tr>
<td>5</td>
<td>18.67</td>
</tr>
<tr>
<td>10</td>
<td>23.53</td>
</tr>
</tbody>
</table>

Figure 3: Shape of the deformed orthotropic membrane.

3 CONCLUSIONS

The solution of the problem of nonlinear deformation of the orthotropic flexible membrane stretched on a rectangular frame and subjected to the transverse uniform pressure was developed in this work. The system of the nonlinear differential equations written in terms of in-plane displacements and deflection was solved using Galerkin method. The membrane displacements and deflection were approximated by the trigonometric functions satisfying the boundary conditions. The problem has been reduced to the algebraic cubic equation and the analytical formula providing the value of deflection at the centre of membrane was derived.

The deflections of the membranes made from the orthotropic flexible glass-fibre fabric have been calculated and the effects of the in-plane stretching loads on the deflection and internal stress resultants have been investigated. The accuracy of the analyses has been verified by comparison with the results obtained using the finite-element method. It has been shown that the analytical solution developed in this work provides an accurate estimate of the
deflection at the centre of membrane and can be successfully applied to the design of composite flexible-membrane stiffened space solar arrays.

REFERENCES


HIGH PRECISION LARGE SPACE STRUCTURES: CHALLENGES IN CABLE NETWORKS DESIGN

STRUCTURAL MEMBRANES 2013

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Key words: Large Space Structures, Cable Network, Form Finding, Structural Optimization, Force Density Method, Dynamic Relaxation.

1 INTRODUCTION

Nowadays space missions such as Earth observation, telecommunication and science require large antenna reflectors in the diameter range up to 20 meters or even larger for high performances. The antenna reflector structure has to satisfy strict requirements like high surface accuracy, low mass, high packaging efficiency, high reliability/stability in hazardous space environments for a long service life among others. These requirements lead to great challenges in developing large space deployable reflector (LDR) structures. One of the challenges is related to providing the parabolic high accurate (sub-millimeter range) shape with a minimum mass. Most of the flown reflectors known use cable networks suspended circumferentially or radially over a deployable framework. The cable networks approximate a parabolic surface with flat triangular facets, creating support for reflective material, e.g. extremely flexible knitted metal mesh.

In some specific missions LDR of about 6 m shall operate in high radio-frequency band (e.g. up to 30 GHz) [2]. This indicates a need of a fine facet size of the cable network of about 100 mm with a large number of cables and a strict surface error tolerance (around 0.1 mm). Moreover, manufacturing and assembling of such complex cable network with high accurate positioning and proper tensioning in cables is a quite challenging task as well. All above mentioned aspects lead the cable network design to be a form finding and design optimization problem with large number of variables, strict goal definition and multiple complex constraints.

A mechanical architecture of a large deployable mesh reflector (Figure 1) contains a deployable peripheral structure, which supports a cable network and a reflecting metal mesh. The cable network usually consists of a front network, a rear network and tie cables between them to form a so called tension truss (see also [12]).
The cable network is a kind of flexible structures, which generally have strong interactions between their geometry and stresses. Different to conventional rigid structures, the configurations of flexible structures are more likely to be found rather than to be designed. The sustainable configurations of such structures are the equilibrium status among used materials, stresses in structures, defined boundary conditions and external loads. From energy point of view, the equilibrium status can be understood as a system with minimum total potential energy, which is the same to minimum difference between internal strain energy to external kinetic energy.

2 FORM FINDING AND STRUCTURAL OPTIMIZATION

Form finding is one of the classical problems in flexible tension structures like cable networks and membranes. Two typical approaches used in form finding are force density method (FDM) and dynamic relaxation (DR). Their basic principles are briefly explained in the following.

The force density method is initialized by Linkwitz and Schek in early 70s, which is used to design the cable-net roofs of Olympia stadium Munich (1972). The core of FDM is linearizing the system of nonlinear equations by introducing a parameter called force density (ratio of force to cable length). These linearized system equations enable an efficient and accurate analytical approach to be performed for finding good initial geometry of flexible structures. In later publications of Schek [1], he has given a more general understanding of form finding and optimization about cable networks. Cable network design is understood as a structural optimization, whose goal is finding an equilibrium initial geometry subjecting to constraints like cable lengths, cable forces and point positions. The expression of the optimization problem is:

\[ g(x(q), y(q), z(q), q) = 0 \]  

Goal: min. system potential energy

Design variable: force density (q) of each cable

Constraints: cable lengths, cable forces and point positions
One of the simplest algorithms for solving the optimization problem is gradient based least squares principle. The gradients and related Lagrange factors can be readily derived through “chain rules” [1]. To improve the convergence behavior of a form finding design optimization particularly in problems with large force densities or shape changing, damping factors are added into functions as well.

The dynamic relaxation (DR) is an explicit solution technology based on finite difference method. Day [13] introduced this method in 60s to tidal flow computation and later Barnes extended it to solve design problems of flexible structures [15]. It traces step-by-step for small time increments of the motion of each node of the structure until it reaches the static equilibrium due to damping. Since the objective is to find the equilibrium status rather than trace the real dynamic behavior, the mass is fictitious here and is set to optimal convergence. In addition, better numerical stability of the analysis can be reached by taking kinetic damping instead of viscous damping. According to Newton’s second law, the residuals between external loads and internal loads cause motion of a system and the velocity and displacement of each node can be obtained by integrating time steps. The update of geometry is performed on each node step-by-step based on center finite difference method. So their computation costs in DR are significantly less than stiffness matrix inversion calculations in FDM. In kinetic damping method, once the maximal kinetic energy of a system is detected, the analysis has to be restarted at the current geometry configuration with zero residuals and acceleration for the next iteration (Figure 2).

\[
R_{lx}^{t+\Delta t} = M_l \cdot V_{lx}^{t+\Delta t} \quad \text{with} \quad \dot{V}_{lx}^{t+\Delta t} = (V_{lx}^{t+\Delta t} - V_{lx}^{t} - \frac{\Delta t}{2}) / \Delta t
\]

\[
V_{lx}^{t+\Delta t} = V_{lx}^{t} + \frac{\Delta t}{M_l} \cdot R_{lx}^{t+\Delta t}
\]

\[
x_{ix}^{t+\Delta t} = x_{ix}^{t} + \Delta t \cdot V_{lx}^{t+\Delta t}
\]

\[
R_{lx}^{t+\Delta t} = p_{lx}^{t+\Delta t} + \sum (\dot{T}_{ix}^{t+\Delta t} \cdot (x_j - x_i)^{t+\Delta t})
\]

\(R\): residuals
\(M\): lump mass at node
\(V, \dot{V}\): velocity, acceleration
\(\Delta t\): time increment
\(P\): external loads
\(T\): tension in cables
\(l\): cable length
\(x\): node position in x component
Equations are similar in corresponding y and z components.

There are three aspects of requirements to the cable network design. First is the high surface accuracy requirement. Generally the surface shape accuracy is characterized by systematic and random errors. Triangular faceted approximation of the parabola determines the extent of systematic errors, which can be controlled by the facet size. The random error is the deviation of the achieved point positions from the nominal positions, which depends on the design and manufacturing quality. The second aspect of requirements is related to tensions in the cables. The cables are to be tensioned and, for the reason of manufacturing
simplification, to be uniform in magnitude. Last but not least, the cable network must be in equilibrium configuration with satisfying the above two aspects of requirements simultaneously.

![Variation of kinetic energy to iterations](image)

**Figure 2: A typical curve of variation of kinetic energy to iterations**

The above mentioned two approaches of form finding are integrated into a structural optimization code of cable networks. Two types of examples are used as testing benchmarks to compare these two design approaches. One uses the frame structure as in Figure 1 and has a front to rear symmetric networks. The other uses a conical ring structure (Figure 3) and has a front to rear asymmetric networks.

![Conical ring structure](image)

**Figure 3: Conical ring structure [ESA patent 568]**

### 3 DESIGN OF CABLE NETWORKS USING FORCE DENSITY METHOD

In the design approach using FDM, FDM is used for form finding and gradient based algorithm (least square) is used for structural optimization. The performance and features of this method are discussed through design examples.

There are lots of design examples of symmetric cable networks in references [4, 6, 8, and 9]. Since it has a symmetric construction, simplifications e.g. using half of cable network, using vertical forces to instead of tie cables are taken in designs. Further simplifications like considering only the internal regular cable network while ignoring the interface cables, frame structures as well as their interactions are taken in these examples. In [9], Morterolle
presented a simple design approach using FDM to design parabolic and uniform tensioned cable networks. In his approach, Z coordinates of points are directly given by analytical parabolic functions according to their in-plane positions and tie forces are calculated by relating to tensions in cables and points’ positions. Therefore a 3D design problem is simplified to be a 2D design problem with only tension constraints. In his later publication [14], he demonstrated this method works also for offset parabolic configurations and other facets like square and hexagon.

Generally, those methodologies in literature have number of simplifications, which in fact have to be followed more accurately. For example, behavior of an interface between internal cable network to interface cables (Figure 4) is better to be taken into account. The over simplified problem leads to difficulties in applying this method to practice applications directly. Therefore in [14], a stepwise approach is performed to design the interface cables based on designed internal cable networks. But actually, the interface cables belong to the same structure as well, whose design can be implemented similarly and simultaneously to the internal cable network.

![Figure 4: A design example of a symmetric cable network with uniform tension](image)

The fact that the number of support truss points is usually less than the number of the edge points of the internal cable network determines the need in having interface cables. The interface cables connect several internal network points to a single truss point. As proved by numerical tests, it is infeasible to find an equilibrium form with identical tensions in all internal and interface cables. Therefore, modification of constraints with respect to the interface cables has to be implemented. It can be done in two ways. One is maintaining uniform tension in interface cables while allowing varying their positions. Then the tensions in interface cables can be allowed to be larger than in the internal cables. The other is maintaining the rim points’ positions of the internal network while varying tensions in interface cables. Examples of using the first modification can be found in [16] and examples of using the second modification are explained in the following.
The simplified method of Morterolle is not capable to handle these multi-types constraints. Therefore the general method described by Schek [11], which is a gradient based optimization, is used. Different gradients from position constraints (internal cable network edge points) and tension constraints (internal cables) are linearly combined by weighting factors for iterative update of the force densities. Four design examples using edge point position constraints are demonstrated in Figure 5. The first two have the same number of points and cables while different edge points’ positions. The latter two contain large number of members and complex interface cable connections.

All these four designs are achieved with uniform tension in internal cables and predefined positions of the edge points are maintained. One highlight of these designs is the narrow varied facet size, which is preferred for surface accuracy reason. The tensions in interface cables vary adding some complexities in manufacturing and assembling.

Different to the symmetric design, the simplifications due to symmetric conditions are not valid and design approaches are to be updated correspondingly in case of asymmetric network designs (Figure 3) [16].

As the number of points and cables increase (the largest number of points is 733 and cables is 2214 in Figure 5) in designs, the computation cost increases dramatically. Besides this, the convergence behavior is not stable due to multiple types of constraints. Damping factors must be appropriately selected to find a compromise between computation robustness and cost. For instance, the last example with proper selected damping factors took around 3 hours for computation on a high performance computer. To find these proper damping factors, several
trial and error tests are needed, which consumes extra time as well.

For high RF applications, the cable networks of medium class LDR require around 4000 to 6000 cable elements for achieving an acceptable level of systematic shape errors. The gradient based optimization with FDM seems quite complicated to accomplish this challenging task due to the following main drawbacks. The first is related to the stiffness matrix inversion in each iteration in the gradient calculation, which is generally time consuming and inaccurate. The second is related to the multiple types of constraints in optimization, which generate multiple gradients and they are difficult to be combined properly for updates. Both drawbacks belong to the used FDM but they can be avoided using DR method addressed in the following.

4 DESIGN OF CABLE NETWORKS USING DYNAMIC RELAXATION

Form finding with dynamic relaxation has significantly lower computation cost especially when using the kinetic damping method, which ensures a rapid and stable convergence of the analysis. DR method for form finding and structural optimization is implemented and tested through different examples discussed below.

Found shapes for several examples with front and rear symmetric networks are presented in Figure 6. Optimization constraints of internal cable network edge points are set to maintain their positions in a very narrow range while varying tensions in interface cables. Given examples in the figure contain different number of support truss points and intermediate points as well as the number of cables, which varies between 4000 and 6000.

Different to the iteratively updated design parameters to approach the goal using FDM, the targeted cable forces and positions are directly assigned to the initial network to cause oscillations (geometry changes) until reaching the equilibrium due to damping. The convergence criterion is set to force residuals on nodes, which determines the accuracy of results and computation cost.

Computation cost of the both design methods discussed has been compared using the same cable network with the same convergence criterions (tension residuals). This particular investigation shows that the FDM needs around 180 minutes for computation, while using the DR 10 minutes are sufficient. Additionally, the point position error in using DR is significantly smaller than it in using FDM (refer to the similar constraints). The design method using DR has shown attractive features of computation efficiency and robustness, which is appropriate to accomplish tasks of designing cable networks for high precision LDR for high radio frequencies.
One of the key points in asymmetric network design process using DR is the way of determination of the tensions in tie cables. The tensions in tie cables are updated in each iteration to be the difference between the actual tensions and out-of-plane residuals in front network points. The purpose of this action is adjusting the tensions in tie cables to set residuals in front network points to zero. So their pre-defined positions can be maintained. The rear network points have almost no position constraints and their geometry is simply updated according to updated residuals.
Various cable networks designs have been investigated using the described method. Results of these investigations show that computation cost is usually higher in designing asymmetric cable networks than symmetric ones due to configuration differences. The front networks of asymmetric cases have no significant configuration difference from the symmetric ones however their facet sizes vary wider than in symmetric cases for smaller rear network constructions.

5. INFLUENCE OF COMPLIANCE OF FRAME STRUCTURES

In above test examples, cable networks are attached to idealized rigid frame structures. A simplified truss frame structure, which is made from carbon fiber reinforced plastic (CFRP) tubes, is implemented to figure out the influence of its compliance to cable networks (Figure 8). The used cable network contains 1670 points and 5815 cables and its convergence criterion for design optimization is set to residual to be less than 1% of tension in cables.

Subject to an idealized rigid frame, the point position error of the designed network in FEM analysis is insignificant (6.2e-5 mm under 1% residuals). With considering the compliance of the frame structure under radially symmetric and statically determinate constraints in FEM, the point position error is increased significantly (Table 1).

The compliance of the frame structure can be reduced by increasing its bending stiffness. For instance, increasing the CFRP tubes diameter by about 25%, the maximal deformation of the frame structure is reduced by 30% and the surface error is also marginally reduced. This example has shown increasing bending stiffness of the frame structure is a useful but not an
efficient method.

![Figure 8: A cable network and its frame structure](image)

On the other hand, the compliance problem can be solved by an iterative method. The deformation of the frame structure indicates the boundary conditions of the cable network have been changed. Therefore an update design of the cable network according to updated boundary conditions is required. The resulting acting forces due to cable network updates change deformation of the frame and in the end influences are reflected back to the cable network design. The iterative procedure is continued until either change of the reaction forces or deformations of the frame are small enough. Using this iterative method, the point position error is close to the value of rigid frame model.

<table>
<thead>
<tr>
<th></th>
<th>Rigid frame model (reference)</th>
<th>Compliant frame model</th>
<th>Increasing bending stiffness of the frame structure on 25%</th>
<th>Iterative update frame structure and cable network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative point position error</td>
<td>1</td>
<td>114</td>
<td>92</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Though the relative change of point position error due to compliance of frame structures is large, its absolute value can be not critical and be accepted in applications. With a sufficient high stiff frame structure, the compliance influence can be ignored even. These investigations have shown another fact of the designed networks, they are quite sensitive to their positioning and small deviations in points locally may cause a relatively large overall point position error.

6. CONCLUSIONS

In this paper, two approaches of cable networks design optimization for high precision LDR were discussed and compared. The design method using DR with kinetic damping is much more efficient and robust as compared to the FDM. This is especially the case when networks contain large number of members and have complex configurations.

A negative influence of the compliance of the frame structure on surface accuracy can be
maintained acceptable, if setting the convergence criteria sufficiently small. On the other hand, relative error increases at least on two orders of magnitude as compared to the assumed rigid boundary. This can be still reduced by the established iterative procedure of taking into account the frame deformations during the optimization process.

In practice, manufacturing and assembling of such high accurate structures is another challenging task. The cable networks accuracy is quite sensitive to deviations of point positioning, which indicates difficulties in implementing the high accurate designs. As a part of future work, the uncertainty study of used material properties, points’ positioning and tensions in cables will be investigated.

REFERENCES


1. INTRODUCTION

Membrane structures have received a wide range of attention in applications of large-scale spacecrafts, such as inflated wing, light-than-air (LTA) airship and membrane antenna reflector, and so on. These spacecrafts need to be designed as high loading-efficiency components or structures, especially with high shape precision [1]. Therefore, some special processing and design should be done on the membranes so as to satisfy with special requirements, such as free wrinkle, ultra-lightweight, high shape precision, and high load-carrying ability, etc..

Several applications may give us good ideas to deal with abovementioned problems in membrane structures. One example is easy to be remembered, that is, the Super-Pressure Balloons (SPB) [2-4] covered by some ropes on membrane surface to make the balloon stable and strong. The similar considerations are also applied to some gossamer spacecraft components or structures, taking the Lunar habitat [5] as an example.

Experiments on small scale ground models have shown that wrinkles are present over a wide range of applications. Thus membrane wrinkling has attracted much interest in the past, starting from the development of tension force filed theory, the bifurcation simulations to the explicit time integration etc.. [6-10]

2. MRM AND ITS PROPERTIES

In order to make the material isotropic, we need to design some special forms of mesh. They are as follows, see Fig 2.

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*Project supported by National Natural Science Foundation of China, 11172079; Program for New Century Excellent Talents in University, NCET-11-0807; the Fundamental Research Funds for the Central Universities, HIT.BRETIII.201209 and HIT.NSRIF.201156.
We classify the reinforced ribbons placed in a parallel pattern as a series and separate each series (suppose it has \( n \) layers), and divide the membrane into a number of layers (each has a thickness of \( 1/n \) layer), with a range of reinforced ribbons and a single-layer film combining into a composite structure consisting of layers. We call this scheme as the "pseudo-laminate method." Taking the reinforced membrane structure by using ribbons in three directions for example, it depicts equivalent process for analyzing the mesh reinforced membrane by using ribbons in three directions. We combine each range of ribbons and a \( 1/3 \)-thin layer of membrane into one composite laminate, then MRM can be divided into 3 composite layers.

Decomposition schematic and its cell element of 3-direction mesh reinforced membrane is shown in Fig.7. Here, the layer with \( 0^\circ \) reinforced belts is named as "standard layer". The other non-standard layers may be obtained by transforming the standard layer. In order to make the analysis easy to understand, this paper analyzes MRM by using ribbons in three directions, using the reinforcing method of "isogrid". We call the smallest periodic unit "cell element".

![Decomposition schematic of 3-direction reinforced mesh membrane and cell element](image)

If the reinforced ribbons in each direction are placed with equal space, assuming there are \( n \) directions, the reinforced ribbons in \( i \)-direction has an angle of \( \theta_i \) with the initial direction. If the membranes are divided into \( n \) layers and we regard each layer and a series of corresponding reinforced ribbons as one laminate, then the corresponding layer can be regarded as a simple laminate. We can get the elastic modulus, Poisson's ratio and shear modulus in the \( x, y \) direction under the same coordinate based on the formula. We remark the material parameters of \( i \)-laminate are \( E_i^x \), \( E_i^y \), \( \nu_{xy}^i \) and \( G_{xy}^i \), and we remark the corresponding volume weight as \( V^i \left( \sum_{i=1}^{n} V^i =1 \right) \). Using parallel model for composite materials, we can obtain the elastic modulus:
\[ \bar{E}_x = \sum_{i=1}^{n} V^i E^i_x ; \quad \bar{E}_y = \sum_{i=1}^{n} V^i E^i_y ; \quad \bar{v}_{xy} = \sum_{i=1}^{n} \frac{E^i v^i_{xy}}{\sum E^i} ; \quad \bar{G}_{xy} = \sum_{i=1}^{n} V^i G^i_{xy} \]  

(1)

The tensile stiffness along reinforce belt can be expressed as:

\[ \bar{E}_x A = A \sum_{i=1}^{n} V^i E^i_x \]  

(2)

For getting the parameter for each laminate, we can divide the laminate into two parts, including the reinforced ribbons and the membrane. We can first get the parameter of the ribbons of the standard layer. By transforming from the standard layer to the layer with an angle of \( \theta_i \), we can get the parameters like \( E^i_x \), \( E^i_y \), \( v^i_{xy} \) and \( G^i_{xy} \). Then using the formulas (1) and (2) and take the ribbons and membrane as a laminate, we obtain its \( E^i_x \), \( E^i_y \), \( v^i_{xy} \) and \( G^i_{xy} \). We assume the elastic modulus directed toward the fiber of a standard layer is \( E_1 \), the elastic modulus perpendicular is \( E_2 \), Poisson's ratio is \( \nu_{12} \) and \( \nu_{21} \) and the shear modulus is \( G_{12} \).

Assuming these two kinds of materials are elastic, and that there is no relative displacement between the membrane and reinforced ribbons. The direction of reinforced ribbons is 1, and the vertical direction in the plane is 2. There is the same strain along the 1 direction between the membrane and the reinforced ribbons. The parallel model of theory of the simple laminates shows that:

\[ E_i = E_m V_m + E_f V_f = E_m \frac{b_m t_m}{b_m t_m + b_f t_f} + E_f \frac{b_f t_f}{b_m t_m + b_f t_f} \]  

(3)

Among them, the elastic modulus of membranes is \( E_m \), width of membrane is \( b_m \), the thickness of membrane is \( t_m \), the elastic modulus of reinforced ribbons is \( E_f \), width of reinforced ribbons is \( b_f \), the thickness of reinforced ribbons is \( t_m \).

We divide the cell layers into two parts, which contains partly reinforced ribbons and pure membrane. The part with reinforced ribbons is a two-layer isotropic compound material which contains reinforced ribbons and pure membrane. We can get its elastic modulus with reinforced ribbons from a parallel model of theory of simple laminates:

\[ E_{mf} = E_m \frac{t_m}{t_{mf}} + E_f \frac{t_f}{t_{mf}} \]  

(4)

Assuming cell layer only bears the load of \( F_2 \) along the 2 direction, we combine this layer with theory of simple laminates model and can get that, the equivalent elastic modulus along the x-axis is:

\[ E_2 = \frac{\sigma_2}{\varepsilon_2} \]  

(5)

For a standard layer of a cell layer in the structure by using reinforced ribbons, the deformation of the overlapping region between the reinforced ribbons and the membrane is coordinated.

We explore the part with reinforcing ribbons and the part without those respectively. According to Hooke's law, if one only subjects to the stress of 1 direction, we can get the equivalent Poisson's ratio by superimposing lateral deformation.
\begin{align}
\nu_{12} &= \left(1-\frac{b_f E_f t_f}{b_m (E_m t_m + E_f t_f)}\right)\nu_m + \frac{b_f E_f t_f}{b_m (E_m t_m + E_f t_f)}\nu_f 
\end{align}

Add the following main stress to the element in the 1, 2 coordinate system
\begin{align}
\sigma_{12} &= \begin{bmatrix} \sigma_{11} & -\sigma_{11} & 0 \\ \end{bmatrix}^T
\end{align}

In the x, y coordinate system, for the standard level that has the rotation angle of \(-\pi/4\), we can convert its main normal stress into tangential stress, namely:
\begin{align}
\sigma_{xy} &= \begin{bmatrix} 0 & 0 & \sigma_{11} \end{bmatrix}^T
\end{align}

According to the relationship between stress and strain of the materials we can get
\begin{align}
\varepsilon_{xy} &= C\sigma_{xy} = \begin{bmatrix}
-\frac{E_1-E_2}{2E_1E_2}\sigma_{11} \\
-\frac{E_1-E_2}{2E_1E_2}\sigma_{11} \\
\frac{E_1+2\nu_2E_2+E_2}{2E_1E_2}\sigma_{11}
\end{bmatrix}
\end{align}

In which, the flexibility matrix in the x, y coordinate system is conversion of the flexibility matrix of the standard layer. At this point, the shear strain can be expressed as:
\begin{align}
\gamma_{xy} &= \frac{E_1+2\nu_2E_2+E_2}{2E_1E_2}\sigma_{11} = \frac{E_1+2\nu_2E_2+E_2}{E_1E_2}\tau_{xy}
\end{align}

For the material parameters are not affected by the status under stress, the shear strain can be expressed as:
\begin{align}
G_{12} &= \frac{\tau_{12}}{\gamma_{12}} = \frac{E_1E_2}{E_1+2\nu_2E_2+E_2}
\end{align}

For the \(i\)-th layer that is disposed randomly, we assume that the layer is rotated from the standard layer with the rotation angle of \(\theta_i\) as shown in Fig.4. We assume that the 1 direction is along the longitudinal direction of the reinforced ribbons while the 2 direction is perpendicular direction. From plane stress problem, we can get that:

![Fig.4 Non-standard layer model](image)

For the model after coordinate conversion, the apparent constants are the following in the non-main direction of the x, y coordinate system:
\begin{align}
E_{1i} &= \frac{1}{\cos^4 \theta' + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\sin^2 \theta' \cos^2 \theta' + \frac{\sin^4 \theta'}{E_2}} \\
E_{2i} &= \frac{1}{\sin^4 \theta' + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\sin^2 \theta' \cos^2 \theta' + \frac{\cos^4 \theta'}{E_2}}
\end{align}
We design several kinds of MRM, as shown in Fig.5. In order to obtain the tension properties of MRM, the tensile experiment of isogrid MRM is performed. The samples are composed of PI (mesh) and PU (membrane). Each sample consists of 4 cell elements. The elastic modulus of membrane (PI, 0.1mm) is 2.99GPa, and Poisson’s ratio is 0.34. The elastic modulus of tape (PU, 0.025mm) is 67MPa, Poisson’s ratio is 0.34. And the cell element is 23mm width, 40mm length. The size of membrane is 40mm×138mm. The tension velocity is 10mm/min using Instron5965 with 5KN sensor. The tension properties of MRM is tested and compared to pristine membrane, shown as follows.

Fig.5 MRM reinforced by different tapes (Membrane/ Membrane, Composite fiber/Membrane, Composite tape/Membrane)

Fig.6 Tension displacement versus load in x and y directions

Fig.7 Comparisons of tension properties between MRM and pristine membrane (y direction)

We also simulate the tension performance of MRM, which shown as follows.

Fig.8 Major principle stress in x and y direction.

Observed from Fig.8, the major principle stress is located fully on the reinforced ribbons which reveal the reinforcement component in MRM.

Surface density of PU is 130g/m², while that of PI is 36g/m². We define modulus/weight as elastic modulus efficiency. Comparing with performance predictions using pseudo-laminate method, we can
get the predictions of elastic modulus of MRM (as shown in Tab.1). From the Tab.1, we can see that the experiments agree well with the predictions.

<table>
<thead>
<tr>
<th>Items</th>
<th>Experiment</th>
<th>Prediction</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus, $E_x$ (MPa)</td>
<td>250</td>
<td>239</td>
<td>4.4</td>
</tr>
<tr>
<td>Modulus, $E_y$ (MPa)</td>
<td>140</td>
<td>150</td>
<td>7.1</td>
</tr>
</tbody>
</table>

Then we can get the modulus efficiency of pristine membrane and MRM as follows in Tab.2.

<table>
<thead>
<tr>
<th>Items</th>
<th>PM</th>
<th>MRM</th>
<th>MRM/PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment EEx (MPa*m²/kg)</td>
<td>515.4</td>
<td>1592.4</td>
<td>3.09</td>
</tr>
<tr>
<td>Prediction EEx (MPa*m²/kg)</td>
<td>515.4</td>
<td>1522.3</td>
<td>2.96</td>
</tr>
<tr>
<td>Experiment EEy (MPa*m²/kg)</td>
<td>476.9</td>
<td>885.4</td>
<td>1.86</td>
</tr>
<tr>
<td>Prediction EEy (MPa*m²/kg)</td>
<td>476.9</td>
<td>987.3</td>
<td>2.07</td>
</tr>
</tbody>
</table>

Based on above comparisons, MRM has higher elastic modulus efficiency than pristine membrane which reveals a high load-carrying ability of MRM.

3. MRM WRINKLING

We simulate the membrane wrinkling using explicit time iteration and verify it using DIC technology. In order to capture the major wrinkles in membrane we choose a special square membrane under corner tension. Square membrane (size and material properties are same with specimen in Part 2) is 100mm side length and 25mm loaded corner width. The wrinkling patterns in simulation and experiment are compared and shown as follows. The simulation agrees well with experiment with 5% difference.

![DIC test system and tension wrinkling test machine](image1)

Fig. 9 DIC test system and tension wrinkling test machine

![Experimental wrinkling patterns and y-strain of pristine membrane (1mm tension)](image2)

Fig.10 Experimental wrinkling patterns and y-strain of pristine membrane (1mm tension)
The simulated wrinkling patterns and y-strain as well as the out-of-plane displacement comparisons are shown in Fig.11.

Then we simulate the wrinkling patterns in square MRM. The wrinkling results are shown as follows.

The simulated wrinkling parameters of MRM, wrinkling wavelength is 14.37mm, wrinkling amplitude is 0.56mm, wrinkling numbers (crest, trough) is 3(1,2), which are smaller than those in pristine membrane, they are 15.05mm, 0.75mm, and 5(2,3), respectively. In addition, for 1mm tension displacement, the y-stress of MRM and pristine membrane are 3.82MPa and 1.69MPa, respectively. We use a loading efficiency (y-stress/total mass) to evaluate the load-carrying ability of these two kinds membrane. We have loading efficiencies of MRM and pristine membrane which are 1.91 and 1.3, respectively. These comparisons reveal the reinforced mesh may make MRM more stable and stronger than pristine membrane.

CONCLUSIONS

This paper presents a concept of mesh reinforced membrane (MRM) as well as simulates its tension properties which verified by the experiment. The membrane wrinkling is then simulated and verified by DIC test. In the end, the MRM wrinkling is simulated and compared with the pristine membrane. The results reveal the reinforced mesh may make MRM more stable and stronger than the pristine membrane.

REFERENCES

EFFECT OF CUSHION GEOMETRY AND CONFIGURATION ON THE EMBODIED ENERGY OF ETFE FOIL CONSTRUCTION

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Key words: ETFE Foil, Embodied Energy.

Summary. The low weight and high spanning capacity of ETFE foil when compared to other translucent cladding materials has potential to reduce the weight of supporting structures and energy embodied in their construction. This paper compares the embodied energy in ETFE cushion panels of different shape, size and configuration, relates these to some built examples. The results are compared with the estimated embodied energy of some built examples of ETFE roofs and from other studies. Factors that influence these are reviewed.

1 INTRODUCTION

The low weight of ETFE foil (typically less than 1kg/m² in three-layer inflated cushions) and high spanning capacity (with examples of up to 10m when used as a cladding material in building envelopes, for example, the Dolce Vita Tejo shopping mall, Amadora, Portugal) has potential benefits for both the extent and weight of supporting structures and the energy embodied in their construction.

A recently published Environmental Product Declaration EPD-VND-2011111-E for the Texlon® ETFE foil roof system¹ includes an assessment of the typical embodied energy per square metre of a three-layer cushion, which assumes an “...average quantity of frame material required for the assembly of the roof construction”¹. This provides valuable information about the overall environmental impact of ETFE foil cushion construction, highlighting the individual contributions to the total of the ETFE foil, aluminium edge profile and transportation. However, this is not particularly useful to designers who may wish to minimise the embodied energy of their building envelopes, as it does not take into account the effect of individual cushion geometry or, where there is more than one, the configuration of the cushions.
To clarify this issue, the results of a recent desktop study into the relative embodied energy in ETFE cushion panels of different shape, size and configuration are described. The results are compared with the estimated embodied energy of some built examples of ETFE roofs and from other studies. Factors that influence these are reviewed.

In conclusion preferred configurations are suggested and the effects that these may have on the configuration of supporting secondary and primary structures are discussed.

2 EMBODIED ENERGY OF ETFE FOIL CONSTRUCTION

The principal components used in the construction of ETFE foil building envelopes are the foil itself and some form of aluminium perimeter profile. Published data was used to assess their relative contribution to the overall EE. However, as is demonstrated by the relatively wide range of values assessed in Hammond and Jones’s Inventory of Carbon & Energy, estimation of the energy embodied in materials from cradle to gate is not as easy as it may seem, even for common and well-established construction materials. For ETFE foil estimates of embodied energy vary considerably, as can be seen in Table 1. The most recent is over ten times that stated by Robinson-Gayle et al in 2001.

Table 1: Comparison of published values of embodied energy MJ/kg for ETFE foil

<table>
<thead>
<tr>
<th>Source</th>
<th>Embodied energy MJ/kg</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robinson-Gayle et al 2001</td>
<td>26.5</td>
<td>[p.325]</td>
</tr>
<tr>
<td>Fernandez 2006</td>
<td>120-130</td>
<td>Cited in Monticelli et al 2009</td>
</tr>
<tr>
<td>Ashby et al 2007</td>
<td>100-120</td>
<td>Cited in Monticelli et al 2009</td>
</tr>
<tr>
<td>EPD Texlon® 2011</td>
<td>337.3</td>
<td>Calculated from 326.2MJ/m² quoted for a three-layer cushion weighing 0.967kg/m² [p.17]</td>
</tr>
</tbody>
</table>

For this study the highest (and most recent) embodied energy value, 337.3MJ/kg, was used to compare the relative efficiency of different ETFE cushion geometries. Although this value applies specifically to foil material supplied by Nowoflon and incorporated in the Texlon® system, due to the limited number of ETFE producers and foil manufacturers, and the generally similar methods of foil cushion fabrication, in the absence of other data, this was considered acceptable. For the extruded aluminium edge clamping profile a typical value of 154MJ/kg was taken from Hammond and Jones, which is assumed to include an average of 33% recycled material.

3 INFLUENCE OF CUSHION GEOMETRY

In contrast to the method employed in the Environmental Product Declaration EPD-VND-20111111-E for the Texlon® ETFE foil roof system, which calculates the embodied energy for a nominal cushion area of one square metre, Chilton, Pezeshkzadeh and Afrin have
investigated the effect of changing cushion geometry and configuration on embodied energy per square metre of cushion. The results, for areas ranging from 5m² to 100m² and aspect ratios (length/width) from 1 to 20, are shown for individual cushions in Figure 1, for cushions sharing profiles on two (longest) edges in Figure 2, and in Figure 3 for cushions sharing profiles along all four edges. Graphs have been deliberately plotted to the same vertical scale to allow direct comparison between the embodied energy (MJ/m²) of the different conditions.

Figure 1: Embodied energy MJ/m² for different ETFE individual cushion areas and aspect ratios (length/width).

Figure 2: Embodied energy MJ/m² for different ETFE cushion areas and aspect ratios (length/width) with two (longest) shared edges.
Embodied energy MJ/m² for different ETFE cushion areas and aspect ratios with four shared edges

Figure 3: Embodied energy MJ/m² for different ETFE cushion areas and aspect ratios (length/width) with all four shared edges.

Although the foil thickness in real building envelopes will be dependent on the size of the cushion, inflation pressure and anticipated wind and snow loading, for consistency, a theoretical three-layer cushion was assumed, which has the same foil thicknesses for any size of cushion. Equally the same edge profile was used in all cases.

Comparing the three graphs, Figures 1 to 3, it can clearly be seen that, for any given cushion area and aspect ratio, isolated cushions will expend the greatest amount of energy per square metre in their production. Of these, as one would expect, square (1:1 aspect ratio) cushions consume the least energy for any given covered area – 1591 MJ/m² for a 5 m² cushion, compared to 3296 MJ/m² for a cushion of the same area but with aspect ratio of 1:20 – over twice the EE/m². These values should be compared with the embodied energy of 853.6 MJ determined for a 1 m² notional cushion (transportation to site excluded) according to the EPD-VND-2011111-E for the Texlon® system. For cushions of 25 m² the equivalent values are 892 MJ/m² and 1655 MJ/m² with a relative embodied energy ratio of 1.86. At 100 m², the area of the square cushions used for the roof of the Dolce Vita Tejo shopping mall in Amadora, Portugal, the equivalent values are 609 MJ/m² and 990 MJ/m² with a relative embodied energy ratio of 1.63.

Allowing edge profiles to be shared between two adjacent on their longest edges has the greatest effect for cushions with higher aspect ratios. For instance, for square cushions of 5 m² the EE/m² reduces from 1591 to 1275 MJ/m², a reduction of 20%, for cushions with 1:5 aspect ratio the decrease is from 2024 to 1316 MJ/m², a reduction of 35%, whilst for the cushions with 1:20 aspect ratio the decrease from 3296 to 1882 MJ/m² represents a reduction of 43%.

When sharing edge profiles on all edges there is only a relatively small decrease in the EE/m² for cushions with high aspect ratio but larger savings for cushions that are closer to square. In this case, for square cushions of 5 m², the EE/m² reduces from 1275 to 959 MJ/m², a further reduction of 20%, for cushions with 1:5 aspect ratio the decrease is from 1316 to
1175MJ/m\(^2\), a further reduction of 7\%, whilst for the cushions with 1:20 aspect ratio the decrease from 1882 to 1811MJ/m\(^2\) represents a further reduction of just 2\%.

Previously the authors have reported the estimated EE/m\(^2\) of ETFE cushions used for three roofs of different configuration: ETFE 1 - 5 x 5 grid of 25 approximately square cushions; ETFE 2 – parallel cushions of varying length but similar width; ETFE 3 - single head-ring supported conic cushion\(^2\).

### Table 2: Estimated embodied energy of MJ/m\(^2\) for ETFE foil and glass roofed atria.

<table>
<thead>
<tr>
<th>Roof</th>
<th>Covered area m(^2)</th>
<th>Length of perimeter profile m</th>
<th>Perimeter/covered area</th>
<th>Estimated embodied energy (MJ/m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETFE 1</td>
<td>616</td>
<td>301.2</td>
<td>0.49</td>
<td>582</td>
</tr>
<tr>
<td>ETFE 2</td>
<td>352</td>
<td>187.2</td>
<td>0.53</td>
<td>719</td>
</tr>
<tr>
<td>ETFE 3</td>
<td>463</td>
<td>99.0</td>
<td>0.21</td>
<td>598</td>
</tr>
</tbody>
</table>

At 582MJ/m\(^2\) the approximately 25m\(^2\) average area cushions of roof ETFE 1, which mainly share edge profiles on all sides, accords well with the predicted embodied energy (from Figure 3) for cushions of this size and configuration (609MJ/m\(^2\)). The average, approximately 35m\(^2\), cushion of roof ETFE 2 (with aspect ratio of around 12) has an EE/m\(^2\) of 719MJ/m\(^2\) which compares favourably with the predicted value of 816MJ/m\(^2\), from Figure 2. Finally the single large cushion of roof ETFE 3 with an embodied energy of 598MJ/m\(^2\) is within 2\% of the estimated value (609MJ/m\(^2\)) for a 100m\(^2\) square cushion.

### 4. IMPACT ON EMBODIED ENERGY OF THE PRIMARY AND SECONDARY STRUCTURE

Significant benefits from using ETFE foil in building envelopes, in roofs in particular, derive from savings in the number of secondary supporting elements, the material required for them and consequent reduction size of the primary supporting structure. The former is due to the spanning capacity of the foil when used as a tensioned surface – synclastic in inflated cushions and anticlastic in single layer applications. Whilst, for the latter, the minimal thickness of the foil and consequent low self-weight, typically around 1kg/m\(^2\) in triple layer cushions, minimize self-weight actions on a roof.

Despite the apparent benefits of using ETFE foil in place of other transparent and
translucent cladding materials, as noted by Cremers\(^7\), there is little published data on the embodied energy savings that can accrue. One of the reasons for this is the difficulty of comparing like with like. Enclosures of similar size and shape often have very different supporting structures (beams, trusses, number of columns), spaced at different centres and with different environmental loading (snow/wind) and support configurations. However, Cremers\(^7\) cites a study by Manara\(^8\) where the primary embodied energy has been calculated for similar roofs of 27 x 33.5m clad in glass and ETFE foil alternatives. In that study the embodied energy of ETFE foil was taken to be 140MJ/kg, similar to that proposed by Ashby\(^7\) and Fernandez\(^8\).

For this paper the authors have estimated the embodied energy for two built examples of ETFE foil covered roofs in the UK, including the cushions, aluminium profile frames and roof steelwork but supporting column steelwork is not included. These are included with those reported by Manara\(^8\) in Table 3, below.

**Table 3:** Comparison of primary embodied energy (kWh) and (kWh/m\(^2\)) for glass and ETFE-covered roofs

<table>
<thead>
<tr>
<th>Structure</th>
<th>Mass (t)</th>
<th>Primary Energy kWh</th>
<th>Primary Energy kWh/m(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Glass roof from Manara <em>et al</em>(^8) (1320m(^2))</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel and substructure</td>
<td>112.4</td>
<td>877,000</td>
<td>664</td>
</tr>
<tr>
<td>Glazing</td>
<td>66</td>
<td>390,000</td>
<td>295</td>
</tr>
<tr>
<td>Glass roof (total)</td>
<td>178.4</td>
<td>1,267,000</td>
<td>960</td>
</tr>
<tr>
<td><strong>2. ETFE foil roof from Manara <em>et al</em>(^8) (1370m(^2))</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel and substructure</td>
<td>78.3</td>
<td>636,000</td>
<td>464</td>
</tr>
<tr>
<td>ETFE cushions</td>
<td>1.3</td>
<td>53,000</td>
<td>39</td>
</tr>
<tr>
<td>ETFE roof (total)</td>
<td>80</td>
<td>689,000</td>
<td>502</td>
</tr>
<tr>
<td><strong>3. ESLC, University of Nottingham (352m(^2))</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel trusses</td>
<td>16</td>
<td>100,400</td>
<td>285</td>
</tr>
<tr>
<td>Aluminium profile</td>
<td>1</td>
<td>43,300</td>
<td>123</td>
</tr>
<tr>
<td>ETFE cushions</td>
<td>0.23</td>
<td>21,500</td>
<td>61</td>
</tr>
<tr>
<td>ESLC Total</td>
<td>17.23</td>
<td>165,200</td>
<td>469</td>
</tr>
<tr>
<td><strong>4. Nottingham Boys High School (616m(^2))</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel trusses</td>
<td>19.4</td>
<td>125,100</td>
<td>203</td>
</tr>
<tr>
<td>Aluminium profile</td>
<td>1.38</td>
<td>59,100</td>
<td>96</td>
</tr>
<tr>
<td>ETFE cushions</td>
<td>0.43</td>
<td>40,400</td>
<td>66</td>
</tr>
<tr>
<td>NBHS Total</td>
<td>21.21</td>
<td>224,600</td>
<td>365</td>
</tr>
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</table>
5 CONCLUSIONS

As demonstrated by Figures 1 to 3, the overall EE/m² of a building enclosure can vary considerably depending on the size, shape (aspect ratio) and overall configuration (number of shared edge profiles) of the ETFE cushions employed in its roof and façade construction. Although the contribution of the ETFE foil relates directly to the area of the cushion, that of the aluminium edge profile is determined by the length of the perimeter. Hence the EE/m² is much higher for small cushions, where the perimeter/area ratio is highest.

The embodied energy in cushions that share edge profiles is less than that for isolated cushions. With two shared edge clamping profiles the greatest benefit accrues when long, thin cushions (with high aspect ratio) are joined on their long edges.

For the designer, a slightly simplistic view may have been conveyed by the EE values quoted in the EPD -VND-2011111-E for the Texlon® system. Taking the EE for the construction of a notional 1m² of three-layer ETFE cushion, could understate the EE in small cushions and overstate the EE in large cushions, depending on their geometry and configuration.

For all three examples of ETFE foil covered roofs (proposed and built) the volume and weight of material consumed and the energy embodied within it is generally less than half that of the proposed glass roof. This is a demonstrable advantage of ETFE foil cladding systems which should perhaps be exploited more.

6 ACKNOWLEDGEMENTS

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ELLIPSOIDAL SHAPE AND DAYLIGHTING CONTROL FOR THE ETFE PNEUMATIC ENVELOPE OF A WINTER GARDEN

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Key words: ETFE, Thermal Indoor Comfort, Envelope engineering, Shading strategies.

Summary. The paper aims to highlight the design and building processes and the interface between the different disciplines, during the design of the pneumatic envelope of the Winter Garden in Verona. Especially the step between the executive design and the structural design of the elipsoidal shape, made of etfe cushions, and the surface characterization and strategy in order to control the solar radiation, in terms of environmental building design.

1. INTRODUCTION

From the initial use on projects such as botanical gardens, zoos, swimming pools, and exhibitions spaces, the ETFE pneumatic technology is now increasingly chosen as envelope design in more traditional buildings for permanent uses and where people spend many hours of the day, i.e. roofing for courtyards, shopping malls, atria and stores. It is more and more used in different climatic conditions, till extreme conditions. Such cases in the Southern Europe, where the solar radiation is intensive for most of the year, exist. Its popularity is mainly due to the daylight transmittance and because of the possibility to generate special shapes. The cushions provide thermal insulation, but are quite totally transparent to long wave radiation. It shows an incomparable transparency equal to 90-95% of the total light and 83-88% of the ultraviolet light, resulting comparable to day light and a marked greenhouse effect (related to high absorption in the infra-red range). Depending on the building use, its design, the site, and the geographical location, some design strategies have to be studied to balance the overheating and the indoor comfort.

ETFE offers a valid alternative to glazing in building envelopes. In comparison to equivalent glazing, pneumatic cushions achieve a comparable level of performance with less than 1% of the weight. This reduces the amount of secondary structure required to support the building envelope with consequent benefits on the primary structure and the foundations allowing unsupported spans up to 10m\textsuperscript{1,2}. The potentialities of ETFE overtake the traditional idea of a static building envelope and allow a new type of building envelope which acts as an adaptive filter able to optimise the interaction between the internal ambient and the surrounding environment that can control the light transmission and heat transfer of the
cushions. Since the first applications dating back some 30 years ago, born as a technological material mainly from agriculture to architecture, shows an evolution in the treatment strategies of the transparency to sunlight, related to the function of the structure and its location. The main optical limit of the material is represented by the level of distortion, related to the final foil curvature, and by the relatively high diffusion (1.5-3%), which results in a slightly milky view through the building element. However, this characteristic turns into a disadvantage in regions of high sun radiation where providing shading elements is necessary. The use of totally transparent ETFE films for the construction of indoor structures with a comfortable winter climate in cold areas (i.e. Mangrove House in the Arnhem Zoo, Netherlands, 1982 Eden Project, Cornwell, UK, Grimshaw & Partners, 1996) looking for modulation of the intensity of solar radiation through some technological strategies: a the use of printing on film by corona treatment and pigmentation with aluminum powders (with different possible intensity); it allows to print geometric shapes that slightly dull the film and partially reduce the intensity of the incoming solar radiation, in order to optimize the bright and thermal comfort inside, especially in summer and in hot climates (Art center for the college of design, Pasadena, USA, D. Genik Architects, 2004; Duxford Visitors Centre, Duxford, UK, HOK International, 2005); b the use of color in the mixture of the base material granules, the variation of glossiness, the density and the number of layers are strategies that allow modular infinitely the optical and thermal properties of the cushions (i.e. Water Cube, Beijing, China, 2003-2008, C. Bosse, R, Leslie-Carter); c the light transmitted from the ETFE envelope can be decreased through the multilayer cushions manufacturing, by combining different frit patterns printed on the layers which can be moved by changing the air pressure of the chambers; d in a less integrated way, but opposite, some embodiments consider on the inner side the installation of additional shielding systems, such as roller blinds, adjustable brise soleil or opaque panels (i.e. DBU Conference and Exhibition Pavilion, Osnabrück, Germany, 2002, Herzog and Partners). Some recent trends provide for the manufacturing of the cushions also different levels of film transparency with respect to the orientation of the North-South, by leaving transparent areas in the north oriented cushions, ensuring the brightness and the perception of the outside, and dulling with a milky color the areas facing south, in order to reduce or block the UV rays and IR, dampening overheating the effect. (Shopping centre project, Portugal, Promontorio architects - the foils are printed depending on their orientation with respect to the sun, half the cushion is transparent ETFE that faces north allowing sun penetration while the other half, made from white ETFE, is facing south to block sun radiation.

The issues related to thermal insulation and solar radiation control in certain times of the year are always to be considered in the design phase. The pneumatic cushions behavior to solar radiation is complex and is the subject of scientific studies; there is still no legislation on this matter, unless documentation resulting from an experimental and often empirical approach.
2. AIM OF THE PAPER

The paper focuses, in a first part, on the review of the state of the art on the available data on the thermal performance of ETFE cushions, then on an overview of the state of the art information on the subject, and, in a second part, illustrates the case study of the multipurpose dome-shaped building, at the Forum Hotel in Verona, designed by Mario Bellini Associati, where the transparent ETFE pneumatic system was chosen as the covering of a multifunctional space for a facility. The objective of this paper is to highlight the decision-making process and the continued mediation within the phases of the executive project related to, on the one hand, to the definition of the technical and structural details for the building envelope and, on the other hand, to design strategies related to the ETFE as facade material, required in order to balance the indoor comfort and the daylighting and to avoid the overheating of the indoor air.

3. INDOOR THERMAL COMFORT IN STRUCTURES COVERED WITH ETFE

It turns up in the field of membranes for architecture the need to understand thoroughly the ETFE thermal and optical properties of the pneumatic systems and, consequently, to have technical information supporting the design on strategies for shading and screening of the transparency. The thermal properties of a building are an important consideration, they impact on the thermal performances and ultimately to the energy cost of the building. A well insulated building will be both easier to heat and easier to cool than a badly insulated building. The U value is an index measuring the heat flux through an element per unit of surface area and temperature difference\(^6\).

Thermal performance is an important factor when dealing with materials of minimum thickness of 0.1-2mm such as ETFE foils as they can’t provide appropriate thermal resistance. Although ETFE does not offer exceptional thermal insulating properties, the use of multilayer solutions allows the achievement of considerable values of thermal insulation, comparable with those obtained by means of glazed envelopes, reducing overheating, internal condensation and the energy required for air conditioning, both during summer and winter\(^2\). Energy strategies to increase the thermal insulation are applied since many years: - by multiplying the ETFE layer (the intermediate pretensioned ones do not assume a structural function in the cushion, but divide the volume of the air in two chambers connected, thereby improving the thermal insulation), - by inserting ETFE layers with shielding printings on the other hand, - besides focusing on the manufacturing of aluminum frames with thermal break profiles, which are clamping profiles usually consisting of several thermally separated parts\(^7\). If the upper and lower layer are fixed independently one from the other on the primary structure, without welding, the U value can be improved of about 20%\(^8,9,10\).

As declared by Knippers, there are two aspects affecting the value of heat transmission. The first is the surface resistance of air/material interfaces which can be controlled by the number of layers and chambers. Each additional layers of material reduces the volume and provides two more surface resistances (air/membrane/air) which improve the thermal resistivity of the system. The second is the convection effects within the cushion caused by the rising warmer air. These effects depend on the cushion orientation and the direction of heat flow whether horizontal or vertical.

In two studies regarding the application of ETFE cushions as transparent cover for areas
between buildings or atriums, it has been deepened the thermal insulation aspect of the system\textsuperscript{11,12}.

\textit{“An ETFE foil roof will have a better rating for insulation, }U\textit{ value at 1.9 W/m}^2\textit{K (horizontal) in comparison to single glazing at 6.3 W/m}^2\textit{K (horizontal) and double glazing 3.2 W/m}^2\textit{K (horizontal)”}. Thermal transmittance }U\textit{ values of ETFE cushions in the literature are resulting from calculations according to DIN 4108 standard and how }U\textit{ value measure has been carried out for 2, 3, 4, 5 layers cushions (}U\textit{-value }= 2.95 – 1.96 – 1.47 – 1.18 \text{ W/m}^2\textit{K})\textsuperscript{6,13}. The methods used to determine the thermal transmittance value can be the following: - in accordance with DIN 4108, - finite element calculation of the thermal transmittance of the film and the cushion - empirical analysis in the Hotbox climate chamber of the }U\textit{ value for ETFE pneumatic cushions \textsuperscript{14}. It has to be considered that the convective motion of the heat in the chambers, among the different layers influences the thermal performance of the cushions and the estimation of the }U\textit{ value is generally complex.

The great difference between a cover or a façade with ETFE cushions, compared to a glass one is in the ratio between the area covered by a cushion and the length of the aluminum frame: for a ETFE system, the ratio is about 0,8 compared to that of the glass which is about 2. This means that the surface area obtainable with an ETFE cushion corresponds to four or more windows in glass with a reduction of the frames and of the discontinuity frame-glass, with higher homogeneity of thermal transmittance for the same surface.

Besides the thermal insulation control of the building in the evaluation of the thermal performances of the transparency of the ETFE, the thermal effect generated by the passage of the radiation of the solar spectrum from the outside towards the inside, and in particular, the long radiation of the electromagnetic spectrum solar, in order not to underestimate or approximate evaluations. The issue is complex and completely open in the field of scientific research there aren’t any software available that consider in the calculation of the thermal performance of the interior of a building the long radiation transmission in the electromagnetic spectrum of the solar radiation (IR) through layers of thin film. A reference in literature, explaining many aspects, but not fully answering the complexity of the matter, is Poirazis et alii (2009)\textsuperscript{14}. ETFE is not opaque to the long radiation of the solar spectrum, therefore, in the design phase it is necessary to pay attention to this aspect; it is not possible to treat such materials and the system as a normal glass even if they are normally considered in the simulations of calculation equal to parallel glass sheets.

The thermal and optical properties of ETFE cushions can be changed significantly by the application of coating, prints and shapes. The application of coating films can determine the
following behavior: a. low-emissivity coating for the reduction of transmission losses of long waves, i.e. to provide during a cold winter's night a reduction of the thermal transmittance; b. a coating for solar control to reduce the transmission of the electromagnetic spectrum.

Compared with glass, virtually opaque to long radiation, ETFE transmits instead a part of this, which has a great influence on the transmission of the energy absorbed in the layers. The relevance of these effects can vary greatly depending on the environmental conditions and the characteristics of the cushion system built. There are many parameters that affect the transmission of long waves, including the temperature of the floor inside, the radiant temperature of the sky, therefore the difference of temperature, the exchange of long waves, the environment around the building (with or without buildings).

The issue is currently treated very empirically, by printing on ETFE film, as shading and shielding of the transmission of solar energy in the internal space, which improves the thermal insulation in winter and reduces the amount of waves that enter into the inner space in summer, reducing the risk of overheating the interior space. The value of solar factor $G$ (indicator of the level of transmission of UV and IR rays transmitted) of the ETFE cushions system can be reduced to 0.48 by means of a printed surface and to a minimum of 0.35 using a system of three printed layers. Furthermore it is highlighted in which way the geometry of the printing impinges on the optical properties and transmission of solar radiation, and also how in the printed parts, the intensity of the printing ink affects the transmission, which means that by increasing the intensity it is reduced the transmission of solar radiation.

4. WINTER GARDEN PROJECT, VERONA, ITALY

Verona Forum (2005-2011), a construction of the side overlooking the street in a compact manner, is a multipurpose complex at the avant-garde as regards solutions for safeguarding the environment. It is also a pioneering design in terms of the technological solutions which control building automation, managing the building so as to increase energy efficiency. Into the complex the Winter Garden structure is one of the first Italian case of an entire envelope, a complete building fulfilled with multilayered ETFE cushions. The translucent building is a charming special structure, which serves the nearly Crowne Plaza Hotel for meetings or just as relaxing area. The Winter Garden has an ellipsoidal form and the 3D curved envelope surface. At one end of the egg-shape, a cubical installation building for technical plants is foreseen, intersecting the ellipsoidal volume. At the opposite end of the egg shape is an entrance area, which should receive a vertical facade.

![Figure 2: Plants and render of the shape of the Winter Garden in Verona (Source: MBA)](image-url)
3.1. Executive Design Phase and Engineering Aspects

This case study allows to understand the continuous feedbacks between different disciplines, different building process actors and the steps of the design phase, the executive ones and the engineering process in order to define and build up the ellipsoidal shape, made of ETFE cushions, and to characterize the surface and the strategy in order to control the solar radiation, in terms of environmental building design.

3.1.1 Structural Design and Analysis

The primary structure of Verona Winter Garden consists of 7 transversal arches, which are hinge connected to concrete foundations. All arches have different shapes to generate the volume of a partial egg. The 7 arches are connected with 11 longitudinal profiles. These profiles are curved, bending stiff, run above the arches and connect all arches to the technical installation building. The steel profiles act also as bearing profiles for ET-foil cushions. The pressure stabilizes the cushions and forms the transparent envelope of the building. This structural system allowed avoiding diagonal bracing, which would interfere the view from inside. The distance between arches, e = 4.50 m., max. arches span is l = 27.1m and the covered area is approximately 625 m².

![3D-Model of Verona Winter Garden (Source: Form-TL)](source: Form-TL)

The timber wood 7 arches have a rectangular section 240x800mm product according to UNI EN 14080 and DIN 1052, belong to the class of H GL28 resistance according to EN 1194, whose function is to transmit the self-weight and the external loads of the covering to the reinforced concrete foundation. The 11 horizontal or inclined longitudinal profiles structures consist of tubular circular (diameter 168.3 mm) calendared in steel type S 355 J2. These elements have the function of supporting the loads acting, transmitting them to other structural parts connected to them. The profiles have a longitudinal plate welded perpendicularly to the covering membrane (ETFE), where the cushion is connected by the use of aluminum profiles. The distance among the profiles varies and follows the "shell" shape characteristic of the building.

The steel arches with rectangular section, placed at both ends of the structure, are made up of S 355 J2 steel type elements with rectangular section 200x800mm (semi-arch towards the
technical building) and a rectangular section of 200x400mm (side of the vertical glass wall with the entrance). The steel arches have the function to transmit the self-weight and the external loads of the covering to the reinforced concrete foundations.

Regarding the closure structure, the Cushions are foreseen as 4-layer cushions with 250µm, 100µm, 100µm and 250µm foil thickness with minimum and maximum volumes of \( V_{\text{min}} = 18.7 \) and \( V_{\text{max}} = 33.7 \text{m}^3 \), respectively. To stabilize the foil cushions against external loads, the cushions receive overpressure against the environmental atmosphere. This overpressure needs to be continuously provided and needs to be modified according to the external load exposure. To reduce the thickness of the cushions, a steel cable is applied above the wood arches to reduce the sag of the foil. The connection detail of the foil along the cushion edge is very unique, which is invisible from outside. From outside, the connection detail is only 10mm thick. Therefore, no plates are visible. The 3m x 4.7m windows are very special being covered with an ETFE cushion, without bracing structure inside the window and a 3d-shaped frame to suit the surface condition of the structure. Minimum frame extension of height above the regular roof surface was possible. The permanent overpressures outside and inside chambers are 300 and 30 Pa, respectively. The winter time overpressure is 600 Pa and the snow overpressure is 800 Pa. Foil Surface area is approx. 3600 m², the surface of 1 ET-foil Layer is approx. 900 m² and the cushion volume is approx. 300 m³. The snow load is \( q_{\text{sk Ce}} = 1.00 \text{kN/m}^2 \) and the wind base pressure is \( p = q_b \text{ Ce Cp Cd} = 0.70 \text{kN/m} \).

![Figure 4: Clamping Profile Detail (Left) and Window Section (Right) (Form-TL)](image)

3.1.2 Day lighting Control Strategies

A study on different patterns and layers in order to reach the best results modulating the shadowing and transparency was undertaken by Canobbio in collaboration with Politecnico di Milano in order to satisfy the requirements and diffuse different lighting rates ensuring the summer thermal comfort, by varying the opacity of the cushions over the whole building. The supplier of the pneumatic envelope, Canobbio company, has undertaken the executive design of the structure finding the right solution for the requirements. The design requirements related to the ETFE pneumatic cushions structure designed for the Winter Garden in Verona were: 

\[ a \] by the designers of MBA, to have the first 3m of vertical envelope almost totally opaque to the outside and reach the almost total transparency in the center cushions (at about 6 m from the ground), generating a gradient effect from opaque to transparent starting from the first lower cushion to the top central cushion; 

\[ b \] by plant engineers verify that the pneumatic system meets some standard requirements related to the U-value of thermal transmittance and G solar factor value which means a \( U \) value = 1.5 W/m²K, and G diffuse
value = 0.3. To have an average diffuse value of 0.3, the G value must be from 0.38 to 0.22. We assume a cushion system with 4 layers and printing on all cushions. To meet also the formal requirements of the designer, printing varies in intensity from the opaque part to the transparent and moreover, it is applied in a different way from cushion to cushion.

Printing have been foreseen on the more resistant layer, therefore in the outer one, for all six cushions, and in the internal one for the first two with double printing; due to technical reasons of the printing company the printing was possible only on a layer thickness of 250 µm. The geometry and the percentage of opaque/transparent were also optimized, by choosing geometries in the catalog of the printing company.

The compositional aspect of the project has great potential given by the technical solutions and the materials with a high degree of innovation, however, some limitations emerge in the pneumatic roofing system in the way it has been designed: a. the hemispherical synclastic shape it is difficult to verify from a physical-technical and an environmental well-being point of view; b. from a formal point of view, this hypothesis answered well to the blending requirements given by the project, but, from the inner environmental wellbeing point of view, this solution did not appear to be the best. Typically for horizontal closed structures with transparent materials it is advisable to apply shading systems and shielding, to reduce the amount of heat coming through; in the specific case it would be therefore desirable to have the greatest shading in the highest part, zenithal to solar radiation. Considering also that the hot air generates an upward movement, it is assumed that extreme conditions in summer a slight overheating of the high part may occur.

Given the considerations and the limits to the designers, a new hypothesis for the transparency of the structure, by inverting the more opaque part with the more transparent one, creating the most dense and opaque prints in the upper cushions, shading towards the lower part, with the fewest at the base. Due to the designers’ request, this hypothesis had to be
checked by the creation of small cushion models of cushion on 50x50cm frames (fig. 6). Thanks to the evaluation of the mock up, which have significantly given the idea of the final result according to the design requirements, designers have defined a new sequence printing, which better fits the physical-technical requirements for an internal well-being. The geometry which considered a greater transparency with respect to the others (30% opacity) was excluded and have been differently distributed and overlapped to the other geometries.

Figure 7: Final sequence of the intensity effect of the screen printing foils and their densities: 70% dark, 50% dark, 50% light and no screen printing (Source: Monticelli, FormTL).

3.1.3 Patterning Aspects

During the engineering phase before the production, cutting patterns were generated with WINNETZ. The distribution of seams was arranged to be in general parallel to the Wood girders.

Figure 8: Layout of screen printing density (Source: Canobbio)

All seams in the 4 layers end at the same position to provide a most harmonic view from
inside and outside. Based on the targeted visual impact given by the architect (fig. 7,8), the printing schemes of ETFE external and internal layers that determined the pigments density had to be specified for each panel after the generation of cutting patterns for supplying the manufacturing data (fig.8).

3.1.4 Management phase of the ETFE pneumatic envelope

The pressurization system for ETFE cushions inflating, placed in technical building joint to the structure in object, consists of two fans with dehumidification function, to pre-dry the outside air and prevent the formation within the cushion of condensation or algae. Among the various cushions there are communication systems to allow the air to move from one to the other and ensure the same static pressure to the whole. The machine produces two different pressures, one for the inside of the cushion (volume A 330 Pa) and one for the other two more external rooms (volume B 300 Pa). From the machine there are two pipe diameters 160 mm coming out to distribute the air to the cushions. The latter have two valves from one side to allow the inflation of the two internal chambers of the cushion and a pressure relief valve to exhaust the chamber on the opposite side. The machine is connected with a weather station that will increase or decrease the internal pressure of the cushions ensuring the stability of the same in the case of wind or snow, increasing the pressure in the cushions: inner chamber of the cushion (pressure type A): from 330 Pa to 630 Pa; side chambers of the cushions (pressure type B): from 300 Pa to 600 Pa.

Figure 11: Volumes for different pressures into the air chambers of the etfe cushions (Source: Canobbio)

Figure 12: Pictures of the realization, from outside and inside (Source: Canobbio)

5. CONCLUSION

Through the design process, the engineering and the production of the building in object, it was possible to understand the experimental and empirical level related to the choice of the system dealing with energy issues, and choosing a technical solution that could meet the
current architectural requirements for high performance coverings in terms of thermal and indoor comfort, answering to the need to reduce energy consumption of buildings and to satisfy the well-being in a closed environment. Definitely a significant and innovative approach from the design point of view, in comparison with the results achieved in the Winter Garden of Verona, could be the simulation of the solar path related to the covering and be of a great help to the strategic definition of the intensity and density of the printing on the ETFE cushions films in the areas mainly stressed by solar radiation. Also it would be necessary to have film or components matched to the ETFE which are able to transmit or reflect selectively the different wavelengths of the solar spectrum, in order to adapt to any application the overall performance of the covering. There are UV and IR filters whose range of treatments can affect the emissivity of the surface or the capability of the structure to radiate energy.

Assuming that the modeling of ETFE cushions, particularly in the case of curved shapes, added to the already curved shape of the cushions themselves, as part of the simulation of the energy performance of the building has not yet been explored, the performance of the system could be evaluated through computational fluid dynamics (CFD) testing or empirically the climatic chamber test (Hot Box), case by case. Besides understanding the optical properties of the transparent ETFE film and printed ones, by using a spectrophotometer², it would be desirable to investigate on one hand the optical properties of the films overlapped in parallel, both transparent and printed, maybe with a solar simulator and on the other hand the optical properties of the cushions with two / three films, transparent and / or screen printed, by using a special spectrophotometer, which enables to position the film with an angle not perpendicular to the incident light flux, or even better with a solar simulator and define the optical parameters (transmission, reflection and emissivity) which allow the simulation of the real behavior of cushions in place. As a matter of fact, for example, the plane mock ups, made for the above mentioned project, have empirically been of great help, even though, the real effect of the printing and of the overlapping of the geometric patterns is again different and currently difficult to simulate.

Figure 13: Installation phase of the pneumatic envelope (Source: Canobbio)

REFERENCES


INNOVATIVE MEMBRANE ARCHITECTURE FOR CARPORT IN MUNICH

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Key words: ETFE film cushions, three-layered, integrated flexible photovoltaic cells
We would like to introduce you some of our most important projects for innovative Membrane Architecture here in Munich – the AWM carport roof.

It’s not always easy being green that’s what we discovered when attempting to build the carport roof. The project, which involved an ETFE film roof with photovoltaic cells, came under close and critical scrutiny of city officials, engineering specialists and the general public. “For us it was a long and hard way, because every step was controlled.”

It was a hard fight to get building approval and then to construct the roof.” The city and public had good reason for its skepticism. The new roof was meant to replace one that had partially collapsed in 2006 after a heavy snowfall - an event that gave the city and architect a lot of bad publicity. The same architect, Munich-based Ackermann and Partner, was hired for the re-construction.

In the summer months in 2011, Taiyo Europe GmbH erected the new roof structure for the carport used for under-cover parking of the trucks belonging to Munich's waste disposal companies. This structure comprises a steel construction with a roof cover made of three-layered ETFE film cushions with integrated flexible photovoltaic cells. This innovative project, located near to the famous Olympic Park here in Munich, is also used for all-year electricity generation at the main headquarters of the waste disposal companies in Munich (AWM).

The new carport roof was planned by the architects, in close cooperation with the city's Building Department and AWM. AWM short-listed two versions from the series of possible solutions presented. The roof variant with integrated photovoltaic system favoured by AWM was then approved by the Municipal Committee and thus resolved.

For AWM, the reconstruction work presented the opportunity of designing the large roof area as an innovative photovoltaic structure. The new roof concept thus makes a significant contribution to sustainability, particularly to climate and resource protection, which AWM has declared as one of its major maxims alongside efficiency.

We are very glad and proud, that we completed this new technology project in October 2011. Since then we follow up this project and since today we are very sufficient with the result.
DESCRIPTION OF THE CONCEPT

One basic pre-condition for the new roofing was the use of existing points of support for the hinged columns made of tubular steel with integrated roof drainage. The column grid is 10.00 x 12.00 m in size. The primary load-bearing structure comprises multi-bay frames comprising columns and 3-corded tie bars which are fanned out at the edges using tensioned braces. The trussing meant that the curved tubes could be kept slim. The total steel weight is 480 tonnes, or 48 kg/m² covered area. Unlike the earlier design, the primary load-bearing structure has been designed to be stable irrespective of the roof covering. The steel structure is coated with the classic Deutsche Bundesbahn colour DB 703 high-gloss paint.
The roof area is made from 220 air-supported ETFE film cushions. The 220 air cushions covering the roof elements are made of ETFE film. This material is very translucent and resistant to the influences of the weather. Each cushion is made of three layers of ETFE film. As is customary in membrane constructions, the layers are termed upper layer UL, middle layer ML and inner layer IL.

The lower film layer is printed to reduce the light transmitted through the film cushions onto the carport deck. There are 12 photovoltaic modules fixed to the middle layer of each cushion by means of mechanical connectors, some of which can be moved, so that the modules are not subjected to any bending, tensile or shearing forces of note even in the event of heavy snow loads. As with bridge supports, for example, one of the PV module attachments is always without a longitudinal hole, in other words it is in a permanently fixed position, preventing the PV module from "floating freely".

The middle layer is mechanically pre-stressed to prevent creasing and is without load in the operating state, since the large ventilation openings in it lead to the same inner pressure above and below the middle layer.

To allow any faulty modules to be able to be replaced easily even in the long term, the upper film layer was fixed separately from the other two heat-sealed film layers in the double-welt clamping profile. This layer can be opened separately and basically works like a service cover.

The load cases pre-tension of the ETFE films, intrinsic weight, snow, wind and change in temperature were considered in the calculation of the roof structure. Whereas the static calculation of the primary load-bearing structure was carried out using a standard calculation program for space bar frames, the calculation software using the force-density method, which
has been especially developed for architecture membranes, was used for the final design, static calculation and cutting layout specification of the ETFE film.

Pneumatic pre-stressed systems become tensioned, pre-stressed structures due to the air overpressure on the inside. The pre-stress in the upper and lower membrane is the result of the difference in pressure between the inside of the cushions and atmospheric pressure. It also depends on the bend radius of the corresponding membrane layer. The overpressure in the cushion is maintained using blowers. The membrane layers are always tensioned under load and are thus kept stable. Cushions are usually designed with two or three layers, depending on the structural-physical requirements.

The design of the air supply to the film cushions through blower units primarily depends on the magnitude of the defined inner pressure, the number of cushions and the size of the total area. An air dryer is included upstream of the blower. The supply air is dried to prevent condensation forming in the film cushions.

The air supply is connected to the lower cushion chamber. Air is exchanged between the upper and lower cushion chambers via overflow openings in the middle film layer (two rows of 12 circular holes with a diameter of 90 mm at the edge of the middle layer of the cushion and one row of 12 holes at the peak of the middle layer). The air escapes via the air outlet.

The flushing rate of the cushion volume (air exchange of the cushion volume was estimated at 3,000 m³ at planning) was specified by the building engineer to 4,500 m³/24 h, in other words 1.5 times per day. The prescribed flushing rate is set using the cross-section area of the air outlet. In the nominal case, the support air pressure inside the cushion is 300 Pascal compared with the atmosphere. In the event of snowfall, this pressure can be increased to 600 Pascal. If the snow load exceeds 0.6 KN/m², the cushion is compressed in a controlled way. In this case, the upper and lower film layers bear the load together.

![Fig. 4 - Cushion design](image-url)
The geometric form of the film cushions chosen means that no water pockets will occur even if the blower fails during a period of rainfall. Pockets of snow can form towards the lower edge in the event of drifting snow. However, the local additional loads do not endanger the structural safety and do not lead to any significant deformation.

The stability of the cushion roof structure is not dependent on support air supply. Nevertheless, it was decided to make this supply particularly reliable. Three blower units supply one third of the roof area each. Each station has two redundantly wired blower motors which alternate on a weekly basis and automatically replace each other if one of the blowers should fail. The air supply is connected to an emergency power supply and a remote warning system.

Each cushion assembly with corresponding blower is integrated in a separate ring air pipe. All three ring air systems are separated from each another by valves which enable the area affected by blower station failure to be supplied by a neighbouring station. In addition, an air pipe system with high density class and non-return valves was chosen for the blower stations, making the whole system extremely airtight.

**PRINCIPLE SECTION**

**AIRMANAGEMENT**

![Diagram showing the air supply in the cushions](image)

Fig. 5 - Diagram showing the air supply in the cushions
Table 1: Technical data:

<table>
<thead>
<tr>
<th>Dimensions:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>120.00 m</td>
</tr>
<tr>
<td>Width</td>
<td>70.00 m</td>
</tr>
<tr>
<td>Column grid</td>
<td>10.00 x 12.00 m</td>
</tr>
<tr>
<td>Eaves height</td>
<td>+8.45 m</td>
</tr>
<tr>
<td>Ridge height</td>
<td>+10.03 m</td>
</tr>
<tr>
<td>Covered area</td>
<td>8,400 m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Steel structure</th>
<th>S355J2H/ E355+AR/St.52.0 S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total coating 280 µ: (base pre-treatment, primer coat, intermediate coat in the factory and finishing coat on site) with Icosit EG Phosphat Rapid and Icosit Eg 5 and 2K-PUR, DB 703 high-gloss paint</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ETFE film cushions</th>
<th>10 x 22 cushions, 220 in total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions 3.33 x 10.40 m</td>
<td></td>
</tr>
<tr>
<td>Thickness of upper and lower layer:</td>
<td>250 µ</td>
</tr>
<tr>
<td>Thickness of middle layer:</td>
<td>100 µ</td>
</tr>
<tr>
<td>Edge clamping profiles made of anodised aluminium</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solar system</th>
<th>12 photovoltaic modules per cushion</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,640 modules in total</td>
<td></td>
</tr>
<tr>
<td>Power: 145.73 kWp</td>
<td></td>
</tr>
<tr>
<td>Specific power yield: 889 kWh/kWp</td>
<td></td>
</tr>
<tr>
<td>Average power yield: 129 kWh/a</td>
<td></td>
</tr>
</tbody>
</table>

Assembly procedure

The static system chosen by the structural planner required assembly to be carried out in several stages. Following erection of the statically stable primary load-bearing structure, the film cushions could be fitted. The film cushions were clamped in all-round aluminium profiles which were then screwed to the sub-structure.

The photovoltaic thin-layer modules fastened mechanically to the middle film layer 100 µ thick were fixed in place on site on a special pre-assembly table before installation of the cushions. So-called welts were heat-sealed at the edge for the linear edge attachment of the film layers. These welts were clamped into the attachment profiles.
Properties of ETFE films

The cushion membranes are made of ETFE film (Ethylene TetraFluroEthylene) – a fluorine-based plastic with outstanding physical properties. The load-bearing films were dimensioned and selected with a thickness of 250 µ due to the loads to be expected. The main reasons the client and architect decided in favour of the version with ETFE cushion roof were as follows:

- High transparency from UV to IR range and a translucency of 90%.
- Flame-resistant and without burning droplets, construction material class B1, additive-free
- Very good separating properties or self-cleaning on the basis of the non-stick surface
- Long service life of at least 30 years

In addition to the standard acceptance test certificates for the material designated to be used, a comprehensive range of tests were carried out on typical details to meet the requirements set in this individual case by the Supreme Building Authorities. There is no generally valid building approval available for Germany for the membrane material ETFE film. For this reason, a new application must be made for every project. In this individual case, approval was granted 4 months after the complete technical documentation had been submitted.

The physical properties of ETFE films are comparatively complex compared to other materials such as steel or concrete. The load-bearing behaviour is non-linear and non-elastic, and the material is anisotropic. The stress-deformation behaviour of the films differs in machine and cross direction, and can vary from one production batch to the next. This means the usual procedure is to determine the specific material properties by means of tests before specifying the cutting layout. In the biaxial test, the load-bearing and deformation behaviour and the compensation values for the pre-stress are determined on the basis of the load to be expected. In the case of the carport project, a mean compensation value of 5% was determined for the cross direction. The machine direction was not compensated.
Table 2: Properties of ETFE films

<table>
<thead>
<tr>
<th>Properties</th>
<th>ETFE NOWOFLON ET 6235, 250 µ, clear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Printing on the lower layer</td>
<td>Negative dot, silver standard, décor no. 920201, Reisewitz printing</td>
</tr>
<tr>
<td>Resistance to high temperatures</td>
<td>Permanently from max. +150°C to a temperature range of up to 230°C for short periods (6-8 h)</td>
</tr>
<tr>
<td>Melting point</td>
<td>approx. 270°C</td>
</tr>
<tr>
<td>Tear-growth resistance N/mm</td>
<td>470</td>
</tr>
<tr>
<td>Mass per unit area (g/m²)</td>
<td>350</td>
</tr>
<tr>
<td>Fire behaviour DIN 4102</td>
<td>B1, flame-resistant</td>
</tr>
<tr>
<td>Transparency</td>
<td>90%</td>
</tr>
<tr>
<td>Anti-soiling properties</td>
<td>excellent</td>
</tr>
<tr>
<td>Fungal development</td>
<td>Not possible</td>
</tr>
<tr>
<td>Service life</td>
<td>30 years</td>
</tr>
</tbody>
</table>

The ETFE films are manufactured using the slit die extrusion method. The rolls of film used are 1.55 m wide and 250 µ thick in the case of the upper and inner layer, and 100 µ in the case of the middle layer. The density of ETFE is approx. 1.75 g/cm³. Due to the limited film width, individual lengths of film are cut and then heat-sealed together to form a film cushion layer (part-surface connection). At the edge of the cushion the middle layer and the inner cushion layer were heat-sealed to form so-called edge welt pockets using the same method. The upper film layer was also given an edge welt pocket. All three layers or the two edge welts were then fixed in the double-welt clamping profile.

![Fig. 7 - Diagram of the clamping profile](image.png)
ETFE film sealing seams are homogeneous i.e. the sealing seam is made of the same thermoplastic (repeatedly meltable) material ETFE as the part-surfaces it connects. The load transmission thus takes place from film part-surface to film part-surface without an intermediate layer as sealing aid.

Quality assurance for the construction products and their production is carried out from film production through to installation through the following measures:

- Acceptance test certificates from the film manufacturer according to DIN 10204-3.1
- Incoming goods checks at the manufacturing company
- Internal quality control accompanying production by the manufacturing company
- External quality control of production by the membrane expert
- Compliance certificate from the inspection, monitoring and certification office

**Description of pneumatically supported film cushions**

Film cushions become load-transferring, pre-stressed membrane structures due to overpressure on the inside. During normal operation, the films are tensioned under load and stable even under outer loads such as wind and snow.

Depending on the structural-physical requirements, air-supported film cushions are designed as two- or three-layer cushions. ETFE fluoropolymer film (ethylene tetrafluorethylene) is mainly used as the cushion material. In this project, a solution with three-layer film cushions was chosen for the roofing, which is open at the side, whereby in this case the purpose of the middle layer was not to improve heat insulation but rather to serve as a carrier for the flexible photovoltaic modules integrated in the cushion. The middle layer can contribute to load transfer in the case of a snow load, but it is not necessary here since the upper film layer and the inner film layer are sufficiently dimensioned.

The film cushions (secondary system) are clamped between the steel arch supports (primary system). The primary system is stable without the secondary system. The secondary system is not required for stabilisation, deformation limitation or load transfer.

The film cushions are connected to the primary system all the way round by means of aluminium clamping profiles. The film cushions are interlocked into the clamping profiles and connected by pressure transfer through contact pressure between the welt and welt profile as well as between the welt profile and the basic profile. The interlocking connection between basic profile and steel structure is made using a screw connection.

The air supply to the film cushions was designed and dimensioned by the applicant in cooperation with the blower manufacturer.

The advantages of ETFE air cushions as space-enclosing components lie in their transparency and low weight, which also affects the efficient and aesthetic quality of the primary structure, for example.

The successfully result of this unique project finally dependent on the cooperation of all who are involved in that project.
Therefore we would like to say thank you to our subcontractors for the excellent team work and the power of endurance during the hard times. Without them it would have been never such a good result since now.

**Table 3:** Those who were involved in the carport roofing project in Munich:

<table>
<thead>
<tr>
<th>Client</th>
<th>AWM, Munich waste disposal companies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Architecture</td>
<td>Ackermann and Partner; Munich</td>
</tr>
<tr>
<td>Structural planner</td>
<td>Dipl.-Ing. Christoph Ackermann; Munich</td>
</tr>
<tr>
<td>Contractor for the complete structure</td>
<td>Taiyo Europe GmbH, Sauerlach, in cooperation with Konstruct AG, Rosenheim and the assembly service LB, Hallbergmoos</td>
</tr>
<tr>
<td>Steel structure production and assembly</td>
<td>Steel Concept, Chemnitz</td>
</tr>
</tbody>
</table>

This project is a milestone in the development for further applications of photovoltaic in combination with architectural membranes for roof and façade structures. Taiyo Europe GmbH and the Taiyo Kogyo Group will continue further developments to combine this technology with other membrane materials offering owners the possibility of combining the superior esthetic properties of architectural membrane structures and unique energy earnings.
NOVEL BIPV/T ETFE CUSHION

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Key words: BIPV/T, electricity, ETFE cushion, pressure, temperature, solar irradiance

Summary. This paper presents a novel BIPV/T ETFE cushion with the potential for solar energy utilization. An experimental mockup mainly composed of a three-layer ETFE cushion and two amorphous silicon photovoltaic panels (PVs) was designed and constructed. Then, series experiments were carried out in the past two years. In this paper, experimental results of two typical tests under the cold winter and hot summer conditions were selected to evaluate the feasibility of this proposed cushion in terms of net electricity and temperature. The net electricity was more than 37.3 Wh, suggesting that this cushion could be a power independent system. Meanwhile, the temperature difference between inside and outside of the cushion was more than 19.2 °C, showing an attractive potential in collecting thermal energy even in winter. Additionally, the temperature distribution and pressure of this cushion were also obtained.

This study has revealed the technical feasibility of the cushion integrated photovoltaic/thermal system. It provides a novel BIPV/T ETFE cushion.

1 INTRODUCTION

The ETFE cushion structures have received increasing attention in the past decades, which meets the demands of novel buildings with good performance on shapes and load-bearing due to the use of new materials and structural forms. In order to study and apply this new structure, it is necessary to investigate the important aspects such as the form-shaping, structural analysis and cutting patterns in theoretical and experimental fields. In the field of theoretical study, Chen¹ and Borgart² developed the theories and numerical algorithms based on the force density and dynamic relaxation methods. For the experiments, Robinson³, Tang⁴ and Zhao⁵ conducted a series of experiments under wind and snow conditions. All the experimental results show acceptable agreement with the theory and numerical algorithms.

When applying the cushion structure, it needs extra electricity to form and maintain the shape due to the inevitable leakage of the air inside the cushion⁶. This is the main disadvantage to limit the application of the inflatable cushion structure. In general, two ways could be adopted to reduce or solve this problem. A passive way is the improvement of the construction technology. In fact, this one is developing slowly by the lack of the proper machines. On the other hand, however, a positive way of utilizing solar energy is promising with the concept that buildings integrated photovoltaic (BIPV)⁷,⁸. As good performance of integrations to a variety of buildings such as single residence⁹, high rise buildings¹⁰, office buildings¹¹,¹² and airport¹³,
the integration of the PV into cushion structures is meaningful. For this reason, this paper presents a new cushion, composed of amorphous silicon photovoltaic panels and a three-layer ETFE cushion, named the PV-ETFE cushion.

In this cushion, as the PV generates the electricity and thermal energy simultaneously, the electricity is provided for the operation of the cushion and the thermal energy leads to the increase of the temperature, thus both of the electricity and temperature need to be evaluated. This could be firstly achieved much more accurately by advanced experiments than by numerical simulations due to the complex factors affecting the performance of the proposed cushion and difficulty of the related multi-field coupling theory.

Generally speaking, the fundamental factors are the solar irradiance, outdoor temperature and cushions forms\(^{14}\). Among them, the solar irradiance has the highest impact and fluctuates dramatically from winter to summer, which means that the cushion should experience the cold winter and hot summer before concluding its overall performance. For this reason, two-year experimental studies on the proposed cushion have been done, especially under the sunny condition. Then, the electricity for the operation of the cushion system and the temperature distribution of the cushion are evaluated to investigate the feasibility of the PV-ETFE cushion.

2 EXPERIMENTAL SETUP

The experimental mockup is designed to investigate the performance of the PV-ETFE cushion structure system. The photos of the proposed system are shown in Figure 1. The system consists of (a) an ETFE cushion, (b) a solar energy control system (SECS), (c) a pressure control system (PCS), (d) a steel structure system. In detail, the ETFE cushion is a three-layer one with five 120 mm holes equally distributed along the center line of the middle layer to ensure the same pressure of the two chambers formed by the three layers. The size of the cushion is 2 m wide and 4 m long as well as 0.125 m rise in the middle. The SECS consists of a solar controller, two 3 m×0.394 m amorphous silicon photovoltaic panels (PV) and two storage batteries, both the PV and batteries are connected in sequence to provide a 24V DC which is needed for the operation of the system. The PCS consists of a pressure sensor, a programmable logic controller (PLC), a blower and two solenoid valves. The steel structure system consists of a circular hollow section (CHS) simple welded steel structure with the dimensions of 4 m×2 m×2 m in length, width and height, as well as aluminum profile clamps and EPDM robber to reduce the leakage of the cushion.

2.1 Working principle of the PV-ETFE cushion system

The SECS provides the electricity for the PCS to inflate the cushion supported by the steel structure to form and maintain the designed shape. Within this working principle, two working flows are designed and described below.

The first one is the working flow of the SECS, where the solar controller plays the key role. When the current in the system is more than 20 A, the solar controller cuts off the electricity circuit to protect the equipment. When the current in the system is less than 20 A, if the electricity generated by PV from the solar irradiance exceeds the demand of the system, the
extra electricity is stored by the storage batteries. If the electricity generated by the PV is insufficient for the system, the solar controller controls the discharge of the storage batteries to compensate the lack of the electricity. Thus, the electricity could be utilized in an appropriate way.

The second one is the working flow of the PCS, where the PLC is the major part. As recommended by Chen\(^1\), the optimal pressure is 300 Pa so that the stress of the ETFE foil could be assumed to be in a linear range, making the design simple. Moreover, to facilitate the operation, 20% of the pressure needs to be added, this means the pressure range of the proposed system is 240-360 Pa.

In this study, when the system operates, if the pressure sensor detects the pressure lower than 240 Pa, the blower blows and the valve near it opens simultaneously to inflate the cushion until the pressure reaches 300 Pa. If the pressure sensor detects the pressure higher than 360 Pa, the PLC controls the other valve open to deflate the cushion until 300 Pa. Therefore, the cushion could maintain the designed shape to resist the external loads.

The specifications of all equipment of the PV-ETFE cushion are listed in Table 1.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Type</th>
<th>Voltage/V</th>
<th>Current/A</th>
<th>Power/W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photovoltaic panels</td>
<td>PVL68</td>
<td>16.5</td>
<td>4.1</td>
<td>68 Wh</td>
</tr>
<tr>
<td>Solar controller</td>
<td>EPIPC-COM</td>
<td>24</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Solar storage batteries</td>
<td>6-CN-100</td>
<td>12</td>
<td></td>
<td>100 Ah</td>
</tr>
<tr>
<td>Pressure sensor</td>
<td>KQ-SPB2088</td>
<td>24</td>
<td>20 mA</td>
<td></td>
</tr>
<tr>
<td>Programmable logic controller</td>
<td>PM564-T</td>
<td>24</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Blower</td>
<td>G1G160-BH29-52</td>
<td>24</td>
<td>5.8</td>
<td>105</td>
</tr>
<tr>
<td>Solenoid valve</td>
<td>ZQDF-1</td>
<td>24</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: the specification of the equipment in the PV-ETFE cushion
3 MEASUREMENT SETUP

As mentioned previously, this system is designed to investigate the performance of the PV-ETFE cushion in terms of the electricity, temperature, pressure and solar irradiance. Therefore, these parameters need to be collected automatically or manually.

3.1 The electricity

Data for electricity, including the voltage, current, photovoltaic electricity and consumption electricity are displayed in the solar controller with the time lag of 0.1 s and recorded manually each 10 minutes.

3.2 The temperature

Temperature of the system is displayed and collected by the temperature acquisition system. The temperature acquisition system consists of 31 thermo-resistances, a temperature patrol instrument and software M400. The platinum thermo-resistance PT100 was pre-calibrated and verified before tests in an isothermal bath using a standard precision thermometer. The distribution of the thermo-resistances in the cushion are illustrated in Figure 2, including the top and bottom layer, the surface and back face of the PV and middle layer, the air inside and outside of the cushion.

The working program of the temperature acquisition system is interpreted as follows. The temperatures measured by thermo-resistances are transmitted to the temperature patrol instrument. Then, the data are logged by the software M400 through two interconnected data loggers as a local network RS485, which is connected to a personal computer through RS232 series port.

3.3 The pressure

As mentioned in the working flow in the pressure, the pressure detected by the pressure sensor is displayed and logged by the software Fameview.

3.4 The solar irradiance
In this study, the TBQ-2 solar irradiance acquisition system is utilized and installed 20 m away from the experimental mockup. The solar irradiance is displayed and logged by the program developed by the authors based on the software LabVIEW. The specifications of all measuring devices used during the experiments are listed in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Instruments</th>
<th>Type</th>
<th>Derivation</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>Thermo-resistance</td>
<td>WZPT-035</td>
<td>0.1 °C</td>
<td>the range is -100-350</td>
</tr>
<tr>
<td>Solar irradiance</td>
<td>Pyranometer</td>
<td>TBQ-2</td>
<td>1 W/m²</td>
<td>the range is 0-2000 and the transmission is 30s</td>
</tr>
<tr>
<td>Pressure</td>
<td>Pressure sensor</td>
<td>KQ-SPB2088</td>
<td>5 Pa</td>
<td>the acquiring time period is 2 min</td>
</tr>
<tr>
<td>Voltage</td>
<td>Solar controller</td>
<td>EPIPC-COM</td>
<td>0.1 V</td>
<td>the transmission time is 10s</td>
</tr>
<tr>
<td>Current</td>
<td>Solar controller</td>
<td>EPIPC-COM</td>
<td>0.1 A</td>
<td></td>
</tr>
<tr>
<td>Electricity</td>
<td>Solar controller</td>
<td>EPIPC-COM</td>
<td>0.1 Wh</td>
<td></td>
</tr>
</tbody>
</table>

4 RESULTS AND DISCUSSION

The experimental observations, results obtained and the interpretations of these results are presented and discussed in this section to investigate the feasibility of the PV-ETFE cushion.

4.1 The electricity

As mentioned in the introduction, the electricity is used for the operation of the system. The electricity includes the voltage, current, photovoltaic electricity and consumption electricity. These parameters are needed to be analyzed separately to understand the performance of the PV-ETFE cushion in detail.

4.1.1 The voltage

Figure 3 shows the voltage of PV under winter and summer conditions. The voltage curve in winter showed much better consistence with the solar irradiance and fluctuated less than that in summer due to the fact that the shading effect of the upper steel member on PV is much stronger in summer. For the value, the voltage in winter was within the range of 26.8-29.9 V, which was higher than that in summer (26.8-27.6 V). This is in line with the conclusion that the voltage of PV decreases with the increase of the temperature. Since this parameter is one of the two important factors to generate the photovoltaic electricity, the understanding of it is essential for evaluating the performance of this cushion.

4.1.2 The current
Figure 4 illustrates the variation of the current of the PV under winter and summer conditions. The curves of the current were not in accordance with that of the solar irradiance. This could be explained as follows. When the storage batteries reach 70% of the charge capacity, the batteries change the charging mode to ensure not to be overcharged. For this reason, after 70% charge capacity of the storage batteries, the electricity could not be fully charged into the storage batteries or consumed by the system. Therefore, the fluctuation was obvious. For the value, the value of the current in summer was about twice greater than that in winter. This could be explained as that the larger proportion of the beam irradiance in summer was obtained compared with that in winter, which enhanced the photovoltaic effect of PV.

![Figure 3: the voltage in winter and summer](image1)

![Figure 4: the current in winter and summer](image2)

4.1.3 The photovoltaic and consumption electricity

After the investigation of the voltage and current, the photovoltaic and consumption electricity are needed to be evaluated for understanding the feasibility of the PV-ETFE cushion, shown in
Figure 5. Both in winter and summer, it was observed that the photovoltaic electricity was much more than consumption electricity, which means the electricity was enough for the cushion, and that the differences, 37.3 Wh in winter and 87 Wh in summer, showed the potential to supply electricity for other equipment such as air conditioning, lighting, appliance. For the photovoltaic electricity, the value in summer was about twice greater than that in winter, which was caused by the greater solar irradiance in summer compared with that in winter. This observation verifies the conclusion that the solar irradiance is the most important factor influencing BIPV/T systems\textsuperscript{16}. For the consumption electricity it seemed positive linear as it was accumulated by the inflating time period due to period variation of the pressure.

4.2 The temperature

Unlike other cushions, since the PV could generates thermal energy when generates electricity, and this thermal energy leads to the increase of the temperature of the cushion. Therefore, it is necessary to obtain the temperature distribution of the cushion. As described in the measurement setup, there were 31 thermo-resistances arranged in the cushion with the aim to collect the temperature data of the PV, membrane and air.

4.2.1 The temperature of the PV

As indicated by Lalovic \textsuperscript{17}, the temperature could reach more than 65 °C in summer when the PV was fixed on the surface of buildings. For this cushion, the PV is installed in the cushion which means higher temperature could be obtained. This temperature is essential for calculating the efficiency, evaluating performance of the PV and designing the cushion.

Figure 6 shows the average temperatures of the surface and back face of the PV. It is found that the temperatures followed the solar irradiance well. This also verifies that the solar irradiance is the most important factor of the PV-ETFE cushion. For the value, the maximum temperature in winter was 36.8 °C which was almost half of that (85.2 °C) in summer. The
reasons are that the beam irradiance accounts for a larger part of the solar irradiance in summer compared with that in winter and the solar irradiance was about twice greater than that in winter. Another interesting observation was that the variation in summer was obvious and the maximum temperature difference between the surface and back face was 4.3 °C in winter while 6.9 °C in summer. This means when the solar irradiance is higher, the shading effect could be more significant.

4.2.2 The temperature of the middle-layer membrane

The middle layer is the area of the direct effect of the PV, because the PV is located on the middle layer. As the two PV are symmetrical, thus there is a blank area between them, which could be used for evaluating the effect of the PV on membrane and the temperature conductivity of the ETFE membrane.

![Temperature of the PV in winter and summer](image1)

![Temperature on the membrane in winter and summer](image2)

Figure 6: the temperature of the PV in winter and summer

Figure 7: the temperature on the membrane in winter and summer

Figure 7 shows the temperature data of the PV and the blank in winter and summer. Firstly, both curves followed the solar irradiance well, though the winter one was relatively much better. For the value, the maximum temperatures in winter and summer were 31.3 °C and 77.3 °C, respectively. The temperature difference of the surface and back face was not as significant as
that of the PV. The temperature differences between the surface and blank of the middle layer were 7 °C and 15.6 °C in winter and summer. This implies that the effect of the temperature of the PV could be confined in some area and the conductivity of the membrane is low.

4.2.3 The temperature of the three layers

To obtain the temperature distribution, the temperature on the membrane is indispensable. The temperatures of the three layers are shown in Figure 8. Obviously, the sequence of the temperature of the three layers was the middle, the top and the bottom layer both in winter and in summer. The reasons for this are as follows. The temperature of the middle layer increases by the solar irradiance penetrating the ETFE-foil and the heated air in the enclosed cushion, while the effect of wind can be neglected. These factors results in the highest temperature. For the top layer, the temperature of it increases by the solar irradiance directly and the heated air in the cushion on the back face. On the contrary, the effect of wind decreases the temperature. For the bottom layer, the temperature of it is affected by the relatively low solar irradiance and the effect of wind, which leads to the lowest temperature.

For the value, the maximum of the three layers in winter were 25.7 °C, 20.9 °C and 16.6 °C, while these values in summer were 64.7 °C, 61.5 °C and 53.7 °C. This means the integration of PV significantly change the temperature distribution of the cushion. This high temperature results in the nonlinearity of the ETFE foil, which leads to different methods for designing compared to the conventional cushion.

4.2.4 The inside and outside temperature of the cushion

The thermal energy of the PV could heat the air in the cushion. If the heated air in the cushion could be used in an appropriate way, the consumption electricity of the air conditioning could be saved. Therefore, to evaluate the potential of the cushion in collecting thermal energy, the inside and outside temperatures of the cushion were measured and analyzed, shown in Figure 9.
The curves of the inside temperature followed the solar irradiance well, with the maximum value of 25.6 °C and 62.2 °C when the temperature outside the cushion were 5.4 °C and 35.7 °C in winter and summer, respectively. This means that the cushion could have an excellent potential to collect and store the thermal energy generated by PV.

In winter and summer, the maximum temperature differences between the inside and outside were 19.2 °C and 26.1 °C. The heated air in winter could be used as the air instead of part of air conditioning, while the heated air in summer could be utilized as the resource for machines which are driven by heated air. In this way, the thermal energy could be used in an efficient way.

4.3 The pressure

The pressure is the parameter to maintain the designed shape of the cushion. With the help of the PLC, the pressure of this cushion was in a stable range, within 264.5-268 Pa in winter and 264-270 Pa in summer. Furthermore, the variation of the pressure in summer was more notable than that in winter, see Figure 10. This is mainly due to the leakage of the cushion after one year use. In general, the PLC controls the pressure of the cushion well and the cushion operates smoothly.

![Figure 9: the temperature of the inside and outside of the cushion in winter and summer](image)

![Figure 10: the pressure in winter and summer](image)

5 CONCLUSIONS
Experimental studies on a three-layer ETFE cushion integrated flexible amorphous silicon photovoltaic panels were carried out within two years to investigate the feasibility of the PV-ETFE cushion. The results show that the PV-ETFE cushion operates smoothly under both cold winter and hot summer conditions. Some typical conclusions are summarized below:

- On typical days in winter and summer, the photovoltaic electricity of this system were 47.6 Wh and 107 Wh, and the consumption electricity were 10.3 Wh and 20 Wh. The positive difference of the two parameters suggests that the feasibility of the proposed system. During the experiments, the temperatures inside the cushion were 19.2 °C and 26 °C higher than that outside the cushion, indicating that this cushion could have excellent potential in collecting thermal energy.

- In summer, the maximum temperatures of PV and membrane were 77.3 °C and 85.2 °C. Therefore, when designing the PV-ETFE cushion, these temperatures should be taken into consideration because the high temperature results in the nonlinearity of the ETFE foil. Moreover, the temperature distribution of this cushion was also obtained.

- In winter and summer, the average pressures of 267.4 Pa and 266.6 Pa were also obtained, which means that the cushion operates smoothly.

This study reveals the technical feasibility of the PV-ETFE cushion structure system. It provides a way to expand the application of BIPV/T to inflated membrane structures.

However, some limitations are worth noting. Although the system works smoothly, the stress and strain of the ETFE foil are not acquired by experiments. Future work will therefore be paid to analyze the stress and strain to evaluate the performance of the PV-ETFE cushion in detail.

Acknowledgements

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REFERENCES

NUMERICAL ANALYSIS OF AEROELASTIC CHARACTERISTICS OF AIRSHIP ENVELOPE

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Key words: Airship, Airship Envelope, Aeroelastic Characteristics, Numerical analysis.

Summary. Stratospheric airship has great advantages, such as long-endurance, large coverage area, low-cost and so on, these advantages make airship be an ideal stratospheric platform and become highly valued. Airship envelope is a large inflatable membrane structure. As a key component, the envelope features flexible and large displacement. There is strong coupling between envelope structure and the ambient air when airship operating in the high altitude sky. The coupling characteristics have great impact on the aerodynamic and structural performance of airship. Aiming at the aeroelastic characteristics of the envelope structure, a fluid-structure coupled computational method is presented basing on a finite element program. As an example, the envelope structure of airship is computed and the S-A turbulent model is used. The envelope drag coefficient under different attack angle is computed. The contrast between experimental results coming from reference paper and numerical results highlight the correctness of this method. With the developed computational approach, the NPL envelope is also analyzed. The changes of length to diameter ratio, max cross section location and Reynolds number are studied and the aeroelastic characteristics of flexible envelope are analyzed. These results can give some valuable information for precise forecast of the overall airship performance.

1 INTRODUCTION

Airship is a kind of lighter than air aircraft. The stratosphere is the earth's atmosphere from the tropopause top to the 50 km height. Airship voyaged in this region has the advantages of long endurance, large converge area and low cost and so on. Several countries are developing stratospheric airship project at present, such as the HiSentinel airship from the United States of America has conducted 3 stratospheric flight tests. Considering the harsh operating condition, the stratospheric airship technology still needs some further study.

There is a strong coupling characteristic between the flexible envelope structure and the external flow field. In order to guide the airship design, many scholars have done some researches on accurate calculation of the airship aerodynamic characteristics. Using a numerical method based on the Arbitrary Lagrangian Eulerian method, Omari[1] carried out some research on the static aeroelastic characteristics of an ellipsoid null in inviscid low Mach number flow field. Basing on ABAQUS software to calculate the stress and deformation of the structural field, and using VSAERO software to calculate airship external flow field.
aerodynamic loads, Bessert[2] analyzed nonlinear aeroelastic characteristics of the airship and five different coupling forms through a step-by-step coupling calculation. Due to the flexibility of the envelope of the non-rigid airship, the variation of the temperature of the inner gas will lead to its structure deformation and affect its flight altitude. Li [5] developed the structural mechanics model, thermodynamic model and dynamic model of the semi-rigid airship, basing on which nonlinear finite element analysis is employed for geometrically nonlinear deformation of the airship upper film in consideration of thermodynamics and structural mechanics coupling. Wang[6] comprehensively took into account influential factors of solar radiation, environmental temperature, wind speed, and constructed stratosphere solar energy airship heat-transfer model, and acquired the change principles of average temperature of airship preliminarily. Ilieva[7] et al have concentrated on a critical overview of propulsion mechanisms for airships. These induce a detailed overview of past, present, and future enabling technologies for airship propulsion. Diverse concepts are revisited and the link between the airship geometry and flight mechanics is made for diverse propulsion system mechanisms.

Now, the research on the airship aeroelastic characteristics mainly concentrated on the numerical method and its accuracy verification. Most of the airship aeroelastic characteristics is calculated based on the a coupling platform or a self-coupling procedures to achieve data exchange, however, the method according to the calculations of the flow field and the structural field in the iteration process cannot rationally adjust the calculated step. In view of the ADINA software has strong advantages on the fluid-structure interaction, basing on this software, an aeroelastic calculation method is proposed, which can automatically adjust the coupled iterative calculation step and overcome the convergence and computational efficiency problem faced by general method of iterative. These results will provide some references for the calculation of the airship aeroelastic characteristics.

2 CALCULATION METHOD OF AEROELASTIC AND VERIFICATION

2.1 Calculation method of aeroelastic

In view of the excellent performance of structure and flow field numerical computing, accurate data transfer in the coupling interface and a reasonable set of coupled iterative step are the keys of the efficiency and accuracy of the calculation of aeroelastic. Based on the force equilibrium and displacement compatibility conditions of the coupling interface, the coupling interface interpolation method is shown in Figure 1. Since the step size can be automatically selected, the ADINA software provides an effective way for the aeroelastic computation.

During the numerical analysis of envelope aeroelastic characteristics, the coupling equation should be established based on the boundary conditions and the equilibrium equation of structure and flow field. Then the coupled equations should be discrete taken into account the integrated compatible condition and the distance from the coupling interface to grid node. Select the automatically step function of the ADINA, and then start the iterative solution. Convergence results can be tested by stress or displacement convergence criteria. Finally, the results considering the flow field and structural coupling can be achieved.
Fig.1 The numerical interpolation of coupling interface [3]

In order to simulate the flow field realistically, turbulent simulation model should be selected. Since the $\kappa$-$\varepsilon$ turbulence model is time-consuming and difficult to convergence, while the Spalart-Allmaras (SA) turbulence model has certain advantages on the computational precision and efficiency, so the S-A turbulence model is chosen as flow calculation model. Considering the FCBI-C element enjoys the advantages such as ease of convergence, suitable for large-scale iterative solution of coupled problems, then this element is chosen as coupling calculation element.

2.2 Test case

In order to verify the accuracy of the calculation method of the envelope aeroelastic, basing on the Lotte airship which has been conducted wind tunnel tests in Germany, numerical calculations and verification are carried out in this section.

According to reference [4], the calculated model is completed. The viscous force and inner pressure, which are taken into account, have great impact on the aerodynamic performance and structural deformation while the airship is running, so the similarity principle of Reynolds number is used. The Lotte airship’s experiments of 1:20 scale model is conducted in the medium-speed wind tunnel, and the volume Reynolds number is $Re_v = \frac{U_v V}{\nu} = 3.9 \times 10^3$, in which, $\nu$ is the kinematic viscosity.

The computation flow field is rectangular parallelepiped, according to the current flow status of the envelope surface and considering the computational efficiency, the length and breadth of the flow field are set 6 and 5 times of the envelope length and maximum diameter respectively. Delaunay meshing method is used and 106,920 four-node 3-D fluid elements are divided with 20163 nodes. According to the Reynolds number similarity criteria, the inlet velocity of the flow field is 1.19m/s, and the density value is 1.225kg/m$^3$, the calculated dynamic viscosity coefficient is $2.5 \times 10^{-5}$ Ns/m$^2$. The boundary conditions are set as speed entrance and free flow export differently and the envelope surface is assumed smoothly. As the material properties of the airship model cannot be accurately obtained in the wind tunnel experiment, the pressure of the envelope is assumed 300Pa, besides, the elastic modulus of envelope is 10 GPa, Poisson's ratio is 0.38. The end of the envelope structure is constraint and the head is free along the axial direction in the calculation of structure.

Airship aerodynamic performance, especially aerodynamic drag has a great impact on energy consuming and weight of the structure. Therefore, this section starts from the volumetric drag coefficient, and verify the reliability of the airship aeroelastic computational
analysis methods. Which, the volumetric drag coefficient is expressed as,

\[ C_v = \frac{D}{\frac{1}{2} \rho U^2 V^2} \]  

(1)

Where \( D \) is the aerodynamic drag of the envelope, along to the opposite direction of the flow speed, \( \rho \) is operating environment atmospheric density; \( U \) is the flow speed of far-field, \( V \) is volume calculation model.

Figure 2 shows the comparison of the calculated results and the wind tunnel experiment results under 20° angle of attack. The left figure shows that there is a certain zone of positive pressure on the windward side of the envelope head and a larger negative pressure zone on the leeward side of the envelope head. The comparisons results indicate that the numerical calculation method is variable.

![Fig.2 Comparison of pressure coefficient distribution (20° angle of attack)](image)

To exploring the relationship between volumetric drag coefficient and the angle of attack, numerical calculation of the volumetric drag coefficient in different angles of attack are carried out. Comparison of the experimental and calculated results is shown in Fig. 3. The data in figure 3 shows that the aeroelastic numerical results and wind tunnel experiments value have a good agreement, and the volumetric drag coefficient is basically parabolic trend with the increasing of angle of attack. When the angle of attack is 0°, the calculated volumetric drag coefficient is approximately 1.24 times larger than the wind tunnel experiment result, the multiples will also slowly increase along with the angle of attack increasing, the deviation is about 29.98% when the angle of attack is 30°. Since the elastic modulus value of envelope material and the internal pressure of the envelope are unknown, the aeroelastic calculation of the volumetric drag coefficient is larger than experimental results. As the material properties parameters of the wind tunnel experiment airship model and envelope pressure data cannot be obtained accurately, these parameters in paper is given based on the experience. The comparison of numerical results and literature results show that the airship envelope numerical calculation method is feasible. At the same time, different parameters of the model, such as the internal pressure and material properties, have great influence on the aeroelastic calculation results.
3 INFLUENCE ANALYSES OF DIFFERENT PARAMETERS ON AEROELASTIC NUMERICAL RESULTS

In order to study the influence of different calculation parameters on the airship envelope aeroelastic results, the NPL envelope is analyzed as example and the parameters, such as the envelope slenderness ratio, the envelope maximum cross-sectional position and the Reynolds number are analyzed. The NPL envelope shape is shown in Fig. 4.

3.1 Influence analysis of slenderness ratio

Slenderness ratio has an important influence on the whole envelope design and subsystem design. In case of fixing the envelope volume and maximum cross-sectional position of the envelope, different slenderness ratio of NPL envelope is computed. The elastic modulus of the envelope material is 4GPa, and Poisson's ratio is 0.38, the wind speed is 10 m/s. The tail of the envelope is fixed and the head has axial degree of freedom when it is computated, and the flow field boundary conditions are velocity inlet and free flow outlet with smooth wall surface.

Figure 5 shows the maximum stress contours of the structure with the 3.5 and 5.5 slenderness ratio respectively. The maximum stress is 12.12Mpa when the slenderness ratio is 3.5; however, the maximum stress is 10.47Mpa when the slenderness ratio is 5.5.
The envelope drag and volumetric drag coefficient versus slenderness ratio obtained by numerical calculation is shown in Fig. 6. Relational curve shows that NPL envelope has the minimum drag and volumetric coefficient when the value of slenderness ratio is about 5, and subsequently along with the slenderness ratio increasing, the envelope drag gradually increases.

![Fig.5 The contour of stress with the slenderness ratio is 3.5(left) and 5.5(right)](image)

![Fig.6 The envelope drag and volumetric drag coefficient vs. slenderness ratio](image)

3.2 Influence analysis of the position of maximum cross section area

As to the impact of the maximum cross-sectional position on the envelope aeroelastic results, different maximum cross-sectional position cases are analyzed. The calculation model is the NPL envelope, and the boundary conditions are set the same as the previous section. The maximum diameter, length and volume of analyzed model are maintained constant. Defining a dimensionless parameter $\eta$ as the ratio of the maximum length of the maximum cross-sectional location away from the head origin and the length of the envelope, the range of $\eta$ is 0.3 to 0.6.

Envelope drag and volumetric drag coefficient along with the position of the maximum cross-section area is shown in Fig. 7. Fig. 7 indicates that there is minimum drag and volumetric drag coefficient when the value of $\eta$ is 0.45. When the value of $\eta$ is less than 0.45, the envelope drag and volumetric drag coefficient will decrease rapidly along with the increasing of $\eta$. When the value of $\eta$ is greater than 0.45, the envelope drag and volumetric drag coefficient will increase gradually along with the increase of $\eta$. 
3.3 Influence analysis of the Reynolds number

Referring to the international standard atmospheric environmental parameters, as altitude increases, the air kinematic viscosity will gradually increase. So in case of the constant wind speed, due to the external atmospheric density gradually decreasing, the volume of the whole envelope Reynolds number will decrease.

Figure 8 shows the relational curve of the envelope drag, volumetric drag coefficient and the Reynolds number. Fig. 8 shows that, the envelope drag increases are proportionate to the increases of the Reynolds number. For volumetric drag coefficient, as the Reynolds number increases, the volumetric drag coefficient will gradually increase, and then there is a gentle process. The main reason of the flat section is that the increase proportion of envelope drag is nearly the same as the increase proportion of the atmospheric density.

4 CONCLUSIONS

- As to the envelope flexible characteristics, a verified method is proposed based on the ADINA software. With the proposed computational approach, the NPL envelope is analyzed. The changes of slenderness ratio, position of maximum cross-section area and Reynolds number are studied and the aeroelastic characteristics of flexible envelope are analyzed.
- Numerical results indicate that there is the minimum drag and volumetric coefficient when the value of slenderness ratio is about 5, and there is minimal drag and
volumetric drag coefficient when the value of $\eta$ is 0.45, and the envelope drag increases along with the increasing of Reynolds number. The work of this paper can give some valuable information for precise forecast of the overall airship performance.

REFERENCES

WIND-STRUCTURE INTERACTION SIMULATIONS FOR THE PREDICTION OF OVALLING VIBRATIONS IN SILO GROUPS

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Key words: Wind-structure interaction, CFD, silo, ovalling

Abstract. Wind-induced ovalling vibrations were observed during a storm in October 2002 on several empty silos of a closely spaced group consisting of 8 by 5 thin-walled silos in the port of Antwerp (Belgium) [4]. The purpose of the present research is to investigate if such ovalling vibrations can be predicted by means of numerical simulations. More specifically, the necessity of performing computationally demanding wind-structure interaction (WSI) simulations is assessed. For this purpose, both one-way and two-way coupled simulations are performed. Before considering the entire silo group, a single silo in crosswind is simulated. The simulation results are in reasonably good agreement with observations and WSI simulations seem to be required for a correct prediction of the observed ovalling vibrations.

1 INTRODUCTION

Wind-induced ovalling vibrations were observed during a storm in October 2002 on several empty silos of a closely spaced group consisting of 8 by 5 thin-walled silos in the port of Antwerp (Belgium) [4]. Wind-induced vibrations can be caused by either forced resonance or aeroelastic effects. In the first case, the forced vibrations originate from turbulent fluctuations in the wind flow around the structure, e.g. from natural turbulence in the wind flow attacking the structure or due to periodic vortex shedding in the wake of the structure. The second, aeroelastic phenomena are typically self-excited and are due to displacements of the structure and resulting interactions with the wind flow.
Flow-induced vibrations can be initiated by a combination of different phenomena and it is mostly impossible to pinpoint one specific excitation mechanism as the cause of an observed vibration [6]. The goal in this work is to investigate if advanced numerical techniques can adequately predict observed vibrations for such wind-structure interaction (WSI) problems. For this purpose, the coupled multiphysics problem considering both wind flow and structural dynamics has to be solved numerically.

The outline of this paper is as follows. First, the methodology for the numerical simulation of the WSI problem is considered (section 2). Both the structural and wind flow solvers are introduced but the emphasis is on the coupling approaches to investigate the influence of aeroelastic effects on the structural response. Afterwards, the coupling approaches are applied to the present application. First, the WSI problem of a single silo is simulated (section 3) and finally, the silo group is considered (section 4).

2 NUMERICAL METHODOLOGY FOR WSI SIMULATIONS

To solve the present WSI problem numerically, a partitioned approach is considered where the flow and structural equations are solved separately. The interaction between both domains is only enforced at the interface between the wind flow and the structure. In the following, first the structural and wind flow solvers are introduced (sections 2.1 and 2.2). To investigate if the observed ovalling vibrations are the result of forced resonance or aeroelastic effects, two different simulation approaches are explained afterwards (section 2.3) and a technique to compare the structural response in both approaches is given (section 2.4).

2.1 Structural model and ovalling modes of a silo

A numerical finite element (FE) model of the silo structure is constructed in the Abaqus software package [2] that allows to calculate the structural response of the silos to applied aerodynamic pressures. This model is also used to determine the ovalling eigenmodes and corresponding natural frequencies of the silos.

Ovalling deformations of a thin-walled shell structure are defined as a deformation of the cross section of the structure without bending deformation with respect to the
longitudinal axis of symmetry [7]. The ovalling mode shapes for the thin-walled empty silos (diameter 5.5 m and wall thickness varying from 0.07 m to 0.10 m along the height of the silo) are referred to by a couple \((m, n)\) where \(m\) denotes the half wave number in the axial direction and \(n\) is the number of circumferential waves (figure 2).

Figure 2: 3D isotropic view and horizontal section at mid-height for a selection of ovalling eigenmodes of a silo: (a) mode \(\Phi_1 = (1, 3)\) at 3.96 Hz, (b) mode \(\Phi_4 = (1, 4)\) at 4.11 Hz, (c) mode \(\Phi_5 = (1, 5)\) at 5.34 Hz, (d) mode \(\Phi_{11} = (1, 2)\) at 7.83 Hz and (e) mode \(\Phi_{14} = (2, 6)^*\) at 8.62 Hz.

To accommodate an easy transfer of the aerodynamic pressures on the silo walls to the mesh of the structural model in the coupled simulations (section 2.3), the mesh of the FE model is chosen conforming to the mesh on the silo walls in the wind flow simulations (section 2.2). Shell elements with linear FE interpolation functions are used for all silo elements and the following material properties for aluminium are used: density \(\rho = 2700 \text{ kg/m}^3\), Young’s modulus \(E = 67.6 \text{ GPa}\) and Poisson’s ratio \(\nu = 0.35\). The silo structures are bolted to a steel framework in the supporting building at 4 discrete points.

A list of the mass normalized eigenmodes \(\Phi\) of the structure corresponding to the lowest natural frequencies \(f_{\text{eig}}\) is given in table 1. Note that most modes come in pairs: e.g. \(\Phi_1\) and \(\Phi_2\) are both classified as \((1, 3)\) but are mutually orthogonal.

The visually detected pattern of vibrations at the lee side of the silo group during the 2002 storm is believed to have been a combination of ovalling mode shapes \((1, 3)\) and \((1, 4)\), corresponding to the lowest eigenfrequencies. Measurements during normal wind loading have also shown that eigenmodes with 3 or 4 circumferential wavelengths have the highest contribution to the response of the silos [4].
Table 1: Structural natural frequencies \( f_{\text{eig}} \) of the lowest ovalling eigenmodes of the silo structure. Every mode \( \Phi_j \) is determined by a couple \((m, n)\) while \( \Phi_{14} \) cannot be clearly classified as a single shape \((m, n)\). The notation \((2, 6)^*\) is adopted because this mode could be classified as either shapes \((2, 6)\) or \((1, 2)\).

<table>
<thead>
<tr>
<th>( \Phi_j ) ( (m, n) )</th>
<th>( f_{\text{eig}} ) [Hz]</th>
<th>( \Phi_j ) ( (m, n) )</th>
<th>( f_{\text{eig}} ) [Hz]</th>
<th>( \Phi_j ) ( (m, n) )</th>
<th>( f_{\text{eig}} ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_1 ) ((1, 3))</td>
<td>3.96</td>
<td>( \Phi_7 ) ((1, 5))</td>
<td>5.70</td>
<td>( \Phi_{13} ) ((2, 5))</td>
<td>8.19</td>
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<tr>
<td>( \Phi_2 ) ((1, 3))</td>
<td>3.97</td>
<td>( \Phi_8 ) ((1, 5))</td>
<td>5.71</td>
<td>( \Phi_{14} ) ((2, 6)^*)</td>
<td>8.62</td>
</tr>
<tr>
<td>( \Phi_3 ) ((1, 4))</td>
<td>3.99</td>
<td>( \Phi_9 ) ((1, 6))</td>
<td>7.72</td>
<td>( \Phi_{15} ) ((2, 4))</td>
<td>8.85</td>
</tr>
<tr>
<td>( \Phi_4 ) ((1, 4))</td>
<td>4.11</td>
<td>( \Phi_{10} ) ((1, 6))</td>
<td>7.72</td>
<td>( \Phi_{16} ) ((2, 4))</td>
<td>9.10</td>
</tr>
<tr>
<td>( \Phi_5 ) ((1, 5))</td>
<td>5.34</td>
<td>( \Phi_{11} ) ((1, 2))</td>
<td>7.83</td>
<td>( \Phi_{17} ) ((2, 6))</td>
<td>9.62</td>
</tr>
<tr>
<td>( \Phi_6 ) ((1, 5))</td>
<td>5.35</td>
<td>( \Phi_{12} ) ((2, 5))</td>
<td>8.18</td>
<td>( \Phi_{18} ) ((2, 6))</td>
<td>9.72</td>
</tr>
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2.2 Wind flow simulations

To determine the aerodynamic pressures acting on the silo surfaces, the wind flow around the silo group is simulated in 3D computational fluid dynamics (CFD) simulations. The governing incompressible Navier-Stokes equations are discretized by means of the finite volume method in the commercial software package Fluent [1].

Because a high Reynolds number wind flow is considered, detached eddy simulations (DES) are used [9]. This hybrid RANS/LES method combines large eddy simulation (LES) in the far field where large turbulent structures are present with the RANS (Reynolds-averaged Navier-Stokes) approach in the near-wall regions. This is an interesting technique for highly turbulent flows, where it is in practice impossible to use LES simulations because of the prohibitive grid requirements to model the small turbulent eddies in near-wall flows. A delayed DES approach (DDES) is applied in which a shielding function is used to ensure that RANS is retained in the entire near-wall region which may lead to doubtful accuracy in separated regions [5].

Since exact atmospheric conditions near the silo group were not monitored at the time of the ovalling vibrations, approximate wind conditions have been set up, based on location and statistical wind data for storm conditions in design codes. The rectangular silo group is therefore rotated at an angle of 30° to the incident wind flow (figure 1b). For the atmospheric boundary layer (ABL), realistic power law velocity and turbulence profiles [10] are imposed at the inlet of the computational domain while for the generation of fluctuating velocity components, a spectral synthesizer method is used as proposed by [8].

In WSI problems, the interface between wind flow and solid deforms. To allow the deformation of a boundary in the fluid solver, the arbitrary Lagrangian-Eulerian (ALE) approach is used for the CFD simulation as incorporated in Fluent [1]. The displacement of the interface is extended into the entire fluid domain by adjusting the fluid grid accordingly which can be achieved by applying a spring smoothing or Laplacian smoothing method. Because of the lower computational cost, the spring smoothing method is preferred. However, when structural displacements are large, the Laplacian smoothing
method has to be used which leads to a drastic increase in simulation time. This is because the diffusion equation of the mesh velocities then has to be solved iteratively in every coupling iteration of the two-way coupled simulation.

2.3 WSI coupling approaches

From a computational point of view, it is important to assess the necessity of performing the computationally much more intensive WSI, two-way coupled simulations. Therefore, more straightforward one-way coupled simulations are performed as well. Performing both coupling approaches allows to investigate the importance of aeroelastic effects for the onset of ovalling vibrations and are discussed in more detail in the following.

A partitioned approach is followed in the simulations. This implies that the structural and flow solver remain separated, black box solvers and the interaction between both domains is incorporated only at the interface. In this framework, the structural FE solver (section 2.1) is denoted as $S[P(t)] = U(t)$, where $U(t)$ are the displacements of the structure on the interface between fluid and structural solver and $P(t)$ are the aerodynamic pressures acting on the structure. Similarly, the CFD solver for the wind flow (section 2.2) is expressed as $F[U(t)] = P(t)$.

One-way coupled simulations

A one-way coupled simulation can be regarded as a special case of WSI simulation in which the structure is considered as a rigid body in the wind flow simulation. The time history of surface pressures on the rigid body structure is determined in the separate flow solver: $F[0] = P(t_i)$. As illustrated in figure 3, the resulting time history of aerodynamic surface pressures $P(t)$ is subsequently applied as an external transient load on the structure and the structural response is computed in the structural solver for the entire time frame considered: $S[P(t)] = U(t)$.

![Figure 3: Schematic representation of the interaction between flow solver (white) and structural solver (grey) in the one-way partitioned coupling simulation.](image)

To avoid a long transitional regime due to the abrupt application of the wind pressures on the undeformed silo structure, a static step preceding the dynamic calculation
is introduced: $\mathbf{KU}_0 = \mathbf{P}_0$ with $\mathbf{K}$ the FE stiffness matrix of the structure. The applied pressures in this static step are taken equal to the pressures in the first dynamic time step: $\mathbf{P}_0 = \mathbf{P}(t_1)$. The effect of such preliminary static calculation where the structural response $\mathbf{U}_0$ is used as an initial condition for the dynamic calculations was found very effective.

**Two-way coupled simulations**

In two-way coupled simulations, the structural and flow solver are coupled in every time step as illustrated schematically in figure 4. As opposed to the one-way coupling approach, these simulations are WSI simulations in the true sense. An implicit (or strongly coupled) partitioned technique is used to ensure equilibrium at the interface in every time step. The IQN-ILS method is applied for this purpose in the present simulations [3].

![Figure 4](image)

**Figure 4**: Schematic representation of the interaction between flow solver (white) and structural solver (grey) in the two-way partitioned coupling approach.

### 2.4 Modal deformation energy

For the physical interpretation of the structural response in both one-way and two-way coupled simulations, the deformation energy per structural eigenmode is considered. The deformation energy $E_d(t)$ of the structural response can be easily calculated from known displacements $\mathbf{U}(t)$:

$$E_d(t) = \frac{1}{2} \mathbf{U}^T(t) \mathbf{K} \mathbf{U}(t),$$

where $\mathbf{K}$ is the FE stiffness matrix of the silo structure. In order to distinguish the contribution of the different eigenmodes in the response, modal decomposition is applied. By inserting $\mathbf{U}(t) = \mathbf{\Phi}\alpha(t)$ in equation (1), the energy content of each eigenmode in the response can be quantified:

$$E_d(t) = \frac{1}{2} \alpha^T(t) \mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} \alpha(t) = \frac{1}{2} \sum_{j=1}^{n_{\text{DOF}}} \omega_j^2 \alpha_j^2(t) = \sum_{j=1}^{n_{\text{DOF}}} E_{dj}(t).$$


These scalar energy expressions where $n_{\text{DOF}}$ represents the total number of degrees of freedom in the FE model, allows to calculate the energy contribution $E_{d_j}(t)$ of every separate mode $j$ to the structural response using only the modal coordinates $\alpha(t)$.

In the present one-way and two-way coupled simulations, the structural response $U(t)$ is determined using direct time integration. The entire basis of $n_{\text{DOF}}$ eigenmodes $\Phi$ would therefore have to be determined to extract the modal coordinates $\alpha(t)$. It is computationally very demanding, however, to solve the entire eigenvalue problem and only the lowest eigenmodes are relevant for a typical low frequency wind excitation. It is therefore desirable to use only a limited subset of $n_s < n_{\text{DOF}}$ eigenmodes $\Phi_s$ with corresponding modal coordinates $\alpha_s(t)$ to determine the deformation energy. It can be easily shown that these expressions hold when a pseudoinverse of $\Phi_s$ is introduced to determine the modal coordinates $\alpha_s(t)$ for this subset of mode shapes: $\alpha_s(t) = \Phi_s^T MU(t)$.

3 SINGLE SILO SIMULATIONS

For the one-way coupled single silo simulation, the modal deformation energy components $E_{d_j}(t)$ of the structural response are shown as a function of time in figure 5 for the 20 lowest structural eigenmodes. Only a limited number of modes contribute significantly to the structural response of the silo but seemingly random fluctuations are observed in the stationary response. A distinction is therefore first made between static and different fluctuating components in the response.

The mean, time averaged modal deformation energy is related to the static excitation of a structural eigenmode. This is e.g. the case for mode shape $\Phi_{18} = (2,6)$ (dashed black line) in figure 5. In the fluctuating parts of the response, a further distinction can be made between two components.

The first is related to large scale turbulent eddies present in the wind flow in vicinity of the silo structure. The fourteenth mode $\Phi_{14} = (2,6)^*$ is e.g. mostly excited by such low
frequency fluctuations (bold grey line in figure 5). These oscillations are referred to as quasi-static or background vibrations because they are free of any resonant effects. While the exact origins of these low frequency fluctuations are not easily identified, it is possible that they originate from the turbulent fluctuations in the incident wind flow. Because the wind flow fluctuations are generated at the inlet of the computational domain including random numbers, these low frequency fluctuations are not exactly the same in different CFD simulations.

The second type of fluctuating components in the response are exactly at the eigenfrequencies of the eigenmodes and are hence related to forced resonance in the one-way coupled simulations.

Based on this description, an alternative way to represent the results is to consider the time averaged and RMS values of the modal deformation energy, $E_{d,j}$ and $E_{d,j}^{RMS}$. Figure 6 shows these quantities for the one-way coupled simulation of the single silo for the 50 lowest eigenmodes. The mean modal deformation energy $E_{d,j}$ (figure 6a) gives an indication of the static excitation of the eigenmodes while the RMS values $E_{d,j}^{RMS}$ contain information on all dynamic excitations, both quasi-static and resonant. These figures allow to qualitatively compare the response in different coupled simulations.

Based on the comparison of figures 6a and 6b, it is clear that structural vibrations will be much smaller than the static displacements. In general terms, the vibration amplitudes are in the order of 0.5 to 1 cm while the peak displacement is approximately 4 cm for this single silo. The eigenmodes that are mainly excited statically are $\Phi_2 = (1, 3)$, $\Phi_6 = (1, 5)$ and $\Phi_{14} = (2, 6)^*$ while mainly the eigenmodes with the lowest circumferential wavenumber $n$ and only a half wavelength across the height ($m = 1$) are excited dynamically. The excitation of $\Phi_{14} = (2, 6)^*$ is mainly quasi-static and related to the presence of a bolted connection at the windward side of the silo.

In the two-way coupled simulation of the single silo, spring smoothing is used for the ALE mesh motion in the CFD solver and in general 5 IQN-ILS coupling iterations.
have to be performed in each time step to ensure equilibrium on the WSI interface. The computational effort for the two-way coupled simulations is therefore said to be approximately 5 times larger than for the one-way coupled simulations.

![Figure 7](image)

**Figure 7:** (a) Mean and (b) RMS of modal deformation energy $E_{d_j}(t)$ of the structural response of the single silo in the two-way coupled simulation. The deformation energy for the lowest 50 eigenmodes is plotted as a function of $n$ while separate mode shapes $(1, n)$ are depicted as a circle ($\circ$), $(2, n)$ as a square ($\square$), $(3, n)$ as a diamond ($\diamond$), and $(4, n)$ as a cross ($\times$).

By comparison of the one-way and two-way coupled simulation results in figures 6 and 7, respectively, differences in absolute terms are less important because of the short simulated time intervals in the two-way coupled simulations. However, while the excited eigenmodes still correspond to the lowest eigenfrequencies, static and RMS values are overall larger in the two-way coupled simulation. Because the structural flexibility is now taken into account, the wake flow is not only influenced by the random incoming wind flow but also by slightly modified separation and wake behaviour that in turn influence the aerodynamic pressures. The simulated time frame is too short to clarify the difference between the one-way and two-way simulations. It can therefore not decisively be concluded whether the ovalling vibrations of the single silo are related to forced resonance or aeroelastic effects.

### 4 SILO GROUP SIMULATIONS

The influence of the location of a silo in the group arrangement is investigated by performing one-way coupled simulations for the silo at the windward corner of the group and for the silo at the leeward side of the group (figure 1b).

First, the structural response of silo at the windward corner of the silo group is calculated. The mean and RMS values of the modal deformation energy are shown in figure 8. It can be observed that the static deformation (figure 8a) as well as the vibrations (figure 8b) of this silo are dominated by eigenmodes $(1, 3)$ and $(1, 4)$. Other mode shapes with low circumferential wavenumbers are also excited, but less pronounced. The largest structural displacements are found in the small gaps between two adjacent silos as a result of the larger wind velocities and negative surface pressures in these narrow passages. The magnitude of the structural displacements is as high as 7 cm at these locations which is large compared to the total distance of 30 cm between two neighbouring silos. Vibration
amplitudes are again much smaller than the static deformation (approx. 1 to 2 cm). At the same time, the RMS values for this silo in the group arrangement are larger than for the single isolated silo, indicating larger structural vibrations.

![Figure 8](image)

**Figure 8:** (a) Mean and (b) RMS of modal deformation energy $E_{d_j}(t)$ of the structural response of the windward corner silo in the one-way coupled simulation of the silo group. The deformation energy for the lowest 50 eigenmodes is plotted as a function of $n$ while separate mode shapes $(1, n)$ are depicted as a circle ($\circ$), $(2, n)$ as a square ($\Box$), $(3, n)$ as a diamond ($\Diamond$), and $(4, n)$ as a cross ($\times$).

The results of the one-way coupled simulation for the leeward corner silo of the 8 by 5 silo group are shown in figure 9. As opposed to silos at the windward side of the silo group, the modal deformation energy for this silo is negligibly small for all eigenmodes. The radial static displacements are only approx. 1 cm and vibration amplitudes are negligibly small for the leeward corner silo.

![Figure 9](image)

**Figure 9:** (a) Mean and (b) RMS of modal deformation energy $E_{d_j}(t)$ of the structural response of the leeward corner silo in the one-way coupled simulation of the silo group. The deformation energy for the lowest 50 eigenmodes is plotted as a function of $n$ while separate mode shapes $(1, n)$ are depicted as a circle ($\circ$), $(2, n)$ as a square ($\Box$), $(3, n)$ as a diamond ($\Diamond$), and $(4, n)$ as a cross ($\times$).

The predicted vibration patterns correspond well with the observed ovalling vibrations in the Antwerp silo group. The largest vibrations are observed at the windward side of the group and the mode shapes that are preferentially excited seem to correspond with observations. Vibration levels are still smaller than the ones observed in October 2002.
This may be due to simplifications in the numerical model but is possibly also related to aeroelastic effects.

In the silo group arrangement, the vicinity of the neighbouring silos may have a significant impact on the structural vibrations. Considering the peak structural displacements up to approx. 8 cm at the windward side of the group with respect to the limited distance of 30 cm between two adjacent rigid silos, very different and possibly aeroelastic effects should be considered in a fully coupled FSI simulation.

The location of these large peak displacements has important implications for the numerical solution of the coupled WSI problem, however. A direct transfer of the large structural displacements inevitably leads to a change in the fluid solver that causes a long and large transition in the structural response. Apart from the fact that the spring smoothing ALE mesh update in the fluid solver cannot handle such large mesh deformations, the amplitudes of structural vibration during this transition are so large in the narrow gaps between two silos, that neighbouring silos collide. The ALE mesh update then obviously fails and it is difficult to calculate the two-way coupled simulation.

To reduce the transitional effects as much as possible, a solution is proposed to gradually increase the pressures that are applied in the structural solver during a limited period of time at the start of the two-way coupled simulation. Furthermore, the Laplacian smoothing method is applied to handle the more challenging ALE mesh updates in the narrow gaps between adjacent silos in the fluid solver. This results in a significant increase of the overall simulation time. Simulating 1 s of wind flow in this two-way coupled simulation of one flexible silo in the silo group, requires about 70 hours of computing time on 16 parallel cores and no results have been obtained for this configuration yet.

5 CONCLUSIONS

It is investigated if observed wind-induced ovalling vibrations in a silo group can be predicted by means of numerical simulations. More specifically, the necessity of performing computationally demanding wind-structure interaction (WSI) simulations is assessed. For this purpose, both one-way and two-way coupled simulations are performed.

Before considering the entire silo group, one-way and two-way coupled simulations of a single silo in crosswind are performed. For this configuration, it cannot decisively be concluded if the ovalling vibrations are due to forced resonance or aeroelastic effects because the simulated time frames are too short.

The results of the one-way coupled simulations for the silo group are in reasonably good agreement with observations. Although vibration amplitudes are significantly smaller, it is found that the structural modes with the lowest natural frequencies are excited at the windward side of the silo group. WSI simulations seem to be required however for a correct prediction of the observed vibration amplitudes. As a result of the geometry of the silo group, aeroelastic effects may enforce vibrations in the group arrangement but no definitive conclusions are obtained yet due to several numerical issues in the WSI simulation of the silo group.
REFERENCES


A PREDICTIVE MODEL FOR THE DESIGN OF FUNCTIONAL TEXTILES

STRUCTURAL MEMBRANES 2013

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Key words: Predictive model, material design, woven fabric, textile, biaxial, yarn geometry, composite, fabric.

Summary: This report proposes a method for the design of a fabric for specified mechanical properties at multiple biaxial-stress states.

1 INTRODUCTION

Functional textiles have a wide variety of uses including large scale roof structures [1], medical applications [2], and as reinforcement for composite materials. Functional textiles are typically manufactured based on simplified engineering requirements (e.g. weight and uniaxial strength), with other properties (such as detailed analysis of stiffness) determined retrospectively through physical testing. The work presented here demonstrates a methodology for the design of bespoke functional textiles to meet detailed engineering requirements, with the focus on the biaxial response of flexible coated woven fabrics. The method employed uses a semi-analytical optimisation routine to determine the optimum fabric geometry and constituent material properties for detailed material stiffness requirements.

Previously developed mechanical ‘unit cell’ models have been shown to provide a good prediction of the response of architectural plain-weave fabrics under biaxial load, and have therefore formed the basis of the work [3, 4]. The derivatives of the unit cell equilibrium equations have been determined and this allows the fabric parameters to be optimised for a detailed set of biaxial and shear stiffness requirements at different stress levels. Initial validation using the model to design feasible, known fabrics has shown good results and demonstrated the potential utility of this approach.

2 SCOPE AND METHODOLOGY

2.1 Biaxial response

Coated architectural fabrics are employed in biaxial stress states and have “negligible bending or compression stiffness” [5] meaning loads are resisted through tension, and as such
the model was required to work with biaxial input and output parameters. Therefore the response characteristics under biaxial load are considered to be the Young’s moduli in both warp and weft directions ($E_{11}$ and $E_{22}$) and the Poisons ratios of the fabric ($\nu_{12}$ and $\nu_{21}$).

Whilst shear response under biaxial load “is crucial in order to build double-curvature tensioned structures”\cite{6} the shear modulus (G) is not considered in the current version of this model as the response has been found to be dominated by the coating stiffness, currently modelled as linear. It is proposed that later versions of this model will include a module for the consideration of shear effects.

### 2.1 Sawtooth modelling

The sawtooth model developed by Menges and Meffert\cite{7} and further developed and used by Bridgens\cite{3,4} was the basis of the work. It was chosen as it allowed for the possibility of truly predictive design, as the equations contain no factors that need to be derived through testing, and the equations themselves lend themselves to differentiation.

The method considers a unit cell of fabric as shown in Figure 3, and idealises this as a set of two orthotropic yarns that are perpendicular, as shown in Figure 1 and Figure 2.
Unlike the previously developed models, the coating is represented by a single Isoparametric Plane Stress element as described by Cook, Malkus and Plesha [8]. This change was made in preparation for the analysis of shear response and the possibility of non-perpendicular geometry. The equations defining the response of the unit cell are therefore published as:

\[
F'_{k1,2} = 2 \cdot L_{2,1} \left( \frac{E_k}{1 - v_k^2} \right) (\varepsilon_{1,2} + v_k \varepsilon_{2,1})(1 + \varepsilon_{2,1}) ,
\]

\[
Y_{1,2}' = Y_{1,2}' \left[ 1 + \frac{F_{y1,2}}{2E_{y1,2}L'_{2,1}} \right] ,
\]

\[
\text{Area}_{1,2} = 2w_{1,2}r_{1,2} ,
\]

\[
w_{1,2}' = \frac{w_{1,2}}{L_{2,1}} ,
\]

\[
r_{1,2}' = \frac{\text{Area}_{1,2}}{2 \cdot w_{1,2}} ,
\]

constrained by the following equations which ensure geometric continuity and force equilibrium:

\[
(r_1 + r_2) = (A_1 + A_2) ,
\]

\[
F_{c1} = F_{c2} ,
\]

\[
F_{1,2} = F_{y1,2} \cos \theta_{1,2}' + F_{k1,2} ,
\]

where the subscripts 1 and 2 refer to the warp and weft directions respectively. The subscripts k and y refer to the coating and yarn respectively. The apostrophe refers to a value after deformation. Other terms included are the yarn radius (r), the yarn length (L) (1/4 the yarn wavelength), force (F), yarn amplitudes (A), yarn widths (w) (1/2 the yarn width), the yarn cross-sectional area (Area), Young’s Moduli (E), and yarn length (Y) (includes out-of-plane distance).
3 RESULTS AND DISCUSSION

3.1 Construction of defining equations

Once the equations defining the unit cell are available it is possible to calculate the response characteristics of the fabric numerically, employing a finite difference method.

However, numerical perturbation does not lend itself to optimisation, which is necessary to design a bespoke fabric. To produce equations that can be used in conjunction with optimisation routines it is necessary to find the derivatives \( \frac{dF_{l,2}}{d\varepsilon_{l,2}} \) (for \( E_{1,2,2} \)) and \( \frac{dF_{l,2}}{d\varepsilon_{2,1}} \) (for \( E_{1,2,1} \)). The derivative \( \frac{dF_{l,2}}{d\varepsilon_{l,2}} \) refers to the Young’s modulus of the unit cell, and must be converted to the value for the whole fabric as shown in equation 3. The derivative \( \frac{dF_{l,2}}{d\varepsilon_{2,1}} \) is needed to produce the Poisson’s ratios, as shown in equation 4.

\[
E_{1,2,2}^{\text{unit cell}} = E_{1,2,2}^{\text{global}} \times L_{2,1} \times 2
\]  

\[
\frac{-v_{12}}{E_{11}} = \frac{1}{E_{12}}
\]

To find the derivatives the applied force was determined in terms of the unit cell variables, and strain as shown in equation 5. Equations 6 through 9 are then necessary to calculate further derivatives.

\[
F_{l,2} = \left( \frac{F_{2,1} - F_{k,2,1}}{L_{2,1}(1 + \varepsilon_{2,1}) \tan \theta'_{1,2}} \right) (r'_1 + r'_2) - L_{1,2}(1 + \varepsilon_{1,2}) \tan \theta'_{1,2} + F_{k,1,2}
\]

\[
F_{k,1,2} = 2 \cdot L_{2,1} \left( \frac{E_k}{1 - v_{k,2}} \right) (\varepsilon_{1,2} + v_{k,2} \varepsilon_{2,1})(1 + \varepsilon_{2,1})
\]

\[
r'_{1,2} = \frac{r_{1,2}}{1 + \varepsilon_{2,1}}
\]

\[
\theta'_{2,1} = \cos^{-1}\left( \frac{(1 + \varepsilon_{2,1}) \cos \theta_{2,1} - \left( \frac{(F_{2,1} - F_{k,2,1})}{2E_{2,1}L_{1,2}(1 + \varepsilon_{1,2})} \right) r'_1 - r'_2}{L_{2,1}(1 + \varepsilon_{1,2}) \tan \theta'_{2,1}} \right)
\]

\[
\theta'_{1,2} = \tan^{-1}\left( \frac{(r'_1 + r'_2) - L_{2,1}(1 + \varepsilon_{2,1}) \tan \theta'_{2,1}}{L_{1,2}(1 + \varepsilon_{1,2})} \right)
\]

To calculate the full derivatives it is necessary to find the partial derivatives for all the variables. There are numerous variables that are inter-related with relation to the defining equations expressed earlier (equations 1 and 2). As such equations 10 and 11 represent the
calculation that needs to be performed to produce the required derivatives.

\[
\frac{dF_{1,2}}{d\varepsilon_{1,2}} = \frac{\partial F_{1,2}}{\partial \varepsilon_{1,2}} + \frac{\partial F_{1,2}}{\partial \theta_{1,2}} \frac{\partial \theta_{1,2}}{\partial \varepsilon_{1,2}} + \frac{\partial F_{1,2}}{\partial r_{1,2}} \frac{\partial r_{1,2}}{\partial \varepsilon_{1,2}} + \frac{\partial F_{1,2}}{\partial F_{k1,2}} \frac{\partial F_{k1,2}}{\partial \varepsilon_{1,2}} + \frac{\partial F_{1,2}}{\partial \varepsilon_{2,1}} \frac{\partial \varepsilon_{2,1}}{\partial \varepsilon_{1,2}} + \frac{\partial F_{1,2}}{\partial F_{k2,1}} \frac{\partial F_{k2,1}}{\partial \varepsilon_{1,2}} \tag{10}
\]

\[
\frac{dF_{1,2}}{d\varepsilon_{2,1}} = \frac{\partial F_{1,2}}{\partial \varepsilon_{2,1}} + \frac{\partial F_{1,2}}{\partial \theta_{1,2}} \frac{\partial \theta_{1,2}}{\partial \varepsilon_{2,1}} + \frac{\partial F_{1,2}}{\partial r_{1,2}} \frac{\partial r_{1,2}}{\partial \varepsilon_{2,1}} + \frac{\partial F_{1,2}}{\partial \varepsilon_{2,1}} \frac{\partial \varepsilon_{2,1}}{\partial \varepsilon_{2,1}} + \frac{\partial F_{1,2}}{\partial F_{k1,2}} \frac{\partial F_{k1,2}}{\partial \varepsilon_{2,1}} + \frac{\partial F_{1,2}}{\partial \varepsilon_{2,1}} \frac{\partial F_{k2,1}}{\partial \varepsilon_{2,1}} \frac{\partial F_{k2,1}}{\partial \varepsilon_{2,1}} \tag{11}
\]

Unfortunately it can be shown that due to the interdependence of the variables it is not possible to produce a fully analytical answer to equations 10 and 11. To produce useable equations one value must be calculated iteratively, as shown in equation 12. This must be calculated independently using the equilibrium model each time a new value is required.

\[
\frac{\delta \varepsilon_{1,2}}{\delta \varepsilon_{2,1}} = \frac{\Delta \varepsilon_{1,2}}{\Delta \varepsilon_{2,1}} \tag{12}
\]

Whilst this is now a semi-analytical method the equations derived do still allow for optimisation to be used to design a bespoke fabric.

### 3.2 The method of optimisation

MATLAB \[9\] was used to produce an optimisation script for the minimisation of the defining equations. Internal functions were used to optimise the equations for a set of targets produced. The optimisation methodology is briefly summarised in Figure 4. The method initially uses a pattern search algorithm to refine the search ‘area’, and then uses an internal MATLAB search routine to find the “minimum of [a] constrained nonlinear multivariable function” \[10\]. If no perfect solution can be found then the script implements a gradually varying allowance of variation from the targets to allow a solution to be found. This could be changed to allow for accurate optimisation for some important targets, and ‘as close as possible’ optimisation for other targets of less significance to the designer.

Using a function that allows for multiple constraints is used to incorporate the constraint equations (equations 2). If no perfect solution is found then bounds are placed on the targets, and these are allowed to vary by a percentage. This allows the script to find results where no realistic solution would be possible.

Five sets of targets are used in the current model to demonstrate how the method can be used to design for multiple material properties for a single fabric at different loads. More targets could be implemented, however the current number demonstrates the method’s utility without making any solution too difficult, or computationally expensive to find. The ‘Shear Module’ shown is currently in development.
3.3 Results for known feasible targets

To demonstrate the functionality of both the method of optimisation and the validity of the equations used an optimisation for a set of targets that were known to be feasible was performed.
The feasible targets were produced with the equilibrium model using a central finite difference method from the geometry shown in Table 1. The results of this finite difference method are shown in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Geometry from which targets are calculated</th>
<th>Optimised geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$ (mm)</td>
<td>0.069</td>
<td>0.071</td>
</tr>
<tr>
<td>$A_2$ (mm)</td>
<td>0.207</td>
<td>0.190</td>
</tr>
<tr>
<td>$\Theta_1$ (Rad)</td>
<td>0.106</td>
<td>0.116</td>
</tr>
<tr>
<td>$\Theta_2$ (Rad)</td>
<td>0.189</td>
<td>0.183</td>
</tr>
<tr>
<td>$L_1$ (mm)</td>
<td>0.645</td>
<td>0.605</td>
</tr>
<tr>
<td>$L_2$ (mm)</td>
<td>1.082</td>
<td>1.022</td>
</tr>
<tr>
<td>$r_1$ (mm)</td>
<td>0.162</td>
<td>0.152</td>
</tr>
<tr>
<td>$r_2$ (mm)</td>
<td>0.114</td>
<td>0.107</td>
</tr>
<tr>
<td>$w_1$ (mm)</td>
<td>0.786</td>
<td>0.824</td>
</tr>
<tr>
<td>$w_2$ (mm)</td>
<td>0.673</td>
<td>0.920</td>
</tr>
<tr>
<td>$E_1$ (kN/m)</td>
<td>860</td>
<td>859</td>
</tr>
<tr>
<td>$E_2$ (kN/m)</td>
<td>710</td>
<td>703</td>
</tr>
<tr>
<td>$E_k$ (kN/m)</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>$v_k$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 2: Feasible targets found at the applied loads $P_1$ and $P_2$.

<table>
<thead>
<tr>
<th></th>
<th>Point 1</th>
<th>Point 2</th>
<th>Point 3</th>
<th>Point 4</th>
<th>Point 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$ (target 1) (kN/m)</td>
<td>514</td>
<td>662</td>
<td>602</td>
<td>377</td>
<td>777</td>
</tr>
<tr>
<td>$E_{22}$ (target 3) (kN/m)</td>
<td>444</td>
<td>554</td>
<td>510</td>
<td>551</td>
<td>484</td>
</tr>
<tr>
<td>$\nu_{12}$ (target 2)</td>
<td>0.434</td>
<td>0.288</td>
<td>0.344</td>
<td>0.317</td>
<td>0.261</td>
</tr>
<tr>
<td>$\nu_{23}$ (target 4)</td>
<td>0.374</td>
<td>0.241</td>
<td>0.291</td>
<td>0.431</td>
<td>0.180</td>
</tr>
<tr>
<td>$P_1$ (kN/m)</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>$P_2$ (kN/m)</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

The results of this are as expected, a near perfect solution is found quickly suggesting the equations appear to correlate well to the sawtooth method which is known to correlate well with the response of real fabrics. It should be noted that the start point of the optimisation was not the geometry used to find the targets; this ensured that the method was in fact finding a solution, and not succeeding having been given the correct geometry.
The optimisation for the feasible values of stiffness and poisons ratio produces good results (Figure 5). Target points 4 and 5 in the plot of $E_{22}$ results show some slight deviation from the targets. In reality this small error, whilst observable in the figure, equates to a difference of 0.89kN/m and 0.90kN/m respectively. This is as a result of the slight deviation from the original geometry that was found. A higher accuracy requirement on the solver may produce more accurate results, but would be more computationally expensive, taking longer.

3.4 Comparison with measured fabric parameters

Target values of stiffness and poisons ratio were calculated from biaxial test data produced from a fabric with the geometry set out in Table 1. The targets are shown in Table 3, along with the numerical results of the optimisation. The points to be analysed were chosen from areas on the response surface that did not include flattening in one of the principle directions. This flattening leads to unexpectedly large or small results when analytical or numerical derivatives of the surface are calculated to give targets. Therefore similar targets to those used in the previous test could not be used in this instance.
Table 3: Measured targets found at the applied loads P1 and P2

<table>
<thead>
<tr>
<th>Targets</th>
<th>Point 1</th>
<th>Point 2</th>
<th>Point 3</th>
<th>Point 4</th>
<th>Point 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$ (kN/m)</td>
<td>700</td>
<td>799</td>
<td>794</td>
<td>668</td>
<td>596</td>
</tr>
<tr>
<td>$E_{22}$ (kN/m)</td>
<td>748</td>
<td>875</td>
<td>799</td>
<td>681</td>
<td>621</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>0.218</td>
<td>0.170</td>
<td>0.197</td>
<td>0.114</td>
<td>0.138</td>
</tr>
<tr>
<td>$v_{21}$</td>
<td>0.305</td>
<td>0.288</td>
<td>0.234</td>
<td>0.379</td>
<td>0.412</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Results</th>
<th>Point 1</th>
<th>Point 2</th>
<th>Point 3</th>
<th>Point 4</th>
<th>Point 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$ (kN/m)</td>
<td>552</td>
<td>652</td>
<td>604</td>
<td>611</td>
<td>591</td>
</tr>
<tr>
<td>$E_{22}$ (kN/m)</td>
<td>676</td>
<td>811</td>
<td>746</td>
<td>837</td>
<td>829</td>
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<tr>
<td>$v_{12}$</td>
<td>0.248</td>
<td>0.153</td>
<td>0.196</td>
<td>0.145</td>
<td>0.152</td>
</tr>
<tr>
<td>$v_{21}$</td>
<td>0.331</td>
<td>0.203</td>
<td>0.261</td>
<td>0.204</td>
<td>0.220</td>
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<table>
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<th>% differences</th>
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<th>Point 3</th>
<th>Point 4</th>
<th>Point 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$</td>
<td>-21.1</td>
<td>-18.5</td>
<td>-24.0</td>
<td>-8.5</td>
<td>-0.8</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>-9.6</td>
<td>-7.3</td>
<td>-0.5</td>
<td>22.9</td>
<td>33.4</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>13.9</td>
<td>-10.1</td>
<td>-6.6</td>
<td>27.0</td>
<td>10.1</td>
</tr>
<tr>
<td>$v_{21}$</td>
<td>8.6</td>
<td>-29.7</td>
<td>11.6</td>
<td>-46.3</td>
<td>-46.5</td>
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</table>

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>$P_1$ (kN/m)</th>
<th>$P_2$ (kN/m)</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>10</td>
<td>10</td>
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<td></td>
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<td>16</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

Figure 6: Results of the optimisation for the measured targets

No perfect solution could be found through the optimisation for the measured targets (Figure 6). Although no perfect solution could be found Figure 6 does show how close the
solutions found were to the targets. Table 4 shows the geometric solution found against the geometry of the original fabric.

Table 4: Optimised geometry for measured targets

<table>
<thead>
<tr>
<th>Variable</th>
<th>Geometry from which targets are calculated</th>
<th>Optimised geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$ (mm)</td>
<td>0.069</td>
<td>0.428</td>
</tr>
<tr>
<td>$A_2$ (mm)</td>
<td>0.207</td>
<td>1.861</td>
</tr>
<tr>
<td>$\Theta_1$ (Rad)</td>
<td>0.106</td>
<td>0.316</td>
</tr>
<tr>
<td>$\Theta_2$ (Rad)</td>
<td>0.189</td>
<td>0.130</td>
</tr>
<tr>
<td>$L_1$ (mm)</td>
<td>0.645</td>
<td>1.039</td>
</tr>
<tr>
<td>$L_2$ (mm)</td>
<td>1.082</td>
<td>0.210</td>
</tr>
<tr>
<td>$r_1$ (mm)</td>
<td>0.162</td>
<td>0.033</td>
</tr>
<tr>
<td>$r_2$ (mm)</td>
<td>0.114</td>
<td>0.334</td>
</tr>
<tr>
<td>$w_1$ (mm)</td>
<td>0.786</td>
<td>0.254</td>
</tr>
<tr>
<td>$w_2$ (mm)</td>
<td>0.673</td>
<td>1.021</td>
</tr>
<tr>
<td>$E_1$ (kN/m)</td>
<td>860</td>
<td>925</td>
</tr>
<tr>
<td>$E_2$ (kN/m)</td>
<td>710</td>
<td>946</td>
</tr>
<tr>
<td>$E_k$ (kN/m)</td>
<td>30</td>
<td>19</td>
</tr>
<tr>
<td>$\nu_k$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The optimised geometry is clearly not the same as the geometry of the fabric from which the targets were derived. The original set of targets may be unobtainable for the sawtooth method with the constraints currently placed on the solution. The constraints (maximum and minimum values of geometric properties, and the constraints on the deformation stated in equation 2) currently being used are very broad to encompass extremes of realistic fabrics. These would be further constrained for more specific and realistic designs.

When the targets are allowed to vary slightly (5%) from the initial input targets a far more successful optimisation is performed.

6 DISCUSSION

The sawtooth model provides a reasonable prediction of fabric behaviour with the model’s deviation from the mean of the strain range of a real fabric being between 5.3 and 5.9% [4] (Figure 7).

The method developed offers close correlation between results for feasible targets. This good fidelity was predicted, as the optimisation equations were developed using the sawtooth model, but demonstrates the utility of the method. Therefore the optimisation works by finding the solutions available from all possible response planes of the sawtooth model, and should eventually find a solution for targets that originally existed on this plane. This does, importantly, show that the method being employed to find the targets is working.
The error found in the optimised geometry for the targets measured from biaxial data can be explained by the difference in the response planes of the real fabric and the sawtooth’s prediction of that fabric’s response. Figure 7 shows the difference in the response planes of sawtooth and the real fabric when a sawtooth model is run using the geometry of the real fabric. These two sets of response planes, whilst similar, are clearly not the same. Over and under prediction of strain will also affect result.

It was unlikely at the outset that the solver would find a solution that perfectly matched the real fabric’s geometry. It is also therefore possibly the case that no feasible solution exists for the sawtooth model where the targets stated in Table 3 could be achieved within the constraints placed on the model. Future work will be needed to demonstrate how much inaccuracy is inherent in the process, and therefore must be expected when attempting to design the geometry of ‘real’ fabrics.

![Figure 7: Response surfaces for the sawtooth model and measured response for one geometry](image)

5 CONCLUSIONS

- The accuracy of the optimisation method with regards to known feasible targets derived from the sawtooth model is good.
- The methodology is slower than hoped as the calculation of \( \frac{\Delta e_{1,2}}{\Delta e_{2,1}} \) must be completed after each iteration.
The accuracy of the optimisation method with regards measured targets derived from real fabric data is acceptable at this stage of development. The actual accuracy of the optimised geometry for the new targets is unknown as it is not currently possible within the bounds of this work to produce a bespoke fabric to be tested.

It is possible that for some targets multiple solutions exist and that for others no solutions exist. The latter has been shown through the results of the measured target optimisation, but the former is as of yet unproven.

Allowing small amounts of variation from the target may drastically improve the model’s utility and allow for a Pareto front of possible solutions to be found.

6 FURTHER WORK

Further work is on-going to allow the optimum design of a fabric’s shear response characteristics as well as biaxial response to loads. The inherent uncertainty in the manufacturing process, and the discrete nature of some parameters, will also be considered and methods for the calculation of the effect of such variability incorporated into future models. In addition it is necessary to further check the inherent inaccuracy of the model when compared to real results obtained through tests. Other possible implications of the model must be further investigated. And the effect of varying one parameter on the optimised result will also be investigated.

7 REFERENCES

FRICTION AND LEAKAGE CHARACTERISTICS OF CONFINED, REDUCED-SCALE INFLATABLE STRUCTURES

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Key words: Confined Inflatable Structure, Friction, Leakage, and Tunnel

Summary: This work is focused on the evaluation of the performance of a small-scale inflatable, or plug, placed in a confined space provided by a circular rigid pipe as a way to contain the propagation of floods. The rigid pipe is a simplified and scaled approximation of an actual tunnel section. The evaluations were conducted using an inflatable plug made of a single layer of coated Vectran® fabric. Friction coefficients of the system were calculated for three different materials lining the pipe so a comparison could be made. These friction coefficients were also compared to laboratory friction machine testing of the same lining materials. This comparison showed that the friction coefficients of the pipe-plug system were lower than the laboratory friction machine tests. Rates of water leakage around the plug were also studied. The leakage rates were recorded for several different plug pressures while varying the tunnel pressure accordingly. It was observed that as pressure differential decreased between the plug and pipe, the leakage rate increased. Results showed also that the plug was able to withstand a pressure differential with manageable water leakage rates.

1 INTRODUCTION

Inflatable technology has become an attractive alternative to several conventional devices used for building temporary or special structures. Inflatable structures offer the benefits of being relatively lightweight and portable, for maintaining the necessary rigidity while in operation, and for having a relatively reduced production cost. These benefits have prompted
the use of inflatables in confined spaces, such as pipes and tunnels, to act as barriers with minimal infrastructure modification.\textsuperscript{1, 2} Some examples include the large-scale inflatable tunnel plugs that were tested and installed in the London subway system to block smoke spread and limit oxygen supply to tunnel fires\textsuperscript{1} and the 23-foot (7 meter) diameter plug, which was filled with water and used in a uranium mine to successfully stop flooding.\textsuperscript{2} Currently, West Virginia University (WVU) is conducting research in the area of confined inflatable structures that can be rapidly deployed and pressurized to stop a tunnel flood created by a natural disaster or man-made event where the tunnel is damaged under a waterway.\textsuperscript{3-5} With plugs installed at key points, the damaged tunnel section could be contained, limiting the potential losses in a catastrophic event. The work at WVU has progressed in stages from a proof-of-concept, air-inflated structure\textsuperscript{3} to full and quarter-scale models pressurized with water and subjected to backpressure for simulations of flooding.\textsuperscript{4, 5}

Understanding the behavior of these structures requires studies that can be difficult or expensive to carry out at large scales. Therefore, evaluations at a reduced scale become necessary as an initial step to understand the characteristics of confined inflatable structures. This work is focused on the evaluation of the performance of a small-scale inflatable, or plug, placed in a confined space—provided by a circular rigid pipe—as a way to contain the propagation of floods. The rigid pipe is a simplified and scaled approximation of an actual tunnel section. A reduced-scale test bed was constructed in which a plug could be inflated inside of a pipe with one closed end. This space between the closed pipe end and the plug was pressurized with water, which applied an opposing force on the plug trying to push it out of the pipe. In order to stop the flow of water, the plug had to be capable of being pressurized and had to apply enough pressure on the pipe walls so that it did not move while being acted upon by an opposing force. The reduced-scale tests were conducted with three pipe (called also “tunnel” in this document) inner surface conditions for a variety of water pressures. The goal was to estimate tunnel/plug friction coefficients and water leakage rates that could be used to predict the performance of a full-size inflatable tunnel plug.

2 INFLATABLE PLUG AND TEST SET-UP

The inflatable plug was constructed from a single layer of a high-strength fabric made of Vectran\textsuperscript{®} fibers. The surfaces of the fabric were protected with a urethane coating on both the inside and outside of the plug. This coating was important on the inside to provide watertightness characteristics and important on the outside for protecting the fabric from abrasion. The plug had an outer diameter of 50 inches (127 centimeters [cm]). It was slightly oversized to the tunnel's 48-inch (121.9 cm) diameter in order to ensure maximal contact between the tunnel and plug. The total length from tip to tip of the hemispherical end-caps was 110 inches (279.4 cm). The plug was designed for a maximum inflation pressure of 40 pounds per square inch gauge (psig) (275.79 kilo Pascal [kPa]) at which it had a volume of approximately 800 gallons (3,028 liters). An overview of the plug characteristics is illustrated in Figure 1.

A layout of the reduced-scale flooding simulation system created for conducting the tests of this work is illustrated in Figure 2. The system essentially consists of two closed circuits
driven by high-flow and high-pressure water pumps as well as pressure regulators that recirculate and pressurize water, respectively. There is one circuit for pressurization of the plug and one circuit for pressurization of the rigid pipe representative of a tunnel section, as shown in Figure 2. Pressure sensors were installed at the same level in the plug and in the pressurized section behind the plug. A displacement sensor was used to measure plug movement. A collection basin was installed in front of the tunnel exit to measure leakage out of the tunnel. Data was sampled at one-second intervals using a LabView® program.

Figure 1: Inflatable plug general dimensions (1 inch = 2.54 cm).

Figure 2: Plan view of the reduced-scale flooding simulation system used for testing.
3 TUNNEL LININGS

In order to investigate the surface effects on the friction between the plug and tunnel, three materials were used to line the interior of the tunnel. In addition to the original smooth concrete interior of the tunnel, two more materials were used. These included a 0.25-inch (0.63 cm) thick soft neoprene pad and also a 0.125-inch (0.317 cm) thick vinyl coating. Both materials were bonded to the concrete surface with high-strength adhesive for the execution of the different tests. There were several factors that influenced the selection of these materials, such as roughness, compressibility, ease of application, and potential future application in full-scale prototypes. The materials chosen could be installed easily in a full-scale application if they provide benefits in terms of better friction characteristics and reduced leakage rates. A close look at the surface characteristics of each surface is shown in Figure 3. An example of the application of the neoprene liner is illustrated in Figure 4(a). Figure 4(b) shows the inflatable plug positioned in the pipe for the tests.

Figure 3: Tunnel linings: (a) Smooth concrete; (b) Neoprene; (c) Vinyl.

Figure 4: (a) Example of neoprene tunnel lining positioned in the cylindrical portion of the deflated plug; (b) Inflated plug positioned for testing.
4 TESTING PROCEDURE FOR SLIPPAGE EVALUATIONS

The plug was inserted into the tunnel and connected to the inflation system. The plug was then filled with water but not pressurized beyond 5 psig (34.47 kPa). After the plug was filled, the tunnel was then filled with water but not pressurized. Once testing was ready to begin, the tunnel pressure was raised to 2 psig (13.79 kPa) lower than the plug pressure. The data collection system was activated once the pressures were within this 2 psig (13.79 kPa) differential. Data was sampled at one-second intervals, collecting values for plug pressure, tunnel pressure, and plug displacement. The data was recorded for five plug pressures: 5, 10, 15, 20, and 25 psig (34.47, 68.95, 103.42, 137.90, and 172.37 kPa, respectively). Each of these pressures was tested for each of the three tunnel linings.

The goal of the testing was to find the point at which the plug would move due to the force acting on it by the tunnel pressure. Because all pressure regulators and switches were manually operated, two people were required for testing. One person controlled the plug pressure while the other person controlled the tunnel pressure. Changing the tunnel pressure had a residual effect on the plug pressure. That is, when the tunnel pressure was increased or decreased, it produced an increase or decrease of the plug pressure, respectively. This behavior is due to the confining effect of the tunnel and the incompressibility of water. The test was performed by keeping the plug pressure constant and raising the tunnel pressure towards the plug pressure until the plug slipped. A loud thumping noise occurred when slippage took place, indicating the test for that pressure could be stopped. The plug pressure was continuously adjusted to keep it as close as possible to the selected test pressure. If the plug pressure was not adjusted, it would continue to increase as the tunnel pressure was increased, potentially exceeding the maximum pressurization capacity of the plug.

The tests started at the lowest plug pressure and continued to the next highest pressure systematically. When testing for one pressure was completed, the data was recorded and then restarted for the next pressure. When all five plug pressures were recorded for a given tunnel lining, the plug and tunnel were deflated and a new liner was installed. The tests then continued with the same procedure for each additional liner.

5 TESTING PROCEDURE FOR WATER LEAKAGE RATE EVALUATIONS

The plug was inserted into the tunnel and connected to the inflation system. The plug was then filled with water but not pressurized beyond 5 psig (34.47 kPa). After the plug was filled, the tunnel was filled with water but not pressurized. Both pressures were then adjusted to the desired positions, making sure the tunnel pressure always stayed at least 2 psig (13.79 kPa) lower than the plug pressure to avoid the chance of plug slippage. One person controlled the plug pressure regulator and another person controlled the tunnel pressure regulator. By using two people, the pressures could be adjusted simultaneously to reach the desired test point. The pressures had to be carefully observed because the change in one pressure affected the pressure in the other.

Leakage rates were recorded for seven different plug pressures ranging from 5 psig (34.47 kPa) to 35 psig (241.32 kPa) in increments of 5 psig (34.47 kPa). The tunnel pressure was set to percentages of the plug pressure: 20%, 40%, 60%, and 80%. These percentages of the plug pressure...
pressure were used to keep the data consistent across the various tests. For each plug pressure, three leakage rates were recorded for each of the tunnel pressures. This equaled a total of 12 leakage rates for each plug pressure. An average leakage for each plug and tunnel pressure combination could then be found. A total of 84 leakage tests were performed for each of the three linings, providing a total of 252 measurements.

6 RESULTS

6.1 Slippage tests

Analysis of the results showed that the data displayed unique trends in the plug and tunnel pressures; these trends allowed the slippage point to be easily seen. When the plug slipped, it caused a sudden increase in tunnel volume. This volume increase caused a sudden decrease in the pressure in the tunnel, which also caused a decrease in plug pressure. An example of the changes in the plug and tunnel pressures at the instant of slippage is shown in Figure 5. The slippage itself created very small axial displacements that were detected by the displacement sensor. As seen in Figure 5, the plug remained relatively steady while the pressures were gradually matched until reaching the slippage point in which the plug moved and reached a new equilibrium position. The oscillations in the displacement data were attributed to static interference and a relatively low-resolution sensor used for this set of experiments. A fitting line shows the tendency of the axial displacement in Figure 5. Similar behavior was observed in all combinations of pressures and for the three lining materials.

![Figure 5: Example of plug and tunnel pressures variation as well as axial displacement up to the point of slippage.](image)

The pressure differential between the plug pressure and tunnel pressure was then used to calculate the friction coefficient for each tunnel lining. A static force balance was used to find the friction coefficient corresponding to the slippage instant in terms of the measured plug and tunnel pressures. The general static friction equation \( F_F = f \times N \), where \( F_F \) is the resisting
tangential force originated by the action of the friction coefficient \( f \) and the normal force \( N \), was used to estimate the friction coefficient of the system at the moment of slippage. Figure 6 shows a free body diagram of the acting forces applied to the tunnel and plug test bed along with their dimensions.

![Figure 6: Forces acting on the testing system.](image)

6.2 Evaluation of friction coefficient

The hydrostatic forces and the pressure forces were superimposed and integrated to obtain the normal force used in the general friction equation to obtain the following equation:

\[
\frac{F_{TH} + F_{ATP}}{F_{PH} + F_{APP}} = f
\]

(1)

Where \( F_{TH} \) is the hydrostatic tunnel force, \( F_{ATP} \) is the force from the applied tunnel pressure, \( F_{PH} \) is the hydrostatic plug force, and \( F_{APP} \) is the force from the applied plug pressure. The effective contact length (\( L_C \)) of the plug was measured and resulted in a value of 72 inches (183 cm) and tunnel diameter of 48 inches (121.9 cm). The forces in terms of measured pressures and geometric properties are:

For the tunnel

\[
F_{ATP} = P_{ATP} \times A_T
\]

(2)

For the plug

\[
F_{APP} = P_{APP} \times \pi \times D \times L_C
\]

(3)

Combining equations (1), (2), and (3), we get:

\[
\frac{1568.3 \text{ lbs} + P_{ATP} \times 1808.64 \text{ in}^2}{1709.65 \text{ lbs} + P_{APP} \times 10857.34 \text{ in}^2} = f
\]

(4)

Where \( P_{ATP} \) is the applied tunnel pressure and \( P_{APP} \) is the applied plug pressure. The pressure acting on the hemispherical end-cap of the plug was assumed to act on the projected
circular area of the plug, which is conservative and gives slightly lower friction coefficients. The resulting tunnel/plug friction coefficients calculated with Equation (4) are summarized in Tables 1 to 3 for concrete, neoprene, and vinyl linings, respectively.

Results summarized in Tables 1 to 3 show that equation (4) predicted the lowest average friction coefficient for neoprene lining with a value of 0.154, while the vinyl lining had the highest value of 0.176. The friction coefficient for a concrete surface was very close to the vinyl with a value of 0.172. These results contradict the assumption that the vinyl covering would have the lowest friction coefficient because of its smoother surface. It is thought that the urethane coating on the plug fabric sticks better to the plastic-like surface of the vinyl lining, therefore creating a higher friction coefficient than in the cases of neoprene or concrete surfaces.

Table 1: Friction coefficient for concrete lining (1 psig = 6.895 kPa).

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Plug Pressure (psig)</th>
<th>Tunnel Pressure (psig)</th>
<th>Pressure Differential (psig)</th>
<th>Friction Coefficient, ( f ) Eq. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.109</td>
<td>5.948</td>
<td>0.161</td>
<td>0.181</td>
<td></td>
</tr>
<tr>
<td>10.535</td>
<td>10.461</td>
<td>0.074</td>
<td>0.177</td>
<td></td>
</tr>
<tr>
<td>15.271</td>
<td>15.160</td>
<td>0.111</td>
<td>0.173</td>
<td></td>
</tr>
<tr>
<td>19.536</td>
<td>18.755</td>
<td>0.781</td>
<td>0.166</td>
<td></td>
</tr>
<tr>
<td>24.136</td>
<td>23.020</td>
<td>1.116</td>
<td>0.164</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>0.172</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Friction coefficient for neoprene lining.

<table>
<thead>
<tr>
<th>Neoprene</th>
<th>Plug Pressure (psig)</th>
<th>Tunnel Pressure (psig)</th>
<th>Pressure Differential (psig)</th>
<th>Friction Coefficient, ( f ) Eq. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.584</td>
<td>4.473</td>
<td>0.111</td>
<td>0.188</td>
<td></td>
</tr>
<tr>
<td>9.518</td>
<td>8.130</td>
<td>1.388</td>
<td>0.155</td>
<td></td>
</tr>
<tr>
<td>15.073</td>
<td>12.060</td>
<td>3.013</td>
<td>0.141</td>
<td></td>
</tr>
<tr>
<td>19.834</td>
<td>16.251</td>
<td>3.583</td>
<td>0.143</td>
<td></td>
</tr>
<tr>
<td>24.706</td>
<td>20.763</td>
<td>3.943</td>
<td>0.145</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>0.154</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Friction coefficient for vinyl lining.

<table>
<thead>
<tr>
<th>Vinyl</th>
<th>Plug Pressure (psig)</th>
<th>Tunnel Pressure (psig)</th>
<th>Pressure Differential (psig)</th>
<th>Friction Coefficient, ( f ) Eq. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.667</td>
<td>6.630</td>
<td>0.037</td>
<td>0.183</td>
<td></td>
</tr>
<tr>
<td>11.998</td>
<td>11.911</td>
<td>0.087</td>
<td>0.175</td>
<td></td>
</tr>
<tr>
<td>15.147</td>
<td>15.023</td>
<td>0.124</td>
<td>0.173</td>
<td></td>
</tr>
<tr>
<td>20.416</td>
<td>20.255</td>
<td>0.161</td>
<td>0.171</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>0.176</td>
<td></td>
</tr>
</tbody>
</table>
These friction values were also compared to other experimental values obtained from the friction machine testing of the same material.\textsuperscript{6, 7} A comparison of these values is shown in Table 4 where it can be clearly seen that the friction values for the plug and tunnel slippage tests are significantly lower than the values predicted from the friction sled tests. This information does not tell us the values are incorrect, but rather that there are other factors influencing the friction characteristics in the tunnel tests that were not present in the sled tests or vice versa. Note also that the sled tests follow the same trend as the plug and tunnel tests in that the neoprene has the lowest friction coefficient and the vinyl has the highest value. It is thought that the leakage pressure or leakage ratio could be influencing the estimation of the friction factor in the tunnel plug system. This relationship between the friction coefficient and leakage rate is summarized in Table 5.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Average Friction Coefficient</th>
<th>Plug Tunnel Test</th>
<th>Friction Machine Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>0.172</td>
<td>0.620</td>
<td></td>
</tr>
<tr>
<td>Neoprene</td>
<td>0.154</td>
<td>0.610</td>
<td></td>
</tr>
<tr>
<td>Vinyl</td>
<td>0.176</td>
<td>0.710</td>
<td></td>
</tr>
</tbody>
</table>

### 6.3 Evaluation of leakage rates

From the results of the leakage tests it was observed that the leakage rate decreases when the pressure differential between the plug and tunnel increases. That is, a larger pressure differential means that the plug and tunnel pressures are further from each other. When this differential increases, the plug is able to exert more force on the tunnel walls, which seals the contact surface better. Therefore, the water in the tunnel is not able to flow around the plug. This trend is observed across all tunnel linings, as shown in Figure 7.

Pressure differential is not the only factor that influences the leakage rate. Figure 7 shows that for a given pressure differential, the leakage rate increases as the plug pressure increases.
This indicates that higher tunnel pressures cause more leakage, despite having the same pressure differentials. A different way of seeing this effect is illustrated in Figure 8 where the leakage rate was plotted in terms of the pressure ratio, defined as the ratio between the tunnel and plug pressures. From Figure 8 it is seen that the higher the pressure ratio (closer to one), the higher the leakage rate.

Because the plug pressure must always be more than the tunnel pressure to avoid unwanted plug slippage, we plotted the pressure ratios with respect to leakage rates; this allowed us to represent how the leakage rate increases with increased pressure ratios. The three linings followed this general trend. However, for the same pressure ratio, the vinyl lining consistently displayed the least leakage rate, followed by the concrete and neoprene liners, as illustrated in the linear trends for each material in Figure 8.

From Figures 7 and 8 it is seen that, depending on the combination of tunnel and plug pressures, the leakage rates varied from a minimum of approximately 0.2 gallons per minute (0.76 liters per minute) to a maximum of approximately 1.2 gallons per minute (4.54 liters per minute). These values are relatively small and were manageable by the draining and pumping system installed in the test set-up.
Table 5: Relationship between friction coefficient and leakage rate.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Average Friction Coefficient $f$</th>
<th>Average Leakage Rate (gpm)</th>
<th>Average Leakage Rate (liter/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neoprene</td>
<td>0.154</td>
<td>1.09</td>
<td>4.13</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.172</td>
<td>0.90</td>
<td>3.41</td>
</tr>
<tr>
<td>Vinyl</td>
<td>0.176</td>
<td>0.61</td>
<td>2.31</td>
</tr>
</tbody>
</table>

7 CONCLUSIONS

The reduced-scale, confined, inflatable plug, tested under three different friction surfaces, was able to withstand the backpressure applied to the end-cap of the plug and only slip when the tunnel pressure approached the plug pressure—that is, when the tunnel to plug pressure ratio approached one. Thus, monitoring of tunnel and plug pressures is an important operational aspect to ensure adequate blockage of the tunnel in the event of flooding. For the three materials used as liners of the tunnel surface, the leakage rate was relatively small and manageable by the drainage system.

The friction coefficients determined from the different sets of tests presented in this work provided guidelines on the magnitude of friction coefficients that might be useful when designing large-scale confined inflatable plugs. Based on experiments, the friction coefficients estimated at a reduced scale were smaller than those obtained when testing the plug fabric alone in a standard friction test. The leakage rate or leakage pressure may be causing the difference; further testing will be necessary to assess their influence.

ACKNOWLEDGMENTS

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REFERENCES


SPECTRA OF COMPUTED FABRIC STRESS AND DEFORMATION VALUES DUE TO A RANGE OF FICTITIOUS ELASTIC CONSTANTS OBTAINED FROM DIFFERENT ESTABLISHED DETERMINATION PROCEDURES

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Key words: Fabrics, biaxial test, material stiffness, elastic constants, structural analysis.

Summary. The aim of the present paper is to present ways how lower and upper limits for the range of possible fictitious elastic constants (tensile stiffness and Poisson’s ratio) can be determined for one fabric material on the basis of different established test and determination procedures. In the structural analysis this range of fictitious elastic constants results in a spectrum of computed stress and deformation values for one and the same structure and load case. The possible range of structural analysis results due to that variety of possible stiffness parameters is demonstrated for one exemplary membrane structure with different magnitudes of curvature.

1 INTRODUCTION

Although woven fabrics show a highly non-linear load-strain-relationship under uniaxial or biaxial tension, it is common in the daily practice, that simple elastic constants are used in the structural analysis of membrane structures. As such a set of maximal three independent elastic constants is not able to cover the complex biaxial load-strain-behaviour of fabrics, elastic constants have to be seen as fictitious stiffness parameters. However, usually biaxial tensile tests are conducted in order to determine these fictitious elastic constants from the resulting load-strain-paths. But: different existing test procedures and different determination procedures may lead to a wide range of values for the resulting fictitious elastic constants. Furthermore, different interpretations of the established procedures are common.

In the recent past it could be demonstrated that the material behaviour is of great importance for the structural analysis of woven fabrics, especially with regard to the nowadays more and more minimally curved or even flat structures. The present contribution will give a state of the art report of the currently published test and determination procedures used for the characterization of the material behaviour of woven fabrics. The aim is to present a way how lower and upper limits for the range of possible fictitious elastic constants can be determined for one fabric material based on test data obtained from the different established test and determination procedures. In the structural analysis this range of fictitious elastic constants results in a spectrum of computed stress and deformation values for one and the
same structure. For one exemplary basic form of a tensile structure – a simple hypar – the spectrum of computed stresses and deformations, which can possibly occur for a Glass/PTFE-material in the design practice, will be determined by means of the obtained ranges of fictitious elastic constants.

2 THE ORTHOTROPIC LINEAR-ELASTIC CONSTITUTIVE LAW

For the use in a structural analysis where the membrane is modeled as a continuum – and not as a cable net –, the actual anisotropic highly nonlinear stress-strain-behaviour of woven fabrics is considered as a linear-elastic orthogonal anisotropic plane-stress structure. For practical reasons, nowadays the design engineers are forced to use this assumption in commercial as well as inhouse design software, although it is known that this procedure is a rather rough approximation. One possible mathematical formulation for the load-strain-relationship – known from classical mechanics – is given with the following elementary equations

\[ \varepsilon_x = \frac{n_x}{E_x t} - \nu_{yx} \frac{n_y}{E_y t} \]  
\[ \varepsilon_y = \frac{n_y}{E_y t} - \nu_{xy} \frac{n_x}{E_x t} \] 

Herein, \( \varepsilon \) are the strains [-] and \( n \) are the loads [kN/m], which are often called stresses in membrane structure analysis. The four elastic constants are: \( E_x t \) as the tensile stiffness in warp direction [kN/m] and \( E_y t \) in fill direction, respectively. Generally, the axes \( x \) and \( y \) refer to the warp and the weft (fill) yarn direction of the fabric. The transverse strains are taken into account by the Poisson's ratio \( \nu \). \( \nu_{xy} \) is the Poisson’s ratio in \( x \)-direction caused by a load in \( y \)-direction, \( \nu_{yx} \) applies analogue in perpendicular direction. Transposed to the loads \( n \) and written with matrices this law becomes

\[ \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \frac{1}{1-\nu_{xy} \cdot \nu_{yx}} \begin{bmatrix} E_x t & \nu_{xy} \cdot E_x t \\ \nu_{yx} \cdot E_y t & E_y t \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} \] 

The linking matrix between the loads on the left side and the strains on the right side of the equation is the stiffness matrix. The stiffness matrix has to be symmetric, what directly leads to

\[ \nu_{xy} \cdot E_x t = \nu_{yx} \cdot E_y t \Rightarrow \nu_{yx} = \nu_{xy} \cdot \frac{E_y t}{E_y t} \] 

It can be seen from eq. (4) that only three of the four elastic constants are independent of each other. Furthermore, the stiffness matrix has to be positive definite, which means that the tensile stiffnesses and the determinante of the stiffness matrix have to be positive. The latter constraint leads to

\[ \nu_{xy} \cdot \nu_{yx} < 1. \]
Another possible formulation of the constitutive law can be given by

\[
\begin{bmatrix}
  n_{11} \\
  n_{22}
\end{bmatrix} = \begin{bmatrix}
  E_{1111} & E_{1122} \\
  E_{1122} & E_{2222}
\end{bmatrix} \begin{bmatrix}
  \varepsilon_{11} \\
  \varepsilon_{22}
\end{bmatrix}.
\] (6)

Herein \(E_{1111}\) and \(E_{2222}\) are the tensile stiffnesses in warp and weft direction, respectively, and \(E_{1122}\) is the stiffness interaction between warp and weft direction. In the notation of eq. (6) the stiffness matrix is directly symmetric. The two mathematical formulations in eq. (3) and (6) of the same constitutive law result in identical analysis results. Special attention has to be paid, as the numerical values of the elastic constants of both definitions are not equal\(^6\). But the elastic constants of both definitions can easily be converted by the following equations:

\[
v_{xy} = \frac{E_{1122}}{E_{1111}},
\] (7)

\[
v_{yx} = \frac{E_{1122}}{E_{2222}},
\] (8)

\[
E_{xt} = E_{1111} \cdot (1 - ν_{xy} \cdot ν_{yx}),
\] (9)

\[
E_{yt} = E_{2222} \cdot (1 - ν_{xy} \cdot ν_{yx}).
\] (10)

### 3 Biaxial Tests and the Determination of Elastic Constants

Many different published and unpublished biaxial test procedures and related evaluation procedures exist today. Two common test procedures are described in the Japanese standard MSAJ/M-02-1995\(^2\) and in the TensiNet European Design Guide for Tensile Surface Structures\(^4\). Additional test procedures are described in [1,12]\(^1,12\). Furthermore, unpublished, office specific as well as project specific test procedures are oftentimes used by the engineering design offices. To every test procedure one or more related evaluation procedures exist to determine elastic constants from the biaxial test results. This situation leads to a confusing variety of elastic constants.

Selected test and evaluation procedures, based on the common recommendations of MSAJ/M-02-1995 and the TensiNet Design Guide, are introduced in the next paragraphs. The objective of both recommendations is to determine one single set of “fictitious” elastic constants from the biaxial test results which are intended to be used for the practical structural analyses of all kinds of structural forms and all load cases.

#### 3.1 The Japanese standard MSAJ/M-02-1995

The main characteristic of the Japanese standard MSAJ/M-02-1995 is that five different predefined load ratios warp:fill – 1:1, 2:1 1:2, 1:0 and 0:1 – are consecutively applied on a cross shaped test specimen with the yarns parallel to the arms of the cross. During the loading and unloading procedure the load ratio warp:fill is held constant. The maximum tensile test load is fixed to \(\frac{1}{4}\) of the maximum strip tensile strength of the material. The result of this test
procedure is a load-strain-diagram as exemplarily shown in figure 1(a). From this complete set of test data ten load-strain-paths can be extracted – one for each yarn direction for the five load ratios –, see figure 1(b).

![Load-strain-diagram](image1)

![10 Load-strain-paths extracted](image2)

(a)                                                                                       (b)

Figure 1: (a) Load-strain-diagram as a result of a MSAJ biaxial test on Glass/PTFE material, (b) ten load-strain-paths (warp/weft at five load ratios), as extracted from the diagram

The commentary of MSAJ/M-02-1995, which is an inherent component of the standard, recommends to determine one single design set of elastic constants from the extracted load-strain-paths stepwise in a double step correlation analysis. In the first step each curved loading path has to be substituted by a straight line. In the second step the slopes of the straight lines obtained in the first step have to be modified in such a way that they satisfy the equations of the assumed linear-elastic constitutive law. The MSAJ-commentary uses the formulation of eq. (1) and (2) or eq. (3), respectively. To determine the “optimum” set of elastic constants several methods are proposed, e.g. “least squares method minimizing the sum of squares of the strain term”, “least squares method minimizing the sum of squares of the stress term” and other simplified methods. The resulting sets of elastic constants differ more or less. As this procedure can not be solved “by hand”, a correlation analysis routine has been programmed at the Institute for Metal and Lightweight Structures. However, the MSAJ-commentary recommends to disregard the zero-load-paths, i.e. the weft-path at 1:0 and the warp-path at 0:1, so that only eight out of the ten extracted load-strain-paths are used for the determination of the elastic constants. According to the commentary, this is because the testing method had low repeatability of test results in the low stress range.

3.2 The TensiNet European Design Guide

The TensiNet Design Guide proposes a completely different loading procedure. An appreciable prestress is initially applied to the test specimen and is hold constant for a – undefined – period of time. After that, one direction of the cross shaped specimen (e.g. warp) is loaded while the perpendicular direction (weft) holds the constant prestress at the same time. This means, that the load ratio warp:weft changes continuously while loading and is not
constant over time like in the MSAJ-procedure. The loading procedure is repeated five times. Afterwards the procedure is inversed, i.e. the weft direction is loaded five times while the warp direction holds the constant prestress. This loading procedure shall approximate a typical loading by wind followed by a snow loading – or the other way around – in an anticlastic structure. The decision on the amount of prestress and the maximum test load is left to the design engineer in each case – although some recommendations are given.

For the determination of elastic constants the constitutive law as stated in eq. (6) is used. In each loading direction, one of the five load steps is chosen to read out the corresponding strain differences $\Delta \varepsilon$ in each fabric direction, see figure 2, firstly for the loading interval $\Delta n_{11}$ (where $\Delta n_{22}=0$) and secondly for the loading interval $\Delta n_{22}$ (where $\Delta n_{11}=0$). On the basis of eq. (6), hereewith the following eqs. (11) and (12) can be filled for the first part and eqs. (13) as well as (14) for the second part of the loading procedure:

$\Delta n_{11} = E_{1111}\Delta \varepsilon_{11} + E_{1122}\Delta \varepsilon_{22}$

$0 = E_{1122}\Delta \varepsilon_{11} + E_{2222}\Delta \varepsilon_{22}$

$0 = E_{1111}\Delta \varepsilon_{11} + E_{1122}\Delta \varepsilon_{22}$

$\Delta n_{22} = E_{1122}\Delta \varepsilon_{11} + E_{2222}\Delta \varepsilon_{22}$

As the result, four equations with the three unknown elastic constants are established. A usual practical approach to solve this – mathematically unsolvable – problem is to determine two sets of elastic constants and average the results.

4 SPECTRUM OF FICTIONAL DESIGN STIFFNESS PARAMETERS

Recently a discussion on the determination of elastic constants had been started and modifications of existing evaluation procedures have been proposed, resulting in a great
A variety of values for the “fictitious” elastic constants. In this paragraph, some selected sets of elastic constants are presented, which can be obtained for one and the same exemplary material. For this purpose, a Glass/PTFE-material with a tensile strength of 140/120 kN/m in warp and weft direction has been tested in the Essen Laboratory for Lightweight Structures (ELLF). On the basis of these tests, different sets of elastic constants have been determined at the Institute for Metal and Lightweight Structures (IML) at the University of Duisburg-Essen. All test specimens were taken from one batch. The results are given in tables 1 and 2, starting with the set of elastic constant obtained from the original MSAJ-test and determination procedure, see determination option (DO) 1.

**Table 1:** Different sets of elastic constants obtained by different determination options from one and the same set of MSAJ-test data for a Glass/PTFE-material with a tensile strength of 140/120 kN/m

<table>
<thead>
<tr>
<th>Determination option (DO)</th>
<th>Tensile stiffness [kN/m]</th>
<th>Poisson’s ratio [-]</th>
<th>( \nu_{xy} )</th>
<th>( \nu_{yx} )</th>
<th>( \nu_{xy}\nu_{yx} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original MSAJ-determination: 8 load-strain-paths evaluated (zero-load-paths omitted)</td>
<td>1300</td>
<td>770</td>
<td>0.55</td>
<td>0.93</td>
<td>0.51 (&lt; 1)</td>
</tr>
<tr>
<td>MSAJ modified: All ten load-strain-paths evaluated (Bridgens&amp;Gosling)(^8)</td>
<td>930 (752)*</td>
<td>590 (611)*</td>
<td>0.82 (0.88)*</td>
<td>1.29 (1.08)*</td>
<td>1.06 (&gt; 1) (0.95 (&lt; 1))</td>
</tr>
<tr>
<td>Particular for plane structure: MSAJ-load-ratios 1:1, 2:1, 4 load-strain-paths (IML, Univ. of Duisburg-Essen)(^7)</td>
<td>1920</td>
<td>1020</td>
<td>0.42</td>
<td>0.79</td>
<td>0.33 (&lt; 1)</td>
</tr>
<tr>
<td>Particular for anticlastic structure and load case with warp stressing: MSAJ-load-ratios 1:0, 2:1, 4 load-strain-paths (IML, Univ. of Duisburg-Essen)(^7)</td>
<td>890</td>
<td>240</td>
<td>0.49</td>
<td>1.82</td>
<td>0.89 (&lt; 1)</td>
</tr>
<tr>
<td>min/max</td>
<td>890/1920</td>
<td>240/1020</td>
<td>0.42/0.82</td>
<td>0.79/1.82</td>
<td>0.33/1.06</td>
</tr>
</tbody>
</table>

\(*\) Values in brackets are directly taken from literature\(^8\). These values were determined by Bridgens & Gosling on the basis of biaxial tests conducted by themselves on the same material but from another batch.

Bridgens&Gosling\(^8\) have emphasized, that the zero-load-paths of the load ratios 1:0 and 0:1 – which are omitted in the MSAJ determination procedure, see above – are highly relevant for the critical design case of anticlastic membrane structures. Based on the biaxial test procedure of the MSAJ/M-02-1995 they have discussed results, which were obtained when taking the zero-load-paths into account. Due to mathematical reasons, the tensile stiffnesses decrease and the Poisson’s ratios increase compared to the original MSAJ procedure (omitting the zero-load-paths), see DO 2. In the present determination, the product of the Poisson’s ratios exceeds 1.0 and therefore, this set of elastic constants cannot be used in a structural analysis. Due to that the exemplary analysis in paragraph 5 will be conducted using the values in the brackets for DO2.

As for the load ratios 1:0 and 0:1 a good correlation between measured load-strain-paths...
and calculated straight lines can (for Glass/PTFE-materials) only be obtained with big values for the Poisson’s ratio ($\nu > 1$) while for other load ratios (1:1, 2:1) considerable smaller values are required (e.g. $\nu < 0.5$), it is impossible to model all load-strain-paths with only one single set of elastic constants. This problem can be solved if the elastic constants are determined particularly for a specific structure and a specific load case. This means, e.g. for an anticlastic membrane structure with predominant warp stressing under one load case, that the load ratios 2:1 and 1:0 might be reasonable. In this case, the load ratios 1:2 and 0:1 have to be picked out for opposite loading. For plane and synclastic structures as well as anticlastic structures with very small curvature, the load ratios 1:1 and 2:1 fit best. For the exemplary analysed structures in paragraph 5 this proposal leads to the set of elastic constants shown under DO 3 for the plane structure and DO 4 for the two anticlastic structures.

To determine elastic constants according to the TensiNet Design Guide, two tests have been conducted in the Essen Laboratory for Lightweight Structures. To enable a direct comparability to the MSAJ-procedure, firstly, a biaxial test with the same maximum tensile load of 30 kN/m as for the MSAJ-test has been chosen. The biaxial test has been conducted with material from the same batch as for the MSAJ-test. The prestress has been chosen to be 2 kN/m in each fabric direction so that it equals the minimum load of the load-strain-path on which the MSAJ-determination is based on. This value is fixed by the MSAJ-commentary to be 2 kN/m for Glass/PTFE-materials. Elastic constants have been determined with the second loading cycle. The results are shown in table 2 for both: as defined in eq. (6) and for a better comparability as defined in eq. (3), too. The determination results reveal much bigger elastic constants than those obtained by the MSAJ procedure, for the tensile stiffness as well as for the Poisson’ ratios. The latter ones give a product $\nu_{xy} \nu_{yx} > 1$, which means, that this set of constants is unfeasible for a structural analysis, see explanations above.

In the second test, the maximum test load range was much smaller. Oriented towards the expected maximum membrane stress, a value of 13 kN/m was chosen. The prestress has been chosen to be 3 kN/m in each fabric direction as supposed to be in the exemplary structure analysed in paragraph 5. From the results in table 2 it can be seen that all values of the elastic constants decrease compared to the first test procedure. However, the product of the Poisson’s ratios still clearly exceeds 1.0.

<table>
<thead>
<tr>
<th>Test N°</th>
<th>Elastic constants according to eq. (6) [kN/m]</th>
<th>Elastic constants according to eq. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{111}$</td>
<td>$E_{222}$</td>
</tr>
<tr>
<td>1</td>
<td>-1180</td>
<td>-640</td>
</tr>
<tr>
<td>2</td>
<td>-3015</td>
<td>-1190</td>
</tr>
</tbody>
</table>

This problem occurs especially for Glass/PTFE-materials, which show considerable transverse strains, and especially for very big and very small values of load ratios warp:weft,
where the transverse strains show big absolute values. In the second conducted test the load ratio at maximum test load was $13:3 = 4.3$ (and $3:13 = 0.23$, respectively). For the analysed Glass/PTFE-material, the test and determination procedure of the TensiNet Design Guide leads even for that relatively low load differences between warp and weft to very high values of Poisson’s ratios. Similarly high Poisson’s ratios could be obtained from MSAJ test results, if tried to determine one set of elastic constants from the four load-strain-paths of the load ratios $1:0$ and $0:1$. It can be realized from this comparison, that it is difficult and probably quite often inappropriate to try to cover all loading situations (wind and snow) of a membrane structure with only one single set of elastic constants.

This enormous spectrum of elastic constants could be used by design engineers for one and the same material – excluding the unfeasible ones (table 1 DO2 and table 2) of course. Consequently, the question arises whether this spectrum of elastic constants has a significant influence on the stress and deformation results in the structural analysis or whether the influence is negligible. This question shall be answered in the following paragraph.

5 INFLUENCE OF THE FICTIONTIOUS STIFFNESS PARAMETER SPECTRUM ON THE STRUCTURAL ANALYSIS RESULTS

The quantitative influence of the spectrum of fictitious elastic constants obtained in paragraph 4 is exemplarily examined by means of a $10 \times 10 \text{m}$ square hypar with two high points and two low points (a saddle shaped example is given by the authors, too\textsuperscript{10}). The edges are fixed. Prestress is chosen to be isotropic with $p = 3.0 \text{kN/m}$ in the main anisotropic fabric directions. The shear modulus is supposed to be $G = 50 \text{kN/m}$. The structural analysis is conducted with the finite element software package SOFiSTiK 2012\textsuperscript{11} applying a third order analysis. The structure is vertical loaded downwards with $q = 0.60 \text{kN/m}^2$.

Three different curvatures are analysed, from $h = 0 \text{m}$ (plane structure) up to $h = 4 \text{m}$, see figures 3 and 4. The warp direction is running between the high points, so that for the curved variations of the structure the warp direction is stressed for a downward load while in the weft direction the prestress decreases. Load ratios of approximately $4:1$ and greater occur in the center of the structure. Thus, the four measured load-strain-paths of the MSAJ load ratios $1:0$ and $2:1$ are picked out to determine the elastic constants for DO 4 in table 1. The plane variation of the structure is characterized by load ratios between $1:1$ and $2:1$. Correspondent to that, elastic constants for DO 3 are determined based on the four load-strain-paths of these two load ratios.

Figure 4 shows the resulting membrane warp stress $n_w$ as the result of the structural analyses for the three sets of elastic constants of DO 1 to DO 3 (DO3 is replaced by DO 4 for the curved structures, respectively) as warp stress ($n_w$)-Poisson’s ratio ($\nu_{xy}$)-diagrams. The stress value is always given for the middle of the membrane, although the maximum stress occurs sometimes at other locations. The $n_w$-$\nu_{xy}$-diagrams emphasize the importance of the Poisson’s ratio. The Poisson’s ratio $\nu_{xy}$ corresponding to each set of elastic constants in table 1 are marked in the diagrams.

In the plane structure, the set of constants of DO1 results in $n_w = 12.3 \text{kN/m}$ while DO2 results with $n_w = 22.5 \text{kN/m}$ in an over 80% greater stress value, although the tensile stiffness is considerably smaller. The reason can immediately be identified in the $n_w$-$\nu_{xy}$-diagram as the
influence of the high value of Poisson’s ratio \( \nu_{xy} \). In the curved structures with \( h = 2.0 \text{ m} \) and \( h = 4.0 \text{ m} \) the set of elastic constants from DO2 also results in 60%-75% greater stresses compared to the results from DO1. The results of DO3 (plane structure) and DO4 (curved structures) lay in between.

On the one hand, it can be seen from the curves closing ranks that with increasing curvature the influence of the material stiffness parameters decrease. But on the other hand, the concrete sets of elastic constants demonstrate their enormous importance, especially the high magnitudes of Poisson’s ratios. This emphasizes the role of Poisson’s ratio as part of a whole set of elastic constants. A comparison or assessment only of the tensile stiffnesses – as done sometimes – is not sufficient.

Figure 5 shows the influence of the spectrum of fictitious elastic constants on the deflection results. In the plane structure \( \text{max } f_z \) varies between 20 cm (DO2) and 39 cm (DO1 and 3), which is a variation of almost factor 2. For the curved structures, the deflections decrease considerably as expected. But the results also show a variation of 60%-70%. That means, that the deflections may possibly be underestimated by a factor of up to 2, which can lead to damages of the membrane in case of hitting the primary structure.

This exemplary structural analysis demonstrates the immense range of stress and deflection results due to a great variety of fictitious elastic constants that could be used by design engineers for one and the same material product. None of the underlying determination options is validated by static load tests on curved structural components, which means that the real stresses and deflections are left unknown to the engineer.

6 CONCLUSIONS

Nowadays, the highly nonlinear anisotropic material behaviour of woven fabrics is simplified to a linear-elastic orthotropic plane stress material in order to conduct the structural analysis. The existing variety of recommendations to determine elastic constants from biaxial test results – which was shown to result sometimes in unfeasible sets of elastic constants –
Figure 4: Maximum membrane stress $n_w$ in the middle of hypar structures with three different curvatures obtained with three different sets of elastic constants from table 1
Figure 5: Maximum deflection $f_z$ in the middle of hypar structures with three different curvatures obtained with three different sets of elastic constants from table 1.
leads to a great spectrum of values, which design engineers can possibly use for their calculations for one and the same material. It was the aim of the present contribution to demonstrate the importance of this stiffness parameter spectrum on the stress and deformation results, which were found to be considerably high. Design engineers should have this issue on mind. The development of an European design standard for membrane structures as well as a European standard for biaxial testing – in which the authors are involved – is currently under way. This, together with the related research, hopefully leads to a better understanding for the determination of elastic constants and a more unified approach in the structural analysis.

REFERENCES

TESTING OF FULL-SCALE CONFINED INFLATABLE FOR THE PROTECTION OF TUNNELS

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Key words: Confined Inflatable Structure, Flooding, Leakage Rate, and Tunnel

1 INTRODUCTION

There are approximately 337 highway tunnels and 211 rail transit tunnels in the United States; many of these tunnels are beneath bodies of water. Every day, more than 11.3 million passengers in 35 metropolitan areas and 22 states use some form of rail transit, either commuter, heavy, or light rail. It is well known that man-made or natural disasters can significantly disrupt the functionality of critical transportation infrastructure. Some examples in the United States include the 1992 Chicago freight tunnel flood; the 2003 flooding of the Midtown Tunnel in Virginia caused by Hurricane Isabel; and the 2012 flooding of New York City, when Hurricane Sandy caused seven subway tunnels under the East River to flood and remain inoperable for several days. Tunnel safety and integrity is a subject of special concern, not only because tunnels are of difficult and limited accessibility, but also because most of the potential threats (e.g. fires, flooding, or noxious substances) compromise the integrity of entire connecting system as the threat can spread along it.

Conventional emergency sealing systems are not always installed or operational during the occurrence of extraordinary events, prompting the evaluation of alternative solutions, such as inflatable plugs. An inflatable plug can seal off and protect an underground system by stopping hazards, such as smoke or flooding. Unlike floodgates, an inflatable plug is fast-deploying, relatively inexpensive, and can be quickly installed in a small space in an existing tunnel or conduit. The concept was demonstrated in 2008 in the Washington D.C. Metro system with promising results. This work describes additional full scale testing performed between late 2011 and 2012 for the development of confined inflatable structures for the protection of tunnels completed at West Virginia University.
2 PREPARATION OF THE INFLATABLE PLUG

The inflatable plug used for tests at full scale consists of a cylinder with two hemispherical end-caps. The cylinder has a diameter of 194.48 inches (4.939 meters [m]) and a length of 182.70 inches (4.641 m). Each hemispherical end-cap has a radius of 97.24 inches (2.469 m).

The membrane of the plug consists of a three-layer system comprised of an internal bladder, an intermediate fabric restraint, and an external webbing restraint. The bladder is the innermost layer of the construction and is in direct contact with the fluid used for inflation and pressurization. The function of the fabric restraint is to act as a middle layer and protect the bladder. The inner bladder and fabric restraint layers are oversized with respect of the webbing restraint in order to minimize membrane stresses generated by the internal pressure. The outermost layer is a macro fabric comprised of woven webbings designed to undertake the membrane stresses generated by the pressurization. Structurally, the outer layer is the most important while the two inner layers contribute to the watertightness and add to the mass and material volume of the plug. The macro fabric of the outer layer consists of a plain weave pattern of Vectran® webbings of 2 inches (0.05 m) in width. Two aluminum fittings are also integrated into the membrane. One functions as either air or water filling port, and the other one only as air release port. The total weight of the plug is approximately 2,100 lbs (953 kg).

An important geometric characteristic of the plug is the length of the cylindrical portion. It was selected based on friction tests run at coupon level on samples of Vectran webbing as well as on small-scale prototypes subjected to induced slippage over concrete surfaces, which can be encountered in typical tunnel sections.

To prepare the inflatable plug for the tests, the following steps were performed: a) Unconstrained inflation for integration of handling and holding mesh; b) Controlled deflation; c) Folding of the inflatable; d) Transportation and placement of folded plug inside the mockup tunnel; e) Storing of folded plug in the container; f) Connection of release mechanism and closing of container door. Figure 1 illustrates these steps.

![Figure 1: Overview of plug preparation procedure for full-scale testing: (a) Unconstrained inflation; (b) Controlled deflation; (c) Folding; (d) Transportation and placement inside tunnel; (e) Storing in the container; (f) Connection of releasing mechanism.](image-url)
3 FULL-SCALE TEST SETUP

The inflation system for full-scale tests was designed to operate with air during the initial inflation and then with water for full pressurization of the inflatable. It was designed also to provide enough water flow to simulate flooding and to recirculate water during the tests so that the entire test operation could be stabilized and measurements could be made from a self-contained water reservoir. Figure 2 shows a schematic of the inflation system showing major components and their function. The testing system consisted of a 50-foot-long (15.24 m) by 16.2-foot-diameter (4.94 m) steel structure and concrete-lined tunnel mockup specially built to replicate a typical rail tunnel section. The initial inflation and positioning of the plug required a high capacity air blower. An 85,000 gallon (321,760 liters) tank provided water for plug pressurization and flooding simulation. Three high-capacity diesel pumps were used for different functions: The water inflation pump was used to pump water from the tank to the inflated plug, replacing the air used for deployment and initial inflation; the flood simulation pump was used to fill the cavity left between the plug and the tunnel end-cap; and the water recirculation pump was used to pump leaking water collected in a dump tank, returning it to the main water tank. A smaller electrical pump and a pressure regulator were used to control the plug pressure while the tunnel pressure was regulated by changing the pumping speed of the flood simulation pump.

Figure 2: Schematic of the flooding simulation system used for full-scale tests.
4 TEST PROCEDURE

The testing procedure consisted of six major steps: 1) Deployment of the plug; 2) Inflation with air; 3) Filling of the plug with water and subsequent pressurization; 4) Tunnel flooding; 5) Stabilization of pressures; and finally, 6) Depressurization and plug removal. The plug deployment and air inflation were automatically controlled, but the water fill was done manually for both the plug and tunnel.

Step 1: Upon successful container door opening, the release system of the holding mesh was activated to deploy the plug.

Step 2: The initial deployment was followed by air inflation using a blower running at 1,800 standard cubic feet per minute (scfm). The inflation continued until reaching a nominal pressure of 0.25 pounds per square inch gauge (psig) (1.72 kilo Pascal [kPa]). When the plug was fully inflated, a constant pressure of 0.25 psig (1.72 kPa) was maintained by the control software. Visual inspection of the plug sealing was then conducted and documented through photographs of the front of the plug and remote video of the rear of the plug.

Step 3: Once visual inspection indicated proper inflation, the blower was turned off and isolated from the rest of the piping system. The main tank valve was opened, allowing water to fill the piping system. Then, the water inflation pump was turned on and the plug filling commenced. During the filling process, air contained in the plug was allowed to escape and the pressure was maintained at approximately 3 psig (20.68 kPa). As the water neared the top of the plug and the air within the plug was purged by the water, a valve installed in the air release port of the plug was adjusted to complete the removal of air. When all air was removed, the small electric pump was turned on and the pressure regulator set to maintain a 17 psig (117.21 kPa) plug pressure to ensure proper system operation.

Step 4: Tunnel fill started with the activation of the flood simulation pump. Tunnel flood pressure was maintained through the diesel throttle adjustment of the pump in order to reach and maintain a nominal pressure of 11.6 psig (79.98 kPa).

Step 5: During the tunnel fill and pressurization, the plug pressure was maintained at a nominal pressure of 17 psig (117.21 kPa) through continuous adjustment of the pressure regulator. As the dump tank water level reached the top, the water recirculation pump was cycled to remove the water accumulated within the dump tank as needed. Plug and tunnel pressures were maintained for the test duration. Measurements of leakage rates were performed during this step.

Step 6: Upon completion of the test, the flood simulation pump was turned off and the tunnel water was allowed to drain to the dump tank through natural leakage around the plug. The plug pressure was maintained at a minimum of 6 psig (41.37 kPa) differential from the tunnel pressure during this step to ensure that the plug did not move. After the tunnel was empty, the plug was depressurized and water was allowed to drain into the dump tank.
the plug was completely deflated, it was removed from the tunnel and prepared for another test.

A total of ten tests were executed. Seven of them consisted of only deployment followed by air inflation at 0.25 psig (1.72 kPa). The remaining three consisted of deployment, air inflation, plug pressurization, and flooding simulation.

5 RESULTS

5.1 Deployment

The deployment consisted of the following steps: First, the container door was fully opened; second, the releasing mechanism of the holding mesh was activated; third, the holding mesh along with the plug fell by gravity to unroll the plug. Figure 3 shows an example of the sequence of door opening and initial deployment of a surrogate plug, which was used for initial trials and adjustments of the process.

Figure 3: Sequence of container door opening and initial deployment.

5.2 Air Inflation

For the air inflation step, the blower was programmed to provide air flow depending on the stage of inflation. The inflation process consisted of three stages: 1) Initial inflation at 2,800 revolutions per minute (rpm) for approximately three minutes or until the plug pressure reached 0.25 psig (1.72 kPa); 2) Reduction of blower speed to 1400 rpm for approximately
one minute or until the plug pressure reached and stabilized at 0.25 psig (1.72 kPa); 3) Maintain blower speed at 1400 rpm in order to keep the plug pressure constant at 0.25 psig (1.72 kPa). These three stages resulted in flows of approximately 1,000 scfm, 400 scfm, and 60 scfm, respectively. Figure 4 shows the sequence of air inflation and Figure 5 illustrates the variation of blower speed and plug pressure.

![Sequence of air inflation](image)

Figure 4: Sequence of air inflation.

![Blower speed and plug pressure](image)

Figure 5: Variation of blower speed and plug pressure.

5.3 Evaluation of Conformity

The global conformity of the plug to the tunnel section was evaluated visually at the end of the initial inflation. The global conformity was considered acceptable when there were not significant distortions on the surface of plug and when the longitudinal horizontal axis of the plug was approximately parallel to the longitudinal and horizontal axis of the tunnel. In most of the tests the concentric circles of the spherical end-cap leaned slightly towards the container side, but did not affect the overall level of global conformity. Figure 6 shows examples of global conformity obtained after the initial inflation with air.
The local conformity was evaluated by visual inspection of critical locations, such as transitions and changes of angles in the perimeter of the tunnel section. Thorough visual inspection of these zones was performed. Local conformity of the plug to the tunnel perimeter was considered acceptable when there were no evident signs of material bridging, visible gaps, or local distortions. The absence of these anomalies was verified before proceeding with the flooding simulations. Close-up views of local contact after initial inflation with air are shown in Figure 7.

The quality of sealing was also tested during the three flooding simulations. During these three tests, different levels of leakage were observed. Particularly on the container side (location E in Figure 7), at the line of longitudinal attachment of the holding mesh to the tunnel, and in the proximity of the container floor, the plug did not seal well. This source of leakage was preliminary attributed to the presence of bridging material that created an opening at the base of the container, allowing leakage of water as illustrated in Figure 12.
5.4 Pressurization

Once the plug was inflated and the evaluation of conformity completed, the test continued with water pressurization of the plug. The process of filling the plug consisted of pumping nearly 1,100 gallons per minute (gpm) (4,164 liters/min) of water into the plug at the same time that air was released through a snorkel pipe located inside the plug. This process took approximately 35 minutes until all air inside the plug was replaced by approximately 35,000 gallons (132,489 liters) of water. Once the plug was completely full, the water inflation pump was substituted with a smaller electric pump for fine tuning and stabilization of the plug pressure at 17 psig (117.21 kPa). An example of the initial plug pressurization process is illustrated in Figure 8. The fluctuations of pressure seen during the filling process are due to the discrete release of air executed manually in order to avoid excessive air pressure in the upper part of the plug.

When the plug pressure was stabilized, the flood simulation pump was turned on to initiate the tunnel filling process for flooding simulation. The estimated volume of the cavity between the plug and the tunnel end-cap was 12,000 gallons (45,424 liters), and filling of this cavity took approximately eight minutes at a pumping rate of approximately 1,500 gpm (5678 liters/min). Once the cavity was full, the same pump was used to stabilize and maintain the tunnel pressure at 11.6 psig (79.98 kPa). The tunnel filing and pressurization process can be seen in the initial slope of the tunnel pressures plotted in Figures 8 and 9. When both the plug and tunnel pressures reached the target values, they were maintained approximately constant for 35 to 40 minutes for evaluation of the leakage rate. Note in Figure 9 the fluctuations of the plug pressure induced by fluctuations of the tunnel pressure. These fluctuations required continuous adjustment of the plug pressure regulator to reestablish a constant plug pressure.
5.5 Evaluation of axial stability

The stability of the plug was verified by continuous monitoring of the axial movement of the plug during the tunnel pressurization. The relative axial movement was measured by a laser range meter pointing horizontally to the tip of the plug for the duration of the entire pressurization sequence. An example of the measurements is illustrated in Figure 10. Results showed that the plug practically did not move when it was subjected to the selected testing pressures. From Figure 10, it is seen that the axial displacement ranged from 0 to 0.05 inches (0 to 1.27 millimeters) during the flooding simulation. Similar results were obtained for the other two flooding tests.

![Figure 10: Plug tip horizontal displacement during flood simulation.](image)

5.6 Evaluation of leakage rate

The water leakage originated from the non-uniform local contact between the external surface of the plug and the inner surface of the tunnel. Leaking water was collected in a dump tank placed in front of the tunnel mockup. The tank was allowed to fill while an ultrasonic depth gauge measured the change of the water level. The change in the water level, along with the known volume of the dump tank, was used to estimate the leakage rate. Once the tank was full, the water recirculation pump was turned on to drain the tank until it was nearly empty. Then, the pump was shut off, allowing the tank to fill again. This process of filling and draining of the dump tank was repeated at least ten times in order to have multiple readings for computation of the leakage rate. An example of recorded data for evaluation of leakage is shown in Figure 11. A summary of leakage rates collected from three flooding simulations is presented in Table 1.

Results summarized in Table 1 show that the average leakage of all tests was approximately 568 gpm (2,150 liters/minute). During execution of the flooding tests, it was noted that the majority of the leakage came from the container side, particularly from the container floor, as seen in Figure 12. As noted previously, it is speculated that bridging created by the plug’s structural membrane in that particular region allowed water to leak. Sealing gaskets consisting of neoprene pads were added to the base of the container to improve the sealing effectiveness of the plug in that particular region. However, this solution did not reduce the amount of leakage and suggested that a different approach would have to be implemented for further tests. Despite the leakage seen in that particular region, the overall
blocking capacity of the inflatable plug was acceptable considering that it was holding approximately 12,000 gallons (45,425 liters) of water pressurized at 11.6 psig (79.98 kPa) with a manageable amount of leakage. This leakage rate was compensated with the flooding simulation pump running at a relatively low speed in order to maintain the tunnel pressure constant. Note that a typical single high-capacity diesel pump can drain a flooded area with pumping rates ranging from 2,900 to 5,000 gpm (~11,000 to ~19,000 liters/min). These results demonstrated the ability of the inflatable system to contain tunnel flooding.

![Graph showing water depth collected in the dump tank for estimation of leakage rate.](image)

Figure 11: Variation of water depth collected in the dump tank used for estimation of leakage rate.

### Table 1: Summary of leakage rates.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Avg. leakage rate [gallons/min]</th>
<th>Avg. leakage rate [liters/min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>450</td>
<td>1,703</td>
</tr>
<tr>
<td>2</td>
<td>526</td>
<td>1,991</td>
</tr>
<tr>
<td>3</td>
<td>729</td>
<td>2,759</td>
</tr>
<tr>
<td>Average</td>
<td>568</td>
<td>2,150</td>
</tr>
</tbody>
</table>

Figure 12: Flooding simulations: Test #1 (left); Test #2 (center); and Test #3 (right).
6 CONCLUSIONS

The preparation work, consisting of folding and packing procedures, allowed the installation of the folded plug well within the available volume of a storage container located against the tunnel sidewall. The restraining mesh, along with the folding procedure, reduced the packing volume so the plug can be accommodated in the interior of a container that can fit in a typical tunnel section.

Plug deployment and initial inflation at a low pressure can be achieved in approximately 3 minutes and pressurization with water can be achieved in approximately 35 minutes with the system configuration used in the testing.

The inflatable plug was able to withstand the selected testing inflation pressure, as well as the external tunnel pressure, originated by the flooding simulation, without slipping. That is, the plug was able to seal effectively a tunnel section in the event of flooding.

The amount of leakage measured during the tests is not negligible but manageable with standard diesel pumps. Improvement of local contact is necessary at the transitions zones of the container in order to reduce the amount of leakage coming from that particular zone.

ACKNOWLEDGMENTS

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FORMFINDING AND STATICAL ANALYSIS OF CABLE NETS WITH FLEXIBLE COVERS

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Key words: Equidistant rectangular nets, Assembly plan, Boundary patterning.

ABSTRACT

Nowadays cable nets are built as roofs, enclosures for animals in zoos, facades, safety barriers in buildings, etc. Some of the advantages are high resistance against damage and high transparency.

In the first part of this paper the problems of the cable net calculation are described. The key problem is the generation of the assembly plan. This procedure is according to the cutting pattern generation process. In case of cable nets we have to create as big as possible fields. The reason for this purpose is the boundary patterning, this means: the patterns are restricted to the region along the boundary cables and equidistant rectangular meshes remain in the inner part of the fields.

The calculation of the formfinding and the statical analysis of equidistant rectangular nets will be pointed out step by step. We also show the specific characteristic of small sized 60°-degree cable nets, produced e. g. by the German company Carl Stahl. This kind of cable net is characterized by a high flexibility and the adaptability to free forms. By the so called S-twist in the cable pieces between the clamps we have to consider special statical properties. The usage of physical nonlinear material laws and the node stiffness caused by the clamps will be explained.

As opposed to the calculation of cable nets we have to consider additional material properties as shear- and crimp- stiffness for membranes and foils. The cutting pattern generation can be performed here as usual with surface seams.

The disadvantage of cable nets is that an additional cover is needed if the usage requires a weather protection. In the last chapter we will show the calculation of the composite material cable net – membrane cover.
1 INTRODUCTION

Because of the simple fabrication and assembly, cable nets have equidistant meshes, in other words, they have constant node distances. Due to the fact that the calculation of the figure of equilibrium with force densities results always in irregular node distances, standard membrane calculation steps have to be used in a modified way. The flattening and remeshing of a pre-stressed 3d geometry (calculated by formfinding) is the first task in cable net calculation.

2 FLATTENING AND REMESHING USING KNOWN MATERIAL LAWS

After the formfinding calculation 3d shapes exist as triangle or polyline surface representations. The forms can be flattened by different theories (also if the formfinding process was calculated by using quadrangular meshes, as is usual for steel meshes). A new developed software module (EASY boundary mapping) maps the complete 3d surface under consideration of the following boundary conditions:

1. Equal area under consideration of minimal distortions energy. The area of the mapped surface in 2d is equal to the surface area in 3d.
2. Equal boundary distance under consideration of minimal distortions energy. The circumference of the mapped surface is equal to the circumference of the 3d surface.

For both procedures additional conditions can be introduced (alignment, specific angles, etc.). After the mapping process the boundary lines are extracted and prepared for the remeshing process automatically. The remeshing is performed under consideration of the given meshing parameters (mesh size, mesh angle, mesh direction, etc.)
The cable net characteristic of the steel mesh can be modelled during the remeshing process and additional adjustment elements for the simulation of complete meshes are introduced. With the help of the remeshed net a new formfinding and statical analysis under prestress and external loads can be performed. Initially a mesh model without additional node rigidity is used (mesh model 1).

3 SHAPE DETERMINATION AND STATICAL ANALYSIS – MESH MODEL 1

Based on the flat model generation (see 2) and linear material laws the cable net can now be hooked in the control point frame and the stresses can be visualized. The following calculation steps have to be performed:

1. Fast model generation (see 2).
2. Linear formfinding in the control frame (for the calculation of initial values of the nonlinear calculation).
3. Definition of force density controlled adjustment elements.
4. Assignment of linear material parameter.
5. Generation of the figure of equilibrium under prestress (nonlinear process).
6. Stress and deformation calculation under prestress and external loads (nonlinear processes).

For the static calculation a characteristic line, determined by the KIT (Karlsruhe Institute of Technology - Fachgebiet Bautechnologie), of the X-TEND steel mesh from CARL Stahl (diameter 1.5 mm, mesh size 60 mm) was used.

The so called S-twist between the nodes was modelled by the nonlinear characteristic curve of the cable. Therefore nonlinear material properties of a defined cable net were introduced and calculated under different loads. In the case of nonlinear material properties the process is equivalent to the points 1-6 (see 3) except point 4 is replaced by “assignment of point-wise defined nonlinear characteristic curves”.

Figure 2: Stainless steel cable mesh X-TEND from Carl Stahl
4 SHAPE DETERMINATION AND STATICAL ANALYSIS – MESH MODEL 2

In mesh model 1 the cable nodes were mathematically assumed as fully hinged. In a next step (mesh model 2) we extended the model by introducing the stiffness of the connection. For the simulation of the node stiffness we examined 2 approaches:

1. Bending stiff beam elements.
2. Springs between the cable net meshes.

We tested and compared both possibilities. Due to the fact that both options lead to almost identical results and the easy usage we implemented Option 2 in our software.

The functional model was extended by springs. The node resistance value can be set individually between 2 cable directions. By doing this a realistic calculation of cable nets is available to users. This is based on the prerequisite that spring stiffness values and nonlinear characteristic curves for the cables with S-twist exist.
5 ADDITIONAL MATERIAL PROPERTIES FOR THE STATICAL CALCULATION OF MEMBRANES AND FOILS

A more precise description of the material behaviour is possible by introducing the shear-stiffness and also the so-called crimp-stiffness, which steers the interaction between warp (u) and weft (v). The extended model allows the statical calculation of membrane and foil structures with transverse extension and shear.

In formula (1) warp- (u) and weft direction (v) are dependent or correlated and shear is active.

\[
\begin{bmatrix}
\sigma_u \\
\sigma_v \\
\tau
\end{bmatrix}
= \begin{bmatrix}
m_{11} & m_{12} & 0 \\
m_{21} & m_{22} & 0 \\
\text{sym.} & \text{sym.} & m_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_u \\
\varepsilon_v \\
\Delta \alpha
\end{bmatrix}
= \mathbf{M} \cdot \varepsilon
\] (1)

\(\sigma_u\): stress in warp direction
\(\sigma_v\): stress in weft direction
\(\tau\): shear stress
\(m_{12}\): crimp module
\(m_{11}\): modulus of elasticity for warp
\(m_{22}\): modulus of elasticity for weft
\(m_{33}\): shear stiffness
\(\varepsilon_u\): strain in warp
\(\varepsilon_v\): strain in weft
\(\Delta \alpha\): shear deformation

6 MEMBRANE PREPARATION FOR THE INTEGRATED STATICAL CALCULATION

In order to be able to calculate a cable net in combination with a membrane the calculation model has to connect both of them. We considered 2 possibilities:

1. The connection is made topologically, means with identical points on the membrane and the cable net.

2. A much more general solution was also implemented: The membrane is connected with the cable net on arbitrary points. Thereby the “connected” membrane point will be enslaved with 3 adjacent cable net points. The enslavement is achieved by retaining the natural coordinates during deformation.
Figure 4: Membrane point 14000429 enslaved on 3 cable net points.

The single working steps for the connection of the cable net and membrane will be listed below:

1. 3d form generation of the cable net.
2. Cutting pattern generation of the membrane related to the cable net geometry.
3. Membrane points triangle enslavement to cable net.
5. Cable net - membrane model generation by using identical boundary points and triangle enslaved seam points.

Figure 5: Triangle enslaved seam points
7 MODEL DEVELOPMENT FOR THE INTEGRATED STATIC CALCULATION

We included the triangle enslaved points in the static programs whereby the calculation of connected models became possible. The connected models are describing the cable net and the membrane in a realistic way. By that we mean that mechanical characteristics of the membrane and cable net can be set in a correct way. Different membrane pre-stress conditions referring to a given cable net can be simulated. If the cable net changes, the process (chapter 6) has to be repeated (to generate new membrane cutting patterns). Particularly the application of external loads (wind, snow, etc.) on the models (chapter 6) and the contact problem of the triangle enslaved points were performed successfully.

The solution strategy for the contact problem can be described as follows: The “flexible” membrane point is connected to the “stiff” steel point case-by-case. That means: If a steel point is compressed by a membrane point contact exists, if a steel point resists tension forces, contact breaks down and the membrane point releases from the steel point. Thus the membrane point is free and no longer triangle enslaved. Compression and tension means in this context: force components are acting into or away from the triangle plane.
8 CONCLUSIONS

The results are based on a research project which was sponsored by the German Federal Ministry of Economics and Technology (BMWI). The Karlsruhe Institute of Technology (KIT) was our partner in this project. The project had some influence in the software package EASY from technet GmbH and the new modules have been proven successfully in practice.

REFERENCES

YARN LEVEL MODELLING OF TEXTILES FOR MEMBRANES

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Key words: Modeling, Yarn Level, Textiles, Woven, Knitted, Structures.

Summary. This document provides information and instructions for preparing a Full Paper to be included in the Proceedings of STRUCTURAL MEMBRANES 2013 Conference.

1 INTRODUCTION

The behaviour and the properties of the textile composites and membranes are determined from three groups of properties – the yarn properties, the matrix (coating, resin) properties and the interlacements of the yarns. The fourth group of influences is the most difficult to be defined and described – it is the combination of these three properties, which explain the type of the interactions between yarns and matrix during deformation.

The yarn and the matrix properties can be tested using standardized testing methods. The adhesion between the yarns and the matrix can be, even with more difficulties, as well investigated and defined experimentally. This work concentrates on the methods for the analysis of the interlacements of the yarns – the definition of the yarn geometry of the textile at meso level, making accent on the numerical models for the geometry generation.

The yarn geometry can be identified as well experimentally, using MicroCT or other image analysing techniques. This allow its re-building and creation of FEM and homogenisaiton models, but is very cost and time intensive method. Even being the most exact method for the preparation of the geometry, it has one significant disadvantage – no parametric studies and optimisations procedures can be started with geometry, generated in this way.

The numerical models do not have this disadvantage, but the yarn geometries, created using these methods differ from the original geometry. These differences are larger for the
geometrical models, which use some parametric description of the yarn geometry\textsuperscript{2}. The newest models minimize these differences and become more accurate using computational mechanics algorithms to find the relaxed state of the structure after the production process. The accuracy of the models has two levels – accuracy of the yarn axis representation and accuracy of the yarn cross section representation. The simpler models, which use constant yarn cross section, can achieve good accuracy, because even if the axis is computed mechanically correct, the local cross section deformations cannot be taken into account.

This paper presents a short overview of the different types of textile structures and some important points for the yarn level modelling of these structures.

2 STRUCTURE TYPES

The textile structures can be classified into several groups, depending on various criteria. For the modelling purpose it is important first to determine if the structure consists of yarns (woven, knitted, braided) or fibers (nonwoven fleece for instance in short fiber reinforcements). The modelling methods for fiber based structures differs significantly from these for yarn based, because in their case the most important property is the homogeneity of the fiber distribution.

2.1 Fiber based structures (nonwovens and yarns)

In the nonwoven structures, build on staple (short) fibers (Figure 1a) the fiber length distribution has to be taken into account. In the structures with endless filaments the orientation of the filaments is as well connected with some probability. In both these cases the models require application of statistical methods. Examples of modelling such fiber based assemblies and the interaction between the fibers and the fluids around these be found in the works of Fraunhofer ITWM\textsuperscript{3,4}. The models for real time simulations are identical with the hair simulation methods, developed for instance by Miralab\textsuperscript{4,5}. The combination of statistical and mechanical methods for determination of the mechanical properties of the products can be found for instance in the works of Dr. A. Rawal\textsuperscript{6,7}.

The modelling of the fiber distribution within the yarns (Figure 1.b) is another field, where the single fibers and their interactions has to be modelled. There are various papers and books in that very complex field, connecting again stochastic and mechanics, where different aspects are covered less or more, but because of the large number of influences incl. chemical treatments, machine settings etc. the task to predict the yarn properties from the fiber properties and the parameters of the spinning technology is still object of intensive research\textsuperscript{9,10,11}.
2.2 Yarn based structures

The yarn based or "secondary fibrous" structures are built of interlaced in some way yarns. These structures are the end product of the textile industry and the "raw material" for the clothing, textile reinforcements, textile membranes, medical textiles and lot of other technical applications. Because the modelling at the fiber level is more complex, usually the properties of the yarns are identified using standard measurements and then the properties and the behaviour of the textiles is modelled based on the yarn properties and the interlacement between the yarns. Depending on the interlacement type these structures can be divided into two main groups: orthogonal structures and orthogonally interlooped structures (Figure 2). The unit cells of the orthogonal structures are based on two systems of yarns, interlaced orthogonally for the woven structures (Figure 2c) or under some angle for the braided structures (Figure 2d). The knitted structures are built of yarns, connected through loops, where for the weft knitted (Figure 2b) each row consists of one (or more) yarns and for the warp knitted (Figure 2a) as many warp yarns are available so many columns are build.
The basic methods of the modelling of such structures are at the geometry level identical, but once the mechanical behaviour has to be predicted, the different behaviour of the yarns within the unit cell require the use of different or at least more powerful mechanical models. The common way for the yarn level modelling of textile structures starts with the definition of some key points, for which some relations about the yarn placement are known. In the most common case these points are the contact points between the yarns. The difference between the models starts after that – from the assumptions about the yarn cross section forms and the type and the accuracy of the approximation of the curve between the points.

3 WOVEN AND BRAIDED STRUCTURES

3.1 Wide woven structures

The woven structures used for architecture membranes, are usually "wide" woven, in the contrary to the narrow tapes, discussed in the next section. These classical woven structure are the most investigated and there are large number of methods and models for the generation of their geometry. The classical, analytical unit cell definition with different cross sections forms and using arcs or splines can be found in famous classical papers, but these models are suitable only for the basic types of structures. The state-of-the-art way of generation of the unit cell of such structures include some geometry generation and following iterative procedures for minimisation of the yarn energy

\[ E = \sum_{i=1}^{L} \int_{0}^{L} (E_b + E_r + E_c + E_t) \, ds \]  

where \( E_b, E_r, E_c \) and \( E_t \) are, respectively, the energy terms per unit length of yarn for bending, torsion, lateral compression and longitudinal tension and \( L_i \) is the length of the i-th yarn piece. Numerical procedures and graphical editor for the input of all required yarn and structure parameters is the software WiseTex, which allows the calculation of the geometry, compressibility and in-plane behaviour of the textile structure.

Because these calculations require information of more mechanical properties of the yarns then the diameter and elasticity modulus, is until know not often used in the industry. The measurement of friction coefficient, bending rigidity, lateral compression behaviour of yarns are not standard tests for the textile industry without these values the calculations can not be performed accurately. Because of this more personal and not more scientific problem, the mechanical models and the software, which use it are still seldom used and more popular are still the pure geometrical models. The pure geometrical models generate some idealized geometry of the structure based on the yarn cross section geometry, space between the yarns and weave type. Such models are included already in some of the professional CAD for woven structure simulation and are available in several packages from universities for geometry generation. One popular such is the TexGen of the Nottingham University (Figure 2c).
3.2 Narrow woven tapes

The narrow woven tapes are used in textile membranes and planes for stabilisation and for tensioning the structure. Generally the unit cells of the simple narrow tapes can be generated with the methods and the software for wide woven structures, but the experience shows that these models and software in the some cases can not represent the specific behaviour and structure of the narrow tapes\textsuperscript{17}. The narrow tapes have from both sides stable selvages, which yarns influence slightly the behaviour of the tape. The narrow tapes consists usually of several layers, which are in some cases very dense compacted, in other cases the connections between the yarns are more soft for lower bending rigidity. The calculation of the mechanical behaviour of the tapes requires exact initial geometry of the yarns, but in some cases the final position of the yarns in the tape differs significantly from the idealized geometry and in such cases currently used iterative methods for minimisation of the energy of the yarns for woven structures do not reach suitable solution. The next difficulty in the modelling is that in the most cases used are multifilament yarns without twist, which change their cross section at lowest lateral load (Figure 3). Using models with constant yarn cross section leads in this case to interpenetrations between the yarns. These interpenetrations do not influence the results of the mechanical calculations, if the numerical methods use only the yarn axis for the calculations, but it leads to significant problems, if the geometry has to be used in standard FEM or CFD software.

![Figure 3: Woven structure of multifilament yarns and its model with WiseTex. Each warp yarn changes its cross section depending on the lateral load. If the correct spaces between the yarns are given, this results to interpenetration of the yarns\textsuperscript{17}.](image)

The experimental verification of the WiseTex for tapes demonstrate good correlation for most standard samples – in the commonly used load for the investigated tape under 5% the force-elongation curves of the simulation and experiment differed slightly.
3.3 Braided tapes

The unit cells of the braided products can be generated in the same way as of the woven structures. Wisetex include generation methods and interface for preprocessing of braided unit cells as well. In the large scale simulations the braided products have to be considered as one complete product, where the "selveges" are as well important and not only the unit cell. In such case the complete braid has to be generated, as presented on the Figure 5. Because the braids are more flexible then the woven structures, during tension they show very low initial shear modulus, but after reaching some level of compaction it increases significantly.

Figure 4: Comparison between the experimental and simulated with WiseTex force-elongation curve of a woven tape.

Figure 5: Generated 3D geometries of braided tapes.
3 KNITTED STRUCTURES

The geometry generation of the knitted structures starts as well with the definition of some key (contact-) points, as at the woven structures\textsuperscript{18,12}. One example of the definition of such points is presented on the Figure 6.

![Figure 6: Basis for the yarn level geometrical model of knitted structure\textsuperscript{18}.

The knitted structures differ from the woven and braided significantly in the behaviour, because the contact zones between the yarns changes during the loading (Figure 7).

![Figure 7: Deformation of knitted loop and depending of the elasticity module from the elongation\textsuperscript{19}.

If in a woven structure the yarn elasticity is the parameter, which is the most determining
the properties, for the knitted structures the friction between the yarns and the type of the structure (pattern) are the determining influences. Because of this, the minimisation of the energy of the yarn pieces can be only used, if some suitable treatment of the friction is included\textsuperscript{20}.

More efficient for computer implementation and for the contact calculations is the explicite FEM method\textsuperscript{21}. To model such effects, an explicit FEM procedure was developed. It is based on the dynamic equilibrium of each yarn piece

\[ M \cdot \ddot{u} + D \cdot \dot{u} + K \cdot u = F \quad (2) \]

where M, D and K are the mass, damping and stiffness matrix of the system, F is the vector of the nodal forces and u is the vector of the nodal displacements with its derivatives.

Based on this method, the initially generated idealized geometry can be "relaxed" until the fabric receive normal state as it is after taking it out of the machine. Figure 8 demonstrates the result of a simulation of tensile test in horizontal direction with application of this method.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Deformation of knitted loop and depending of the elasticity module from the elongation\textsuperscript{19}.}
\end{figure}

\section{4 CONCLUSIONS}
- Short overview about the different type of textile structures and the actual methods for their modelling is presented.
- For the woven and braided structures usually the use of the geometry based methods and additional minimisation of the energy of the yarn can be used for modelling of the structure and the tensile behaviour with sufficient accuracy.
- The knitted structures are more flexible and the contact zones are changing during deformation. In such case, after the geometry generation an explicite FEM procedure is used.
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KINETIC GEIGER DOME WITH PHOTOVOLTAIC PANELS

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Summary. In this paper we describe the kinetic transformation of a Geiger dome in order to create a (large span) roof surface with photovoltaic elements that can be faced to the sun during the day. The purpose of this structure is to combine a sun tracker with an architectural building.

1 INTRODUCTION

For solar cells, such as photovoltaic panels, the angle of incidence between the incoming light and the solar cells determines the efficiency. By placing the cells on trackers the amount of energy produced can be increased significantly compared to fixed arrays of solar cells. Dual-axis tracking systems are relatively expensive and complicated compared to single-axis tracking systems, and have a relatively small benefit. The integration of kinetic sun-tracking systems in building concepts is the challenge of this research. The objective is to create as much energy as possible with limited solar cells on a kinetic roof surface. This research in kinetic domes aims to achieve the same objectives as responsive architecture as described by Tristan d’Estrée Sterk, Geoffrey Thün and Kathy Velikov. In our case the structure is not programmed by computers or responsive in a digital way. We focused on optimizing the harvesting of solar energy (day-night cycle, seasons, different azimuth and altitude) on a kinetic tensegrity dome.

1.1 Research methodology

A large part of the research was done in ateliers with Master students Building Technology (TU Eindhoven). After defining the research question, each student researched literature. They made design proposals from which the most interesting ideas were chosen or combined. Testing of crucial parts was done before the production of a 1:1 prototype. The structural and
kinetic behavior and principles of several options were researched, for example the Geigerdome and the so-called “Leonardo dome”. Different models of scale 1:20 were made and a sunlight study was done to see the relation between the path of the sun and the deformation of the dome. The students have been in a continuous iterative process of observation, induction, deduction, testing and evaluation (empirical cycle according to A.D. de Groot).

2 SOLAR ENERGY

Most buildings, and their geometries are fairly static which limits the possibilities for adaptation of the building surface to optimal energy performance. The research programs CABS and FACET show that different types of skins perform better in different seasons. If we like to achieve optimal behavior during the four seasons, during the day-night cycle and for different inhabitants (users) there is a need for adaptable kinetic skins and/or kinetic geometries.

2.1 Solar cells

We did not research the exact output and price of different systems, we only focused on the output by tracking the sun. An important aspect is the weight, as the objective of this research is to create large spans with a light-weight kinetic geometry. Including the support structure crystalline panels are 3 to 4 times as heavy as solar foils. The crystalline cells have a lifespan of approximately 25-30 years with an efficiency of 15–20%, while the amorphous foils have a lifespan of 15 years with an efficiency of 6–10%. While solar foils have an advantage in weight and form freedom, the output and lifespan is lower. Foils can be integrated in a membrane roof.

2.2 Sun trackers

The orientation of the solar cells is an important factor in its energy efficiency. The best sun-tracker therefore would be one that constantly measures the position of the sun and changes the angles of the solar cells accordingly, so that they are always perpendicular towards the sun. Tracking systems allow solar cells to follow the path of the sun. There are single-axis trackers and dual-axis trackers. With tracked solar cells it is possible to have the same output with less solar cells. This means less weight, less construction and a smaller inverter. The effect of trackers is much higher in the summer. Tracking is feasible one hour after sunrise and one hour before sunset. A misalignment of 10 degrees will reduce the output by only 2%. A bigger misalignment, however, reduces the output significantly. Solar cells with a misalignment or which are partially in the shadow deliver less power output and these cells, with a bad performance, determine the output of the whole system. A dual-axe tracked array of solar cells can achieve an extra energy output of 40-50% compared to a fixed roof array that is tilted ideally for the latitude. For example, according to the 2010 report of Adrian Catarius and Mario Christiner, “Azimuth-Altitude Dual Axis Solar Tracker”, “increases of power output can be achieved up to 43.87% for the two axes, 37.53% for the east–west, 34.43% for the vertical and 15.69% for the north–south tracking, as compared with the fixed surface inclined 32 degree to the south in Amman”. 
The word ‘tensegrity’, invented by Buckmister Fuller, is a combination of tension and integrity. The first-known three-dimensional tensegrity system is the one by Ioganson in 1920. He made a stable structure of three bars and nine strings. Because the bars did not contact each other it is a so-called first-class tensegrity system. Snelson developed the tensegrity in 1948 as a new structural typology for lightweight space structures. As first-class tensegrity structures are difficult to install and calculate they have not been used very often as structural elements. Snelson made many structures as art objects, for instance the 1948 “free ride home” sculpture in New York and the needle tower in the Netherlands. Tensegrity or tensegrity-like structures have been used in architecture for circular roofs. The first dome was designed by Fuller and is called after him. The domes have a triangle deviation of tensile strings and vertical bars within a circular compression ring. Geiger improved Fuller’s design by changing the triangular grid into a rectangular grid. In the Geiger dome loads are carried from a central tension ring through a series of radial ring cables, tension hoops and intermediate diagonals.

In 1960, Snelson designed a tensegrity structure with a tension and compression form similar to woven fabric. Between 1998 and 2000 Motro et al. made this experimental double-layered grid of about 80 square meters with a weight of 12 kg per square meter. Their challenge was to prove that this structure can be built as easy as a regular space frame (see Fig. 3). By varying the length of the strings or rods Geiger domes or other tensegrity structures can be made kinetic, like the movable mast created by Frei Otto in 1976.

4 DEVELOPING TENSEGRITY SYSTEMS

The roof of the Geiger dome is covered with membranes. The membranes are not meant to influence the tensegrity structure of the dome although this might be possible. Within a tensegrity structure it is possible to replace elements or to combine the tensegrity system with:

(i) inflatable membrane;
(ii) mechanically pre-stress membrane; and
(iii) doubly curved surface (shell).

The first option (i) is the combination of an inflatable with a tensegrity. This combination can be divided into three types:
(a) an inflatable membrane with an air-supported outer surface;
(b) an inflatable membrane supports bars against buckling; and
(c) an inflatable substitutes the bars.

In the first type (a) the bars are replaced by the overpressure in the inflatable like in air mattresses, for instance a distance fabric (Fig. 4). If the strings form a 3D space structure such as in the inflatable cloud (Fig. 5) by Pronk, Lindner and students, the air mattress will be much stiffer. In the second type (b) the bars are supported by the surface of the inflated membrane against buckling. This typology is applied for the first time for a military bridge in 1965. In 2004, Pedretti called the structure “tensairity” and improved the structure by replacing the surface of the bridge by a bar within a seam of the membrane. Luchsinger researched the working of this typology in depth. In the third type (c) the bars are replaced by inflatables. Koops studied in a Master thesis at the TU/e the replacement of bars by inflatables of a Geiger dome. Pronk and Luchsinger researched the replacement of bars by inflatables of a tensairity.

The second option (ii) is the combination of a tensegrity with a mechanically pre-stressed membrane. The replacement of strings by membranes was used for the first time in the rigid zeppelins by F. Zeppelin. The main rings of the zeppelin (Fig. 6) have been braced with strings against buckling in the radial direction. In the axonal direction the zeppelin structure is not braced with strings (Fig. 7) but covered with a membrane. As the bars are too slender and will buckle, the membrane must have fulfilled the bracing in the surface of the zeppelin similar to the structural support of the bars in the membrane surface of a tensairity.

Fig. 4. distance fabric Fig 5. Inflatable cloud
Fig. 6. Rings of Zeppelin Fig. 7. Covering structure of Zeppelin with fabric
Maritza researched the replacement of all the strings of a first-class tensegrity by membranes as shown in the models in Fig 8. She also searched for applications of this typology by designing and engineering a tensegrity dome structure. The third option (iii) is to replace a bar and some strings by a hyperbolic shell surface (Fig. 8). The cable net with red borderlines and bar (red arrow) (Fig. 9) can be replaced by a doubly curved surface (Fig. 10). Students of TU/e researched the application of this typology by designing a second-skin façade for the rehabilitation of buildings.

Fig. 8. Tensegrity with membrane

Fig. 9. Tensegrity with membrane in Berlin
Fig. 10. Tensegrity with doubly curved glass for second-skin facade

5 KINETIC GEIGER DOMES

Kinetic deformation can be used to create a sun-tracking roof surface based on the Geiger dome typology. By using a combination of flexible and rigid components it is possible to transform the overall shape of the dome, so that, for example, the optimal sunlight radiation of the solar cells can be acquired (see Fig. 11).

Fig. 11. Dome tracking a large part to the sun
In Fig. 11 the section of a regular Geiger dome is given. The bars (c) are vertical and the other parts are strings. The strings in (d) will form a regular hoop. The membrane surface is not structural and placed at the outside over the strings in (a). By changing the length of the strings it is possible to make different configurations as shown in the pictures above. It is also possible to vary the length of the bars. In this way Geiger domes or other tensegrity structures, like the movable mast created by Frei Otto in 1976,18 can be made kinetic. This mechanism is used to deform a dome based on the Geiger dome typology. The way a Geiger dome can be made kinetic can be achieved by (1) changing length of the bars or by (2) changing the length of the strings. The change of length can result in a sliding (a) or hinging (b) movement. The combination of those parameters results in four ways to make a Geiger dome kinetic:

(1a) changing length of the bars resulting in a sliding movement of bars and strings;
(1b) changing length of the bars resulting in a hinging movement of bars and strings;
(2a) changing length of the strings resulting in a sliding movement of bars and strings; and
(2b) changing length of the strings resulting in a hinging movement of bars and strings.

Option two is generally harder to achieve and more expensive therefore we only researched option (1a) en (1b) changing the length of the strings. For both options we have designed a structure with sliding elements (1a) and with hinging elements (1b). This paper is limited to those two options. We did not research the turning of the roof surface around a vertical axis.

Fig. 12. Geigerdome in different configurations 21 Fig. 13. Deformations of Geiger dome by hinging and sliding

In the figure above a regular Geiger dome is deformed by hinging (yellow) and sliding
(blue). In the third picture both options are compared. The cantilevering angle is specified by the ratio between the length of the bars (H) and the distance between the rings (L). The higher the ratio between H/L the steeper the surface can be. The maximum angle of the roof surface is the tangent of H/L minus the sag of that ring.

With hinging it is possible to reach a higher point as the center of a regular dome. With sliding the center is only shifted and therefore hinging can give a stronger cantilever with the same ratio between H and L. But sliding might give a bigger surface turned to the preferred side. Regular Geiger domes are designed as lightweight structures, therefore striking elements like the compression bars are designed as slender as possible with a low H/L ratio and therefore a slender curvature with limited angle of the roof surface. In our case we need a maximum roof surface angle and therefore a high H/L ratio.

5.1 Kinetic tensegrity dome by means of hinging

The top and bottom net of a regular Geiger dome consists of concentric quadrangular net connected with straight bars. To introduce the forces to achieve a kinetic frame we have connected the upper and lower net with tetrahedrons. In the so-called “hex-tri-hex” configuration the upper net consists of hexagons and the lower net consists of a combination of triangles around a hexagon (Fig. 14).

The hex-tri-hex configuration was chosen because the upper hexagon grid is able to adapt deformations easily, the configuration of the lower grid is more stable and the connection between the layers with tetrahedrons is completely stable. The vertices of the tetrahedron are connected by hinges. The ground surface of the tetrahedrons of the first and third ring is made
by strings, the one of the second ring by bars. In this configuration pulling or releasing the strings will make the tetrahedrons hinging. The circular strings in the lower net will provide stability. To realize the second tension ring in a regular way we had to deform the grid slightly. We proved the working of this configuration by making a physical model with sticks and elastics.

5.2 Kinetic tensegrity dome by means of sliding

For the sliding of a Geiger dome we did not have to change the morphology of the structure. As long as the strings are tensioned the structure will adapt the changes in length. To prove the sliding option we made some physical models with paper rings, sticks, strings and elastics. The models worked surprisingly well and easy. The distance between the rings is equal. While sliding the ratio of the distance between the rings is also equal (see Fig. 17).

![Fig. 17. Concept of sliding dome for low and high sun top view of kinetic tensegrity dome](image)

Therefore the rings in the middle have a bigger movement as the outer rings. Within this context the ratio between the movement of the rings is equal to the mould of sliding rings. For example, with three rings the inner ring moves three times more as the outer ring. This is solved by varying the thickness of the spills (see Fig. 18).

![Fig. 18. Movement of the radial tension cables to achieve sliding](image)

5.3 Digital Geiger dome

For this project we used several digital programs as a supportive tool to generate geometrical and structural properties and to simulate the dynamic behavior of the dome. We
used Rhinoceros as the main program. Within Rhinoceros we used a plugin called Grasshopper, which makes it possible to program visually. Within Grasshopper we used a plugin ISS [27] to export the data to GSA. GSA is used for the FEM-analysis.28 The main focus in this research project was to produce a physical model. Structural analysis is used to require the structural properties of the elements.

Rhinoceros and Grasshopper can be used to generate parametric models. In this project, we used Grasshopper to generate the structure and simulate the kinetic Geiger dome. The geometry of the Geiger dome is exported from grasshopper to GSA with a plugin called SSI. The data from SSI is used in GSA to perform a structural analysis (FEM-analysis).29 GSA allows non-linear and buckling analysis.28 The Loads used in GSA for the dome are according the Dutch Eurocode.30 The results of GSA were used to relocate the nodes and the members of the Geiger dome in Grasshopper (equilibrium based). The maximum displacement of the nodes were up to 15 cm.

With Grasshopper we made a script to generate the physical properties of the elements of the dome, like the coordinates of the nodes, the length of the members between the nodes and the amount of nodes and members.
These physical properties are exported to an excel file, and used to order materials and for creating the physical model.

6 BUILDING THE KINETIC GEIGER DOME

The building of the first prototype was conducted in 10 weeks, after modeling the kinetic geometry in Rhino and building models scale 1:20. In the dome we developed the rigid components which were made from a combination of wood and steel (see Fig. 26). The flexible wires were adjusted with hand-powered pulleys.

![Fig. 26 Top view of the dome, model and detail](image)

After producing the parts the prototype was mounted. Due to a poor mounting plan and a relatively large weight the joints deformed. The dome missed a balance and its full erection was halted. Next steps where to reduce the weight further, make a better mounting plan and to keep the move-ability of the dome under control. Learning from the first prototype the assignment for the students for the second prototype was to design and build a kinetic Geiger dome. By changing the length of the diagonal cables while keeping the circular cables and struts the same length this kinetic Geiger dome can create a large and relatively flat surface which can be directed to the sun. Therefore the change of the shape of this prototype will enhance the output of the solar cells or can deliver the same output with less square meters of solar cells. The Geiger dome is lighter as the first prototype and as the tension rings and struts are kept as they are the geometry is stable while being kinetic. The diagonal tension cables are either pulled or slacked, while under tension. The prototype showed that the movement is realized as the model scale 1:10 predicted. The solar cells will be integrated in a membrane that can slide over the movable geometry. The angle of the cells can be between 27 and 65 degrees, which is sufficient for Portugal. The prototype of the dome was scaled down because of the available poles from the first dome prototype, a maximum span of 4 meters was possible. Next step in the research is to increase that to 10 meters. A research was done in 6 different ways to slide the cables through the struts. A production manual was made how to build the dome.

![Fig. 27. (a + b) movement of the dome, (c + d + e) details of the dome and the hand-powered pulleys.](image)
7 CONCLUSIONS

The objective was to make a kinetic geometry in order to let the solar cells follow the path of the sun. That objective was met although the solar cells were simulated and the spans of the prototypes are limited. The final kinetic Geiger dome has a span of approximately 4 meters. Next step is to make a larger span, and to research how to use electric motors instead of hand-powered pulleys to be able to control the movement better and keep the pre-tension in the geometry. Point of attention is that the forces upon the motors will increase enormously when the dome is scaled up. With a larger span a further reduction of the dead load is necessary, other materials for the struts have to be considered. The length of the cables should be exactly correct or adjustable. The sliding of a skin with solar cells needs a better design idea. And last but not least the anchoring to the ground must be redefined.

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MULTIFUNCTIONAL ADAPTIVE FAÇADE AT IBA 2013; DESIGN STUDIES FOR AN INTEGRAL ENERGY HARVESTING FAÇADE SHADING SYSTEM

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Key words: Membrane, GFRP, bending-active, adaptive

Summary: As part of the international exhibition ‘Bauausstellung’ IBA 2013 in Hamburg, Germany, architects from KVA MATx team and engineers from Knippers Helbig Advanced Engineering have developed an integral energy harvesting façade shading system for their ‘Softhouse’ project. Its overall concept includes an energy harvesting hybrid textile roof featuring flexible photovoltaics, which contributes to create a micro-climate for the building as a shading roof for the terrace and glass façade. This responsive façade is based on a textile hybrid system, using textile membranes and glass fibre reinforced plastics (GFRP) in an intricate form- and bending-active structure. This paper will discuss the multiple design studies that were undertaken to develop a system that satisfies the, at times, diametrically opposed demands from architecture, building physics, structural engineering and technical approval. Furthermore, detailed information will be given on the design specifications for using GFRP in bending-active elements and the Finite-Element simulation techniques used for the form-finding and structural analysis.

1 INTRODUCTION

The textile façade of the ‘Softhouse’ undergoes two modes of shape adaptation: in a yearly cycle, the GFRP boards on the roof top change their bending curvature and therefore adjust the PV cells to the vertical angle of the sun, while the daily east west sun tracking and daylight harvesting is achieved by twisting the vertical membrane strips in front of the façade. The membrane strips are attached to cantilevering GFRP boards acting as compound springs compensating the change in length of the membrane strip through twisting. The form-finding and simulation of the initial system as well as its shape adaptations and the performance of all positions under wind and combined snow loads set a particular challenge to the engineering of the project. The basic shape changing modes are illustrated in (Fig. 1).

The adaptive façade shading system consists of a parallel arrangement of 32 individual strips which are combined in sets of 8 per housing unit. Each strip is a textile hybrid system with a 4m x 0.6m pre-stressed form-active membrane attached to a bending-active 6m
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pultruded GFRP Board (500mm x 10mm). Flexible photovoltaic cells are attached to the upper third of the membrane, continuing to the apex of the shape-adaptive GFRP board.

2 DESIGN VARIATIONS

The two modes of shape adaptation described above can be achieved by various mechanisms. In all cases, a system had to be developed that is able to compensate the nonlinear change in length of the membrane strip due to twisting (Fig. 2).

An intricate system was developed in which a cantilevering GFRP board works as a compound spring to the attached membrane strips and therefore freely compensates the nonlinear change in strip length during twisting. On top of the roof, the cantilevering board
continuously evolves into a bending-active arch system which offers a change in rise and curvature due to the kinematics of the underlying steel structure. For the twisting of the membrane strips, various mechanical systems were simulated and compared. Fig. 3a shows a kinematic cable rigging that induces a twisting motion to the membrane strip when two diagonally opposite cables are contracted. This very simple and efficient system, however, only allowed for a twisting range of 90°. In Fig. 3b the cross bar at the bottom of the membrane strip is directly actuated by cables. Here, the partially unproportional change in cable length of the two sides had to be compensated by springs, or else necessitated two separate cable winches. For reasons of enabling a twisting range beyond 180°, a directly actuated system with a turning drive was chosen as shown in Fig. 3c.

![Figure 3: Various twist actuation mechanisms](image)

3 MATERIAL
Several types of GFRP boards were investigated in the design and pricing phase. Even though hand lamination could have been a valid alternative in terms of costs and mechanical properties on paper, comparative 3-point bending tests quickly revealed that stiffness and strength reserves of industrially pultruded GFRP boards are significantly higher and more consistent than those of hand laminated elements. For the final structure, a GFRP product with national technical approval was selected (see below).
For the membrane strips, a non-coated open weave glass fabric was chosen, which offers excellent shading properties combined with translucency (see Fig. 8 right). The absence of a stiff coating also allows for shear deformation of the fabric and therefore enables twisting without wrinkles.

3.1 Design codes for GFRP

While building structures that include FRP in their load bearing elements still need individual technical approval in Germany (ZiE), there is an emerging development in the standardisation of FRP as building products. Next to the German guidelines, e.g. BÜV-recommendation [1], the Danish based company Fiberline Composites has been granted national technical approval (abZ approval) for their pultruded GFRP products in German building projects [2]. Both consider the influence of loading duration and environmental conditions by means of safety factors. Generally, three strength and stiffness influencing factors are recognized:

Load duration
Ambient media class
Member temperature

The safety concept of BÜV and abZ is based on a general material safety factor and a set of influence coefficients, resulting in different overall safety factors for various loading scenarios. The permissible stress is generally therefore given by equation (1):

\[ \sigma_{Rd} = \frac{f_{k0,05}}{\gamma_m (A1+A2+A2)} \]  

(1)

With \( f_{k0,05} \): 5% quantile of strength, \( \gamma_m \) (partial safety factor) 1,2 (machined) or 1,5 (hand laminated) [1] and \( A_i \): influence coefficients. Within abZ, the influence factor \( A_1 \) is directly applied on the action side, while all other factors are considered on the resistance side. In general praxis, the design of bending-active structures should consider three main scenarios:

Dead load + residual-stress: long load duration, max. pos. temperature: \( \gamma_{tot} \approx 4^* \)
Wind load + residual-stress: very short load duration, mean temperature: \( \gamma_{tot} \approx 1.9^* \)
Snow load + residual-stress: mean load duration, low. neg. temperature: \( \gamma_{tot} \approx 2.3^* \)

*Average values for the ‘Softhouse’ Project from BÜV and abZ.

Note that the dead load + pre-stress scenario limits bending stress to 25% of the limit stress in the form-finding of the curved geometry.

In terms of the material stiffness, it must be noted that, especially for polyester resins, a loss of modulus has to be considered for ambient temperatures above 30°C, with a considerable influence for temperatures higher than 50°C. Here, the aforementioned guidelines introduce a secondary safety concept in which material stiffness is reduced based on equation (1) with stiffness specific influencing factors.
3.2 Breaking strength and material stiffness

For industrially produced profiles, mechanical properties are usually given based on standardised material tests. For practical reasons, the same values are given for all structural profiles with a typical longitudinal bending strength of 240MPa [3]. L-shaped profiles usually exhibit the lowest strength in the cross-sections of pultruded profiles with very small 5% quantile values due to production inaccuracies. Round bars, pipes and flat sections, on the other hand, offer the highest strength and reach bending strengths above 350MPa in all tests known to the authors. Since the profile shapes used in bending-active structures are predominantly round and flat sections, in a project it may be profitable not to rely on the standard values suggested by companies, but instead perform your own material tests. This, however, necessitates technical one time approval.

The elastic modulus, too, is usually given as a uniform value for all profile shapes which represents the lower limit of the various actual moduli. However, it was found that flat sections, as they are often used in bending-active structures, may have a lower modulus than other larger sections. This is due to the fact that all profiles must have at least two outer layers of fibre mats which have a much larger influence on the sectional properties for thin flat sections than for other cross-section types.

3 FORM-FINDING

The form-finding of the continuously shape-adaptive system is divided into several sub routines, starting with a straight GFRP board which is pulled onto its given support using the elastic cable approach [4] (Fig. 4c). Simultaneously, uniform pre-stress is assigned to the membrane strips which are coupled to the bent GFRP boards in a last form-finding step where equilibrium and stress distributions are harmonised by an equalisation routine that reiterates the equilibrium of the system without additionally applied loads. For the shading system of the ‘Softhouse’, only the support at the eaves was pre-defined in the geometry. For the other supports only the heights were defined. By attaching the cables to horizontally sliding supports, the form-finding guaranteed minimal constraining forces.

For the twisting membrane strips it was important to control symmetry and equidistance of the cross bars. This was difficult to maintain in a simultaneous form-finding with the cantilevering GFRP boards. Therefore, the membrane strips were form found separately (Fig. 4b). In a second step the membrane was coupled to an already elastically deformed GFRP board (Fig. 4c). The subsequent equalising calculation lead only to minimal change in the equilibrium position since the position of the cantilevering beam was already known from previous simultaneous form-finding investigations. In order to include the winter position of the GFRP boards with maximally bent arches in the FEM model, the kinematics of the steel structure were included to simulate the shape adaptation (Fig. 4d).
4 STRUCTURAL BEHAVIOUR

The varying structural system and shape led to a highly differentiated load simulation, adapting snow loads according to the varying degree of incline and $c_p$ pressure values for the various wind directions to the different twisting positions of the membrane, as well as the inclination of the GFRP boards on the roof. For the safety of the structure, a storm position was defined where the membrane strips are twisted 90° and therefore offer maximum stiffness due to double curvature. In the twisted position, the membrane strips are less susceptible to flagging due to continuously changing $c_p$ values along the strip. On the roof top, the winter position of fully bent GFRP boards may only be adopted at wind speeds below 12m/s. Overall, the system is characterised by highly nonlinear behaviour which excluded superposition of loads and therefore led to a very involved and time consuming structural analysis. A FEM model of the system including the supporting kinematic steel structure was built which enabled both form-finding and simulation of the system in all modes of shape adaptation (Fig. 5).
Wind tunnel tests were performed to define minimal pre-stress and maximal wind speed in the untwisted position in order to control flagging of the membrane strips (Fig. 6). It was found that 0.3kN/m pre-stress provides sufficient aerodynamic stability to the twisted membrane strip. No critical flagging was observed in the untwisted configuration for wind speeds up to 10m/s.

5 FINAL RESULT
The building was completed at the end of March 2013 for the opening of the IBA. First tests showed that twisting of the membrane worked perfectly, in which the cantilevering beams where able to adjust to the shortening of the membrane strip as predicted. In combination with the open weave non-coated glass fibre mesh membrane, the twisting of the membrane strips
does not lead to wrinkles (Fig. 7 and 8). The deflection of the cantilevering boards and the harmoniously pre-stressed membranes show that the form-found geometry and predicted behaviour match the FEM analysis.

Figure 7: Testing various modes of shape adaptation on the finished structure (Pictures by Textilbau GmbH)

Figure 8: Finished ‘Softhouse’ building with multifunctional adaptive facade at IBA Hamburg Mai 2013
6 CONCLUSIONS

While the adaptive basic system of twisting membrane strips and bending-active GFRP boards displays a high degree of structural and functional integration, its connection to the building structure was very challenging. Continuing changes to the function and design of the adaptive shading system had to be adapted to the, at times, diametrically opposed demands from architecture, building physics, structural engineering and technical approval. Overall, the project was able to prove that GFRP in the unconventional context of a hybrid bending-active system can be realised within the strict rules of German building codes and individual case approval.

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Structural Engineer: Knippers Helbig Advanced Engineering
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REFERENCES


Behavior of One – Way Concrete Slabs with Edge Beams Reinforced/Strengthened by CFRP Rods under Uniformly Distributed Load

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Key words: One-Way Concrete Slab, CRFP Rods, Uniform Load, Strengthening.

Summary: This research presents an experimental investigation on the flexural behavior of eight one-way concrete slabs with edge beams under uniformly distributed load. The models are reinforced or strengthened using CFRP rods and two control models reinforced by deformed steel bars. The dimensions of one-way slab is 1.05 m width, 1.25 m length and 0.1 m thick., while each edge beam is of length 1.25m and depth 0.2m by width 0.1m. Different reinforcement ratios were used. The models were tested under universal testing machine and supported at corners on four stiff steel columns. The models were tested up to failure to study their flexural behavior including load-deflection curves, crack patterns and mode of failure. Among the conclusions obtained, the models reinforced by CFRP rods can attain flexural strength higher than those reinforced by deformed steel bars of same amount. This increase is about (38-44%).

1. INTRODUCTION:

Fiber reinforced polymer (FRP) composites are currently used as reinforcement or strengthening for concrete structures where durability is the controlling parameter. Carbon fiber reinforced polymer (CFRP) rods reinforcement represents a suitable replacement for steel reinforcement in some concrete structural members subjected to aggressive environmental conditions that accelerate corrosion of the steel reinforcements and cause deterioration of the structures.
2. RESEARCH SIGNIFICANCE:

This paper presents the experimental results of eight one-way concrete slabs, including two RC one way slab with edge beams reinforced by CFRP bar as a main reinforcement, two RC models reinforced by steel reinforcement tested for comparison purposes, two RC models reinforced by CFRP bar as a main reinforcement and strengthened using near surface mounted with CFRP bar and two RC models reinforced by steel reinforcement and strengthened using near surface mounted with CFRP bar. The models were tested up to failure under static and repeated loading conditions. The research investigates various limit states behavior including pre-cracking behavior, cracking pattern and width, deflections, ultimate capacities and mode of failure. The behavior of concrete slabs reinforced with CFRP rods is compared with the behavior of a slab reinforced with steel reinforcements. The information obtained throughout this investigation is valuable for future field application and development of design guidelines for one-way concrete slabs reinforced with FRP rods.

3. MATERIAL PROPERTIES OF FRP RODS:

The Aslan 200/201 series provides designers the greater modulus and tensile strengths of carbon fiber in a non-metallic reinforcing bar. Aslan 200/201 can be used for both new construction and as a strengthening material for the novel technique known as "Near Surface Mounted" or NSM strengthening. With a proprietary end anchorage, the Aslan 200/201 bar can be used in un-bonded post tension or pre-stressing applications. The Aslan 200 series features a textured surface whereas the Aslan 201 series is a sand coated surface. Both versions have the same physical properties.

Table (1) contains properties of Aslan 201 FRP 6 mm diameter rebar as measured or supplied by the manufacturer.

<table>
<thead>
<tr>
<th>Bar Diameter (mm)</th>
<th>Cross Sectional Area (mm²)</th>
<th>Nominal Diameter (mm)</th>
<th>Tensile Strength (MPa)</th>
<th>Tensile Modulus of Elasticity (GPa)</th>
<th>Ultimate Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>31.67</td>
<td>6</td>
<td>2704</td>
<td>163</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Table (1) : Physical Properties of Aslan 201 CFRP Bar, (Hughes Brothers, 2010)
4. EXPERIMENTAL PROGRAM

4.1. Slab Models

The one way slab with edge beams symbols are represented as follows:

\[(C \text{ or } S)^*-N^{**}\]

C  CFRP reinforcement.
S  Steel reinforcement.
*  Number of (Ø 6mm) CFRP bars as main reinforcement.
N  Near surface mounted with CFRP bars.
** Number of (Ø 6mm) CFRP bars for strengthening.

The test slab-beam system models had a rectangular slab 1.05 m wide and 1.25 m long with 100mm thickness and for edge beams had cross section 100 mm wide by 200 mm deep with an effective depth \((d)\) of 171 mm for steel RC and FRP RC one way slab with edge beams and these dimensions are the same for all models, as shown in Figure (1). The properties of the tested models are summarized in Table(2).

![Figure (1) One Way Slab Model](image)

a. Details of model under Distributed Load

b. Edge Beam Detail
Table (2): One Way Slab Models Details

<table>
<thead>
<tr>
<th>Model</th>
<th>Main (transverse) reinforcement in slab</th>
<th>Secondary (longitudinal) reinforcement in slab</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Bottom</td>
</tr>
<tr>
<td></td>
<td>Bars</td>
<td>As, mm²</td>
</tr>
<tr>
<td>(C6)</td>
<td>5 No. 2</td>
<td>158.35</td>
</tr>
<tr>
<td>(C7)</td>
<td>5 No. 2</td>
<td>158.35</td>
</tr>
<tr>
<td>(S6)</td>
<td>5 No. 2</td>
<td>141.37</td>
</tr>
<tr>
<td>(S7)</td>
<td>5 No. 2</td>
<td>141.37</td>
</tr>
<tr>
<td>(C3-N3)</td>
<td>5 No. 2</td>
<td>158.35</td>
</tr>
<tr>
<td>(C4-N3)</td>
<td>5 No. 2</td>
<td>158.35</td>
</tr>
<tr>
<td>(S6-R5)</td>
<td>5 No. 2</td>
<td>141.37</td>
</tr>
<tr>
<td>(S7-R6)</td>
<td>5 No. 2</td>
<td>141.37</td>
</tr>
</tbody>
</table>

$f'c = 40.35 \text{ MPa}$

4.2. Instrumentation

The models were positioned in the universal testing machine and supported on four stiff steel columns at their corners and tested up to failure under uniformly distributed load, as shown in figure (2).

To accomplish the required boundary conditions the following setup has been used:

Rigid steel supporting frame is designed as a supporting system and placed on the top face of the testing machine base. Four rigid steel plate (100×100×120) mm in dimensions are welded
at the corners of the rigid steel supporting frame that can be assumed as pin support on four stiff steel columns. The clear distances between these steel columns are 1050 mm in long directions and 850 mm in short directions.

To have a uniformly distributed load subjected on one way slabs a hydraulic jack of the testing machine is used through the following setup:

The sand furnishes a good media to distribute the load uniformly, a box of steel plate of thickness 5 mm with inside dimensions (depth 100 mm and same surface area of one way slab) is used to hold the sand to be placed over the slab as a method to uniformly distribute load. The box coated on the inner surfaces by a sheet of nylon to reduce any possible friction that can result from the contact of sand with steel and concrete.

To maintain more distribution for loading the single point load from the universal machine was distributed equally into nine points load approximately on a steel plate of (1230×1030×5) mm that capping the supporting layer of sand by using 3 I-section beam in longitudinal direction of slab over 3 I-section beam in transverse direction.

At each increment the manual measurements were recorded, which included the following:

1- the applied load are measured by a hydraulic machine with capacity of 2000 kN as mentioned above, the load was applied with a loading increment rate of about 150 N/sec.

2- The deflections are measured using a dial gauge with a capacity of (50) mm and accuracy of (0.01) mm, beneath the center point of the slab and, at the two quarter points of slab and at mid span of the two edge beams in the slab-beam systems. The dial gauge is fixed in such a way that it can contact the lower surface of models. The deflection readings of dial gauge are taken at each 5 kN/m².

3- The crack width is measured at each 10 kN/m² by crack meter (Electrometer 900), in addition to that, the cracks are detected and drawn on the bottom face of the tested slabs and the edge beams.

5. TEST RESULTS AND DISCUSSION

All models were designed with a clear cover to the reinforcement of 20 mm, All details reviewed in previous section. The slab-beams models were designed to fail in flexure. The general behavior of the tested slab-beams models can be summarized as below. For the control models, at early stages of loading, the deformations were initially within the elastic ranges (linear), then the applied load was increased until the first crack became visible which was observed at the center line of slab in long direction and at mid-span of edge beams. As
the load was further increased, several flexural cracks were initiated in the tension face at intervals throughout the slab and beams, gradually increased in number, became wider and moved upwards reaching the compression face of the slab and beams. As the load was increased further, a loss of stiffness occurred and one mode of failure appeared which can be classified as flexural failure in tension by yielding of the steel reinforcement followed by crushing of concrete. The CFRP models also showed similar behavior, but not yielding of steel occurred, the CFRP reinforcement contributed mainly in resisting the loads and increased the stiffness of the concrete models up to failure by crushing of concrete in beams and diagonal shear cracks near the edge of slab.

5.1 Ultimate Loads

Crack formation was monitored throughout testing to assess the behavior of the CFRP one way slab with edge beams in comparison with the behavior of steel reinforced concrete control models. Figures (3) to (4) show samples of crack patterns for some tested models.

Table (3) shows the ultimate load, first crack load and ultimate deflections in slab and beams for all models.
Table (3): Ultimate load and deflection for all models

<table>
<thead>
<tr>
<th>model</th>
<th>First Crack Load in beam</th>
<th>First Crack Load in slab</th>
<th>Ultimate Load $W_u$ (kN/m²)</th>
<th>$\Delta u$ (mm) under slab center</th>
<th>$\Delta u$ (mm) under slab quarter point*</th>
<th>$\Delta u$ (mm) at beam mid-span*</th>
</tr>
</thead>
<tbody>
<tr>
<td>S6</td>
<td>30.5</td>
<td>38</td>
<td>146</td>
<td>17.79</td>
<td>15.455</td>
<td>11.70</td>
</tr>
<tr>
<td>S7</td>
<td>38</td>
<td>45.7</td>
<td>153</td>
<td>19.56</td>
<td>16.565</td>
<td>13.015</td>
</tr>
<tr>
<td>C6</td>
<td>45.7</td>
<td>53.3</td>
<td>202</td>
<td>17.21</td>
<td>15.68</td>
<td>10.725</td>
</tr>
<tr>
<td>C7</td>
<td>49</td>
<td>55.6</td>
<td>221</td>
<td>19.55</td>
<td>17.655</td>
<td>10.59</td>
</tr>
<tr>
<td>C3-N3</td>
<td>42</td>
<td>49.5</td>
<td>191</td>
<td>15.37</td>
<td>13.895</td>
<td>11.095</td>
</tr>
<tr>
<td>C4-N3</td>
<td>45.7</td>
<td>53.3</td>
<td>210</td>
<td>16.506</td>
<td>16.08</td>
<td>12.5645</td>
</tr>
<tr>
<td>S6-R5</td>
<td>30.4</td>
<td>38</td>
<td>149</td>
<td>12.73</td>
<td>11.345</td>
<td>8.715</td>
</tr>
<tr>
<td>S7-R6</td>
<td>38</td>
<td>45.7</td>
<td>160</td>
<td>14.50</td>
<td>13.5015</td>
<td>11.24</td>
</tr>
</tbody>
</table>

*Average value.

Fig.(3): Cracking Pattern at Failure for Model (S6)
5.2 Maximum Crack Width

The main observations which can be made from crack width measurements are listed below:

1- At the same load level, steel reinforced concrete models (control models) ((S6) and (S7)) showed greater crack width than CFRP models ((C6) and (C7)) of similar reinforcement ratio respectively.

2- The smaller crack width is true for NSM models (C4-N3) and (C3-N3) in comparison with control models.

3- However, Control models ((S6) and (S7)) showed rather smaller crack width than repaired model ((S6-R5) and (S7-R6)) respectively at same loading stage.

Figures (5 to 7) show load versus crack width for some tested models. It is clear, with using CFRP reinforcement of percentage (1-1.2)$\rho_b$ , as in the present study will control more the cracking width and deflection up to failure.
N. A. Alwash and H. M. Al-Nafakh

Figure (5) Load Verses Crack Width of (S6) and (S7)

<table>
<thead>
<tr>
<th>Load (kN/m²)</th>
<th>Crack Width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0.2</td>
</tr>
<tr>
<td>40</td>
<td>0.4</td>
</tr>
<tr>
<td>60</td>
<td>0.6</td>
</tr>
<tr>
<td>80</td>
<td>0.8</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>120</td>
<td>1.2</td>
</tr>
<tr>
<td>140</td>
<td>1.4</td>
</tr>
<tr>
<td>160</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Figure (6) Load Verses Crack Width of (C6) and (C7)

<table>
<thead>
<tr>
<th>Load (kN/m²)</th>
<th>Crack Width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>0.2</td>
</tr>
<tr>
<td>100</td>
<td>0.4</td>
</tr>
<tr>
<td>150</td>
<td>0.6</td>
</tr>
<tr>
<td>200</td>
<td>0.8</td>
</tr>
<tr>
<td>250</td>
<td>1</td>
</tr>
</tbody>
</table>

a. Max. Crack Width at Edge Beam (mid span)  
b. Max. Crack Width at Slab (center line)
5.4 Deflection Distribution Plots

Figure (8) show locations of dial gauges in short direction of one way slab with edge beams. Deflected shape of the slab with edge beam at beam mid-span versus distance at different load stages are presented in Figures ((9) to (10)).
Figure (9) Deflection Distribution Along Short Direction of slab For (S6)

Figure (10) Deflection Distribution Along Short Direction of slab For (C6)
6. CONCLUSIONS

Main conclusions drawn from experimental work can be summarized as given below:

-It was found that CFRP reinforced specimens can achieve flexural strength values higher than those of similar steel reinforced models by about (38 % - 44 %), and for NSM specimens about (31% - 37 %) and, for repaired specimens about (2. % - 5%).

-The CFRP RC models showed about (34% - 47%) for beam and (29 % - 30%) for slab lesser deflection than control models. Also, the near surface mounted RC models showed about (43% - 50%) for beam and (43% - 51%) for slab lesser deflection than control models. For models under repeated load, the repaired RC models showed about (20% - 34%) for beam and (32% - 34%) for slab lesser deflection than the control models.

-Using CFRP rods as tensile reinforcement or strengthening in RC slab-beam systems had a significant effect on the crack width of tested models. The low modulus of elasticity of CFRP rods was substituted by using balanced and over reinforcement ratios and was found it reduced significantly the max crack width of reinforced concrete one way slab with edge beams under (UDL ).

REFERENCES


MEMBRANE FORM FINDING BY MEANS OF FUNCTIONAL MINIMIZATION

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Key words: Form Finding, Textile Composites, Membrane, Architecture, Aided Design.

Abstract. This study deals with the methods of architectural design of structures made of textile membranes. We consider the problem of form finding of membranes strained by rigid skeletons and cables installed along the free edges of the membrane. First, we recall the methods used to solve the simple problem of finding form in the case of constant surface tension. Then we propose a method based on the minimization of the total potential energy. The problem is discretized using membrane triangular finite elements.

The potential considered is an energy density per unit area of uniform and constant surface tension. The minimization of this potential leads to the minimal surface solution. However the problem is singular with respect to the in-plane displacement. To handle this problem, the potential is enhanced by an elastic energy in order to regularize the numerical scheme and prevent the mesh degeneration. It is also enhanced by the elastic energy due to the cable tensions. The solution is obtained by minimizing the potential energy using the conjugate gradient method.

1 Introduction

The flexible structure like cables and membranes are characterized by form follower internal forces; the stress vector remains axial to the cables and remains in-plane in case of membrane. The shape of such structures, when they are uniformly taut, is essentially defined by force equilibrium considerations. Conversely, the loads distribution in the membrane are strongly governed by the attained geometry. The structures without bending stiffness obey this principle.

The literature on the form optimisation may be classified into two main topics: structural optimization and form finding [8]. The first topics focuses on the search of the initial shape of the structure through a kinematic criterion or a resistance criterion, whereas the form finding focuses on the final form that can reached by a structure under a prescribed
stress field. The first method is general and applies to any kind of structure as an inverse problem, whereas the second method is more than often used for structures made of stretched membranes and cables subjected to large deformations. It is necessary to clearly distinguish the objectives of these two approaches.

Membrane structures are characterized by a pure tensile in-plane stress state (i.e. without bending stress). The pure tension is governed by local equilibrium. When the stress state is in-plane, uniform and isotropic, the resulting geometry is defined by a minimal surface. This is the case, for example, of the soap film which exhibits a uniform surface tension and a minimum surface area.

Bletzinger [2] and Veenendaal [13] summarized methods of form finding developed in the last decades in three main families:

- Stiffness matrix methods that are based on using the standard elastic and geometric stiffness matrices [11, 6, 12].
- Geometric stiffness methods which are material independent, based on the force density method concept with some extensions [5, 1, 10].
- Dynamic relaxation methods which solve the problem to reach a steady-state solution, equivalent to the static equilibrium solution.

In this study, we will show that the force density method in the case of prescribed stress field can be formulated as an energy minimisation problem. Use will be made of the conjugate gradient method, which is a first order method, to minimise the total potential energy. It will be shown that this method is robust and efficient to solve the form finding problem.

2 Geometric model

In the force density approach, the membrane is represented as a geometric surface and not as a material one. The surface represents the midplane of the membrane and serves only to define the force field domain. The optimal form is defined by this surface when the local equilibrium of the force field is satisfied at each point of the whole surface. Seeking for the optimal form requires the definition of an initial surface \( S \subset \mathbb{R}^2 \) which defines the surface state at time \( t_0 \). This surface evolves towards the optimal form \( s \subset \mathbb{R}^2 \), at time \( t (t > t_0) \), by a geometric transformation \( \Phi \).

We use the bijective mapping function \( \Phi \) to relate a point \( X \in S \) to a point \( x \in s \):

\[
S \ni X \mapsto x = \Phi(X, t) \in s
\]

The initial surface \( S \) is an approximation of the optimal solution \( s \), in the sense that \( s \) is independent from the choice of \( S \). Time \( t \) is any kinematic parameter. The material curvilinear coordinates \((\xi^1, \xi^2)\) are introduced to describe the surface of the membrane, the third dimension is not represented geometrically but taken into account through the thickness denoted \( \xi^3 (h/2 \leq \xi^3 \leq h/2) \) and assumed to be uniform.
Representing the membrane by a surface requires both stress and strains fields constant along thickness $\xi^3$. Integration along the thickness is equivalent to multiplying the integrated quantity by $h$.

We denote $G_\alpha = \frac{\partial X}{\partial \xi_\alpha}$ the curvilinear base in the initial configuration, and $g_\alpha = \frac{\partial x}{\partial \xi_\alpha}$ the mapped base in the final configuration, where Greek indices take the values $\{1, 2\}$.

The metric tensor in the initial configuration is defined by $G_{\alpha\beta} = G_\alpha \cdot G_\beta$ and that in the actual configuration is defined by $g_{\alpha\beta} = g_\alpha \cdot g_\beta$. The deformation gradient tensor $F$ writes

$$F(X, t) = \frac{\partial \Phi(X, t)}{\partial X}$$

and the Green tensor is

$$E = \frac{1}{2} (F^T F - I)$$

We can also write the strain tensor as a function of the metric tensors in the current and reference configurations:

$$E_{\alpha\beta} = \frac{1}{2} (g_{\alpha\beta} - G_{\alpha\beta})$$

### 3 Minimal surface method in form finding

A membrane uniformly and isotropically stretched on its rim takes the form which minimizes its surface. An example of such membranes is soap films. The surface tension of the film ensures a membrane retraction as much as possible until reaching the minimum area. The method of minimal surface amounts to investigate the shape of the membrane that achieves the minimum total surface $s$. The problem to solve is formulated as follows:

$$x = \arg \min_x s = \int_s ds = \int_S J dS$$

where $J$ is the determinant of tensor $F$, $s$ the surface in the current configuration and $S$ the surface in the initial configuration. The surface stationarity condition is
\[ \delta s = \int_S \delta JdS = \int_S JF^{-T} : \delta FdS = 0 \] (5)

It should be noted here that the area \( s \) is not necessarily material and the transformation \( F \) is the mapping function that merely connects the two configurations occupied by the considered surface.

4 Potential energy method for prescribed stress field

In this section, we show that minimum surface finding – which is a purely geometrical method – can be formulated as a static equilibrium problem using the theorem of potential energy minimum. The energy considered results from a constant transversely isotropic stress field (e.g. a uniform surface tension on the membrane). One therefore seeks the form achieved by the membrane when it is stretched by a known plan stress field, represented by a Cauchy stress tensor \( \sigma \) prescribed on the whole membrane, of the form:

\[ \sigma = \tau \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \tau I \quad \sigma_{ij} = 0, i = 1, 2, 3 \] (6)

The stress field being prescribed on the current configuration, we seek the position field \( x = X + U \) that makes the membrane in equilibrium position under this load. The potential energy associated with \( \sigma \) is

\[ \Pi^s = h \int_s \sigma : u, x ds \] (7)

The equilibrium configuration makes the potential energy stationary for any displacement variation \( \delta u \). The corresponding deformation variation is

\[ \delta u = \delta u \cdot x \cdot x = \delta x \cdot x \cdot x = \delta F \cdot F^{-T} \]

It follows that when the tension field is isotropic, \( \sigma = \tau I \), the stationarity condition of the potential energy can be written as

\[ \delta \Pi^s = h\tau \int_s I : \delta u, x ds = h\tau \int_s det FF^{-T} : \delta FdS \] (8)

which is exactly the minimal surface condition established in Equation (5).

Let us define the potential \( \Pi^s \) whose minimum defines the equilibrium surface configuration:

\[ \Pi^s = h\tau s \] (9)

The problem to solve is then formulated as follows:
\[ \mathbf{x}_{\text{sol}} = \arg \min_{\mathbf{x}} \Pi^x = h \tau \arg \min_{\mathbf{x}} \int_{\mathbf{s}} ds = h \tau \arg \min_{\mathbf{x}} \int_{\mathbf{S}} JdS \]  \hspace{1cm} (10)

This result shows the equivalence, in the case of an isotropic stress field, between the minimum surface approach and the minimum of potential energy theorem. However, in the energy method, one can add other potentials of various loads like elastic potentials of deformable bodies (e.g. cables and flexible supporting structures).

5 Regularization of the form finding methods

The problem of form finding is to find the position \( \mathbf{x} = \mathbf{X} + \mathbf{U} \), i.e. the actual configuration \( \mathbf{s} \), that makes \( \Pi^s \) minimum for any variation \( \delta \mathbf{U} \).

This method has the particularity of being singular for degrees of freedom within the plane of the membrane. Indeed, for a given meshed surface, an arbitrary movement of nodes in the tangent plane of the membrane does not change the total area. From a numerical point of view, this can lead to an optimal solution with a very distorted mesh.

To avoid degeneration of the mesh (coincidence of two nodes for example), we should regularize the problem by limiting their in-plane movements. There are several methods to regularize the problem. One of them consists in projecting the displacements obtained at each iteration along the normal to the membrane.

In our case, we have supplemented the quantity to minimize \( \Pi^x \) with the elastic strain energy of the membrane \( \Pi^e \), which plays the role of springs between nodes in the plane of the membrane. For this energy, the material is assumed hyperelastic and governed by the quadratic elastic potential of Saint-Venant Kirchhoff with a surface energy density \( \Psi^e \):

\[ \Pi^e = \int_{V} \Psi^e(\mathbf{E})dV \]  \hspace{1cm} (11)

In plane stress condition, the out-of-plane stress components vanish:

\[ \Sigma_{33} = \frac{\partial \Psi(\mathbf{E})}{\partial E_{33}} = 0 \]  \hspace{1cm} (12)

Equation (12), \( \Sigma_{33} = 0 \), establishes an implicit relationship between the components of the strain tensor.

\[ \Sigma_{33} = \Sigma_{33}(E_{11}, E_{12}, E_{22}, E_{33}) = 0 \]  \hspace{1cm} (13)

From this equation, the normal component \( E_{33} \) can be expressed in terms of the in-plane components of \( \mathbf{E} \) as:

\[ E_{33} = f(E_{11}, E_{12}, E_{22}) \]  \hspace{1cm} (14)

It is then possible to reduce the volume energy density \( \Psi^e \) into a surface density. For this, we rewrite the potential \( \Psi \) as a function of \( E_{11}, E_{12} \) and \( E_{22} \) in the form:
\[
\Psi^e (E_{11}, E_{12}, E_{22}) = \Psi (E_{11}, E_{12}, E_{22}, E_{33})
\]  
(15)

The plane stress condition has enabled one to eliminate \( E_{33} \) from the expression of the elastic potential which depends therefore only on the in-plane components of strain tensor \( \mathbf{E} \). The elastic potential energy of the membrane can be written as

\[
\Pi^e = \int_V \Psi^e (\mathbf{E}) dV = h \int_S \Psi^e (\mathbf{E}) dS
\]  
(16)

The minimisation problem involves the quantity \( \Pi = \Pi^s + \Pi^e \) and is the rewritten as

\[
x_{\text{sol}} = \arg \min_x (\Pi^s + \Pi^e)
\]  
(17)

Adding the elastic energy \( \Pi^e \) to the energy of the surface tension \( \Pi^s \) preserves the structure of the mesh as long as the mesh is far from the optimal shape. However, when approaching the solution, this energy must be deactivated so that the optimal resulting shape is not altered by the added elastic energy.

6 Stain energy canceling

The addition of strain energy to the minimizing quantity introduces in-plane stiffness that disturbs the solution. The minimization leads, as can be seen in Figure 5 below, to the formation of wrinkles orthogonal to compressive stresses. It is therefore necessary to ensure that the quantity minimized in (17) leads to the minimum area of the membrane. To cancel the elastic energy at the end of the iterative process, we simply cancel the strain tensor. For this, we modify the strain tensor defining \( \Pi^e \) by updating the reference configuration. This idea was first proposed by Bletzinger [8] and proves to be efficient and robust. The process is repeated until convergence. For each minimization step \( n \), the strain tensor formula (2) is modified by replacing the metric tensor \( \mathbf{G} \) in the reference configuration by that in the configuration reached at the previous step \( n - 1 \). At the end of the iterative process, the two configurations \( n - 1 \) and \( n \) are close enough to each other, they become asymptotically the same and the strain tensor vanishes. We write

\[
E_{kl}^{(n)} = \frac{1}{2} \left( g_{kl}^{(n)} - g_{kl}^{(n-1)} \right)
\]  
(18)

with \( g_k^{(0)} = G_k \). At convergence, the strain energy \( \Pi^e \), activated for the sole purpose of the problem regularization, will be automatically canceled.

7 Minimisation of the potential energy

The minimization formulation of the energy due to a uniform and isotropic stress field, allows extension of the formulation by adding any kind of potential energy to the quantity to be minimized. Thus, one can easily includes the energy \( \Pi^c \) due to deformable cables
supporting the membrane edges, the energy due to the deformable elements bearing the structure, or the energy due to dead loads uniformly distributed over the membrane such as the snow. The problem then writes

\[ x_{sol} = \arg \min_x \Pi^{tot} = \arg \min_x (\Pi^s + \Pi^e + \Pi^c + ...) \]  

(19)

The total potential energy \( \Pi^{tot} \) is discretized using the finite element method and is written as a nonlinear function of the nodal unknown displacements \( \{U\} \). Minimization is done either by the first order minimizing methods as the conjugate gradient or the second order which requires the linearization of the energy using the Taylor series expansion:

\[ \delta \Pi^{tot} = \frac{\partial \Pi^{tot}}{\partial \{U\}} \{\delta U\} + \frac{1}{2} \{\delta U\}^T \frac{\partial^2 \Pi^{tot}}{\partial \{U\}^2} \{\delta U\} + O(\|\delta \{U\}\|^3) \]  

(20)

\[ = \{\nabla \Pi^{tot}\}^T \{\delta U\} + \frac{1}{2} \{\delta U\}^T [K] \{\delta U\} + O(\|\{U\}\|^3) \]  

(21)

When using the second order methods of the Newton or quasi-Newton-type we simply require that the energy is stationary.

When using the first order methods, the solution is sought for with descent directions oriented in the opposite direction of the potential gradient \( \{\nabla \Pi^{tot}\} \). This type of algorithm converges in all cases to a minimum whenever it exists. Their main disadvantage is that its convergence is linear.

The second order methods, like quasi-Newton ones, are preferable to the first order methods because of their quadratic convergence. However, these methods lose their advantage when the stiffness matrix is ill-conditioned as in the case of a significant loss of stiffness. In this situation, the first order methods take an advantage in that the algorithm works well, even if the critical points exist and making the stiffness matrix singular.

We used the conjugate gradient method to seek for the equilibrium position. This method uses only the gradient vector of the total potential energy and does not require specific processing of hypostatic kinematic modes. Numerical developments are implemented in the Brakke’s Surface Evolver program [4].

8 Numerical examples

8.1 Scherk’s problem

We consider here the classical test of the Scherk form finding problem. It is a minimal surface with boundaries described by the unit cube. The initial surface is made of three flat squares which is each meshed using 1024 linear triangular finite elements.

In order to accelerate the evanescence of the elastic potential, we introduce a weighting factor \( \alpha = \lambda^n \) with \( \lambda \in [0, 1] \) and \( n \) the computational step. The problem to solve is then written as
2.45 2.55 2.65 2.75 2.85 2.95 3.05
0 2 4 6 8 10
Surface
Step n
a = 0.25
a = 0.50
a = 0.75
a = 1.00
a = 1.50
Figure 3: Convergence of central point z-position with respect to parameter $\alpha$.

For $\lambda = 1$, the problem is the same as in (19), for $\lambda$ close to zero the coefficient $\alpha$ tends quickly to zero which reinforces the elastic energy cancellation. Figure 3 shows the effect of the coefficient $\alpha$ on the middle point position convergence.

8.2 Tent structure

The numerical example presented in what follows is a tent structure composed of an elastic membrane supported by cables, fixed anchors and rigid hoops. The structure has a wheelbase on the ground in rectangular form $2l \times 3l$ in the $(x, y)$ plane, blocked on 8 anchors (A, B, ..., H), and surrounded by cables on the free edges. Two hoops are prescribed at $y = l$ and $y = 2l$, having the parabolic form $z = 2l x^2 (x - 2)$ with $l = 0.5$ m.
Figure 4 shows the mesh used and the displacement boundary conditions prescribed on the hoops.

The problem data are fixed as follows:
- For the membrane: surface tension $\tau = 1 \text{ Pa}$;
- For the elastic strain energy used for regularization: Young's modulus $5 \times 10^5 \text{ Pa}$; Poisson's ratio 0.3; thickness $10^{-4}$ m.
- For cables: Young's modulus $\times$ section $ES = 10 \text{ N}$.
- For the mesh: 1601 nodes, 3072 triangular finite elements.

Figure 5 shows the mesh obtained by minimizing functional $\Pi_{tot}$ without canceling the elastic energy of the membrane $\Pi_e$. The deformed configuration, at this stage, has folds due to bifurcations arising from the compression of the membrane in certain directions.

By using the updated initial strategy, the elastic energy can be canceled iteratively. The membrane will gradually tend to a uniform and isotropic stress state. $\sigma = \tau I$. Figure 6 shows the shape obtained after total cancellation of $\Pi_e$.

9 Conclusions

In this study we have transformed the stress field approach used in the form finding method to an energy minimisation problem. We have shown that the case of a uniform isotropic stress field is equivalent to the surface minimization. We have used the initial
configuration updating strategy to regularize the numerical scheme. This is done by modification of the metric tensor of the initial configuration.

The conjugate gradient method used to seek for the energy minimum proves to be very efficient to correctly handle hypostatic instabilities associated with membranes. Indeed, the mechanical model used is a pure membrane without bending stiffness. It is precisely this property of in-plane stress field that is formulated here as the criterion for form finding.

RÉFÉRENCES


A novel method for reducing the amount of material used for structural components in a building is the use of structural optimization in the design process. Structural optimization integrates structure and form in a way similar to natural or biological optimization. Weight reductions of structural members cascade through the structural system of a building decreasing design loads on other structural members leading to an additional reduction in material use. Currently, there are no efficient production methods which are suited for producing the organic shapes distinctive for structurally optimized elements and contemporary architecture mainly focuses on zeroelastic, multi faced bodies. The best results thus far have been achieved with fabric formworks. These mechanically pre-stressed membranes transfer loads solely by linear tension, reducing the volume of the necessary formwork up to 1/300. Subsequently, transportation, storage, landfill and thus embodied energy is also reduced.
Despite the achievements, the method can be complicated since often a large quantity of additional falsework is needed. In a highly industrialized manufacturing process this would not be an issue, since the formwork and additional falsework can be reused many times. However, the custom and unique nature of shapes resulting from a structural optimization analysis call for a more flexible formwork system. In that case the formwork of the three-dimensional structure consists entirely of inflatables and is therefore completely based on form-active principles.

2 METHODOLOGICAL APPROACH

This paper discusses the preliminary findings of an ongoing multidisciplinary research into the possibilities of using inflatable membranes as formwork which can be rigidized to produce structurally optimized section active structure systems. The research consisted of an experimental study and in depth literature reviews into the state of the art of structural optimization, inflatable structures and rigidizable materials. The formfinding process initiated with the topological optimization of the four section active systems defined in a theoretical framework by Engel. Its objective was to derive a case on which the proposed production method is based, by identifying the general morphological features of the optimized section active structures. Thorough empirical case studies were performed using solidThinking Inspire 9.0 to determine which optimized structure reflected the most of the general morphological features. Subsequently, shape and size optimization was performed using the ParaGen method to complete the formfinding process.

3 STRUCTURAL OPTIMIZATION

Structural optimization is a technique to minimize the material use to a given loading. Research to structural optimization has a long history and is studied intensively.

Structural optimization begins with the earlier work of Galilei, (Fig. 1.) Later on, Bernoulli, Lagrange, Navier sought for the ‘best’ shapes for structural elements to satisfy strength requirements. In the 1960, Lucien Schmidt’s seminal paper introduced structural optimization.

Optimal structures –in an architectural context- can be generated by applying an optimization process, this process is already well known method in the car industry, mechanical engineering and aeronautical engineering. Preconditions for the optimization process are; i) the boundaries, ii) load case, iii) material conditions. Using this conditions, the
optimization process, generates an optimal structure, based on maximum strength and minimal amount of material.

The optima forma is comparable to the optimization process of mechanical engineering. Rozvany\(^\text{14}\) described how to optimize structures in different load cases.

According to Rozvany, it is possible to optimize a structure by three types of optimization: i) size, ii) shape and iii) topological\(^\text{14}\). They address different aspects of structural design problem, e.g., the goal may be to find the optimal thickness distribution of a linearly elastic plate or the optimal member areas in a truss structure. The aim of shape optimization is to find the optimum shape of its domain. Topology optimization aims at finding the optimal layout of a structure within a specific domain. Structural optimization seeks to achieve the best performance for a structure while satisfying various constraints such as a given amount of material\(^\text{15}\).

To perform a structural optimization 3 variables are required:

i) **Objective function**: a function used to classify designs: for every possible design, \(f\), returns a number which indicates the goodness of the design\(^\text{17}\).

ii) **Design variable**: a function or vector that describes the design, and which can be changed during optimization\(^\text{17}\).

iii) **State variable**: represents the response of the structure\(^\text{17}\).

\[
\begin{align*}
\text{minimize } f(x, y) \text{ with respect to } x \text{ and } y \\
\text{subject to } \text{behavioral constraints on } y \\
\text{design constraints on } x \\
\text{equilibrium constraint.}
\end{align*}
\]

Fig. 3. The Structural optimization process\(^\text{17}\).

There are several well-established techniques for the generation of solid-void optimal topologies such as solid isotropic material with penalization (SIMP) method and evolutionary structural optimization (ESO) and its later version bi-directional ESO (BESO) methods\(^\text{16}\).

In this research we used the SIMP method. The SIMP method is numerical FE-based topology optimization method. It stands for Solid Isotropic Microstructure with penalization. The basic idea was proposed by Bendsoe\(^\text{15}\).

Common to these well-known topology optimization techniques is that they produce organic looking shapes that cannot usually be cast using conventional techniques. There is a
gap between structurally optimized forms, and those developed intuitively by fabric casting, can be bridged (flexible membrane).

Producing organic shapes in concrete (outcome of optimization) has been a challenge problem since complex freeform buildings became a major trend in contemporary architecture.

Optimal structures can be widely used in architecture, developing new production techniques are necessary. This results in more efficient (smarter) and sustainable buildings.

Optimal structures require non-orthogonal geometries. Fabric formwork should not be seen as replacement for conventional orthogonal planar formwork but as a disruptive technology that offers a new paradigm with a clearly emerging parametric requirements for form, process, precision and complexity 27.

Producing optimal structures is possible with flexible moulding; fabric formwork.

Fabric formwork can be used to create durable, visually appealing, optimized concrete structures. Fabric formwork provides a means by which architects and engineers can create low-carbon concrete structures to facilitate a sustainable future in concrete construction.

Examples structural optimization in practice.

Example 1; A cantilever with a concentrated load at its free end. The cantilever structure having an inner hollow with two plate-like parts which have two holes on them.

Example 2; An optimized beam produced with fabric formwork using two rigid panels sandwiched, see figure 5. This method with ‘pinch points’ makes it possible to produce concrete trusses in relatively easy ways 18.

4 PARAGEN METHODOLOGY

To find a suitable geometry for the beam, a method was chosen which combines parametric form generation with multi-objective shape optimization. The method used was ParaGen, a genetic algorithm (GA) based program developed at the University of Michigan.
ParaGen uses commercial software on a cluster of PCs connected through a web interface to a server that maintains a solutions database from which new solutions are bred. The optimization cycle shown in (Fig. 7) is divided into two parts: the server side and the client side.

On the client side there are basically two steps: 1. parametric form generation and 2. performance evaluation (in this case structural). For Step 1. Generative Components (GC by Bentley Systems) was used as the parametric modeler. To generate a new solution, the GC transaction file reads in a set of variables from an Excel file (the child solution) which was bred by the GA on the server and passed through the web interface to the client PC. Once the new child form is generated in GC, the geometry is passed to the analysis program through a DXF file.

In this case a structural analysis was performed using STAAD.Pro to find internal forces, deflections and modal stiffness, and to determine the size of the elements.

In addition to these performance values, descriptive images were also generated in both STAAD.Pro and GC. These include the base geometry, deflected shape, axial force diagrams, depiction of the member sizes, and a rotatable, 3D VRML image. These images were also available later to aid in choosing which solutions to physically model.

For reasons of expedience, the model was initially generated in GC using straight rather than curved elements. In STAAD.Pro the model was designed using steel pipes in order to make relative comparisons of how the geometry affected the structural performance. Once a geometry was selected in ParaGen, additional detailed investigations were made using the curved geometry and concrete sections (see Section 8.).

In Step 3. all of the parametric variables along with the accompanying performance values and images are uploaded through the web interface to the SQL database on the server. The images are given id names which allow them to be linked to a specific solution in the database. Because the database contains only performance and variable values, it can be quite large and still be searched very effectively using standard database search and filter techniques. For this reason there is no need to limit the number of retained solutions to some predetermined population size as is normally the practice in GAs.

Step 4. is comprised of the GA parent selection. The population from which the selection of parents is made is composed on the fly by using SQL filters to create a limited population out of the entire database of solutions. The SQL filters can be very simple sorts to produce a
set of solutions. For example the top 40 solutions sorted by least weight. Or they can be more complex limit sets, for example, the top 40 solutions where modal frequency is > 20Hz and weight is < 160 kg then sorted by least deflection. After creating a population set a parent solution is selected at random from the population. Actually this operation is performed twice, once for each parent, and a different filter can be used for each of the two populations. This filter method gives different results from simple objective weighting since particular areas of the solution space can be defined. Once two parents have been selected, the last step in the ParaGen cycle is the breeding of the parents to obtain a new child solution. In Step 5. the CHC breeding algorithm developed by Eshelman and Schaffer is used for crossover of the genetic variables. Only one child is produced and passed through the web interface back to the waiting client PC where the cycle begins again.

The cycle is initialized by generating some number of random solutions. At some point with a sufficient number of solutions in the database, the populations can begin to be developed using the SQL sorts and filters. As the run progresses these population sorts and filters can also be tuned or altered to better search specific areas of the solution space. Finally, with a sufficiently developed database of solutions, interactive searches can be performed to explore and compare solutions. Figure 8 shows a selection set made by setting the filters at modal frequency > 30Hz and weight < 147 kg and sorting by least deflection.

Also multiple objectives can be compared and plotted to perform a Pareto investigation. Plots of two objectives as least weight and highest modal frequency can be shown. A mouse over of the data points will reveal the data values, and clicking on the points will display a small image of the solution. Using these techniques allows the designer not only to simply find the best performing solution, but to actually explore the solution space and perhaps better formulate some aspects of the problem.

5 INSPIRE CASE STUDIES

The first step of the form finding process included the topological optimization of section active structure systems. The optimization of these structure systems was assumed to lead to the largest reduction in material, since form-, vector- and surface active systems can already
be considered as lightweight structures. Empirical case studies are performed on the four section active structure systems;

- beam structures
- frame structures
- beam grid systems
- slab structures

The goal of these case studies was to reveal the morphological features of the four separate section active structure systems and the morphological features of topologically optimized section active structure systems in general. Ultimately, the structure that reflects the most of these general morphological features served as a case for the development of the production method. It has to be noted that beam grid systems were not elaborated, since it can easily be shown that an optimized beam grid is equal to a collection of optimized beam structures.

Empirical case studies were performed using solidThinking Inspire 9.0 8. The density method is the main solving strategy used in Inspire, which uses Altair OptiStruct 31 and HyperMesh 31 in the background. During optimization, the material density is the only design variable and is allowed to vary continuously between 0 and 1. The relation between the stiffness and the density of the material is assumed to be linear. Fictitious values of intermediate density are penalized using the power law representation of elastic properties to make the result behave more like an ISE topology 14. For validation purposes, topological optimization on the section active structures was also performed using Topostruct 28, which uses the homogenization method as a solver.

Case studies were carried out using a morphological overview describing all the different constraints that can be applied on a given design space in Inspire 9.0, and the possible or characteristic values they can assume. Varying one parameter at a time, an empirical case study is performed revealing the influence of the individual constraints on the outcome of a topological optimization. The initial geometries were sized using rules of thumb which apply for the Dutch construction industry. The entity exerting the load on the design space was assumed to have its own stiffness. Therefore, material was allowed to be removed throughout the design space. (Fig. 9) displays a characteristic result of topology optimization on a one-bay beam, constrained by fixed supports and a centre point load of 5 kN. The mass target was set on 20% and the material used was standardized C20/25 concrete. Manufacturing constraints such as symmetry, draw direction and thickness control were not used since the manufacturing method has yet to be developed. Also, no frequency targets were specified since Inspire 9.0 always maximizes stiffness and thus natural frequency.

![Fig. 9. Characteristic result of an optimized one-bay beam](image)

The type of supports used, i.e. the degrees of freedom and location, have the most influence on the resulting morphology of topologically optimized section active structure systems. Less material is needed where the moment tends to zero, reducing the area of the cross section locally. In the case of frame structures the type of support has less influence,
especially when the height to length ratio is equal to or larger than 1/10. With these larger spans, an optimal design space would allow the structure to form an arch at the inner side. With larger spans, larger moments occur which need to be transported via the corners of the frame towards the supports, leading to larger cross sections near the corners. In this context, the type of support has little influence on the moment distribution in the horizontal part of the frame. It has to be noted that fixed support generally lead to more clear and stable topologies than rolled and pinned supports.

Point loads lead to denser member distributions than distributed loads, except in the case of slab structures. With respect to beam and frame structures, forces caused by a point load will be transferred by two diagonal members towards the bottom flange. These members, as well as most other members in optimized structures, connect at an angle of about 45 degrees since forces also disperse at this angle. When incrementally increasing the number of point loads, the resulting morphology will move towards the morphology of an optimized structure constrained by a distributed load, meaning that the middle diagonal members move further apart.

The slenderness and height to width ratios were also studied to gain insight in the behavior of the optimization routine when the proportions of the design space change. The separate structure systems that were studied are mathematically speaking closely related. When increasing the width of a beam structure while maintaining a constant height, the geometry will move towards a slab structure. When increasing the height of a slab structure while keeping the width constant the result will be similar to an optimized frame structure. For the intermediate height to width ratios in between the three structure systems no general morphological features were found. Increasing the slenderness, i.e. height to length ratio, of beam and frame structures does not result in new topology. The result of its one-bay counterpart merely gets stretched. However, there are certain limits where further increase of the slenderness will lead to unclear topologies. These limits are mainly determined by the type of supports and type of load that is used. In addition, making a beam or frame structure continuous also does not result in new topology. Here, the result of its one bay counterpart gets copied. The same generalization can be made regarding slabs with high widths.

Mass targets, material choice and the value of the load(s) used have little influence on the resulting topology of an optimized structure, but influence component attributes that mostly deal with size and shape optimization. (Fig. 10) shows two optimized elements with an identical design space and constraints, with a mass target of 20% on the left and 40% on the right.

![Fig. 10. Influence of alternative mass targets](image.png)

The relation between structure and form, i.e. the structural morphology, of these optimized elements is very strong. The resulting topology and morphology of an optimization routine is determined by the force distribution through the design space and the different constraints and performance requirements that act on that specific design space. The morphology of an optimized one-bay beam can be recognized in every optimized section active structure.
system. Many morphological features that apply for an optimized beam structure therefore also apply for frame structures, beam grid systems and slab structures. The production method that is proposed in this paper will therefore be based on an optimized beam structure. The final structure derived of SolidThinking Inspire is a hollow circular beam that conforms to the funicular shapes of inflatable structures, (Fig. 11).

6 PARAGEN RESULTS

The optimized three dimensional beam was used as a basis for the development of the parametric model, which in turn forms the basis for the ParaGen method. The model consisted of 46 nodes, of which 42 were parametric. The nodes were bound to the surface of a hypothetical tube, limiting the number of free variables to 2; the elevation in the x-direction \( (x) \) and the angle \( (\alpha) \). Compared with a model using 3 free variables, this lead to better output with respect to the proposed production method, and also reduced the necessary computational capacity. In addition, due to computational limitations the model was schematized using linear instead of curved members. The FEA model used a center point load of 50 kN, fixed supports, continuous members with fixed moment connections and the material properties of ASTM A-36 steel. Pipe profiles were used with selected diameter and wall thicknesses resulting from the structural analysis.

With this set up a total population of 1276 individual solutions were created algorithmically and stored in the SQL database. ParaGen optimized for two different fitness functions, i.e. minimal weight and highest modal frequency (stiffness). With the plot function a scatter diagram was created to find the most promising solutions which performed well for both objectives (Fig. 12).
The three most promising solutions were analyzed according to their morphology and the proposed production method. Solution 1269 was assessed to be the best solution, since it deviated the least from its curved counterpart.

The solution 1269 is also realized with 3D software to indicate the final result of the prototype.

7 RIGIDIZING INFLATABLES

To reduce expenses the models we have made and tested are rigidized with polypropylene fibre concrete. Finally the inflatable core will be rigidized with polymer composites in strings at the outer surface. The rigidization mechanisms with polymer composites developed over the last 50 years, including advanced mechanisms for use in space engineering, is impressive.

In 2002 Pronk realized a curved concentric arch consisting of a synthetic matrix of fabrics around an inflatable cured by injecting a thermoset resin. By varying the thickness, direction, structure and material of the fibers a range of E-moduli can be obtained. In the 2002 beam a change in layer composition was introduced in order to be able to produce the arch with the stiffness determined. The use of fibers with the E-module of 210 GPa (210,000 N/mm², comparable to steel) was used, but turned out not to be sufficient. By adjusting the E-module from 210 GPa to 60 GPa a new stiffness was found ($EI_y = 5.14 \cdot 10^{12} \text{Nmm}^2$). This led to the use of carbon fibers at the top and the bottom of the beam section and glass fibers in the other parts. (see Fig. 15).

The form of the membrane is influenced by the curvature and demands to realize slender sections at the ends of the arch. The optimization of this beam was achieved by:

- a combination of different materials within the sections;
- a compensated curvature of the beam before comprehensive loading; and
- variable moments of inertia ($I_y$) at different sections.
In the 2002 project an inflatable mould has been used to realize this beam. The mould was rigidized with a polymer composite by vacuum infusion. The material properties and production methods of polymer composite match the arch requirements. The advantages of polymer composite are amongst others: rigid and light-weight construction possibilities, fatigue resistance, chemical and corrosion resistance, freedom in design and form and the possibility to integrate parts. Disadvantages are the relatively high cost prices of material, mould, production (labor) and engineering. In the case of complex shapes, for example a conical arch, approximately 50% of the production costs consist of moulding. By using an inflatable mould the moulding costs can be reduced considerably. A rigidizable inflatable structure can be described as a structure that is flexible when inflated and becomes rigid after exposure to an external influence. After rigidizing it is not necessary to maintain the overpressure. There are several ways to rigidize and new techniques are being researched. They can be divided into three categories: thermosetting composite systems, thermoplastic composite systems and aluminum/polymer laminate system. In the following experiments we focused our research to form optimization. Budget wise we have realized the models with cement bound composites.

8 SCALE MODELS

As a first step in the development of the production method, several different geometries derived using ParaGen were fabricated on a scale of 1:5. The solutions used were the following and represent the criteria which are leading in this research:

- Id:1269 Highest specific stiffness
- Id: 799 Low specific stiffness
- Id: 1249 Lowest deflection
- Id:1259: Highest stiffness

The models were tested using a 5-point bending test to determine the influence of curved members versus linear members. For example, id 1269 has a lower theoretical modal frequency than id 1259. However, id 1259 deviates more from its curved model which is disadvantageous with respect to its stiffness.

8.1 Manufacturing method

The manufacturing method was developed in a way to control the different parameters as much as possible. In this way, the only difference between the models is the geometry. Every geometry was manufactured three times to give significant results. The models were cast in 2 weeks, casting one model per day, and were tested using a 5-point bending test in sets of 3 after 13, 14 or 15 days. Also, standardized mortar bars were made with every cast to test inconsistencies in the concrete composition. The low viscosity concrete mixture was as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Amount [Kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEM I 52.5 R</td>
<td>9.5</td>
</tr>
<tr>
<td>Limestone flour</td>
<td>3</td>
</tr>
</tbody>
</table>
The production technique used for the fabrication of the scale models was based on a method described by Dominicus 25 for producing concrete bone structures. The moulds for the concrete beams were constructed of multiple layers of PVC piping. Initially, clamps which represent the inverse of the beam were sawn out of standard PVC pipes with a diameter of 160 mm. Two layers of these clamps were mounted to a PVC pipe of 1.4 meters long. Then, two layers of high tenacity Lycra fabric were pulled over the PVC, followed by a third and final layer of clamps. The mould was then mounted in a steel bracket keeping it in place during the cast, and allowing the model to rotate about its axis to prevent sagging of the uncured concrete. Treaded rod was mounted into the top and bottom ends of the model, allowing every model to be fixed identically in the bending test. Since the model was pinned at the top and bottom of each side, the two form a force couple.

### 8.2 Results

All the scale models were tested in a 100 kN pressure bench at the van Musschenbroek laboratory at the TU/e campus. The weight of the beams varied between 23 and 33 kg due to member size fluctuations in the moulds. However, the distribution of the material within the model influenced the outcome more than the weight. The results of the mortar bars showed no significant variations with respect to the material density and yield strength. However, the flexural strength of the concrete mixture for id: 1249 was 27% less than the mixture for the other geometries due to the use of a different plasticizer (Glenium 51 con. 35%).

ID: 1269 failed in all the beams in the tension zone which runs helically around the beam. This is natural given the properties of concrete, and means that there are no clear weak parts in the geometry. In every geometry, the members always fail near the nodes. The structural analysis performed in GSA and Staad PRO showed that stresses are the highest near the nodes due to the fixed moment connections, which explains the crack pattern of the scale models. ID: 1259, and to a lesser extent id: 1249, showed a concentrated crack pattern near the cross members at the supports. The morphology of the members prevents the formation of a stable cross locally,
causing the node to fail due to buckling and not because the yield limit of the material is reached. Therefore, large angles between members should be avoided to prevent the nodes from local buckling, and the member length in transverse direction should be minimized.

To determine the stiffness of the four different geometries, the linear elastic region of every beam was determined using trendlines. In this region, according to Hooke’s law, the deformation $\varepsilon$ is proportional to the force $F$. All the beams showed this linear behavior until a first crack occurred, which caused the deformation-force diagram to behave unpredictably due to the use of polypropylene fibers. Figure 17 shows the average linear elastic region of the four geometries, where the angle $\alpha$ is used as a measure for the stiffness. This confirms that ID: 1259 indeed has a disadvantageous geometry with respect to curved members, since it was initially optimized for straight linear members. Now, ID: 1269, which is also the case for the remainder of the research, has the highest stiffness. It has to be noted that ID: 799, which had the lowest theoretical stiffness, is the second stiffest geometry according to these results. This might be due to the theoretical optimization with steel and linear members, while the scale models have curved members and are produced with concrete.

![Fig. 17. Average linear elastic regions of the tested geometries](image)

9 CONCLUSION

The aim of this research was to develop and refine new production techniques for producing optimal structures. Optimal structures can be described as a structure with a minimal material use and maximal strength and characterized as an open cell and complex structure. In this research we use different techniques to generate optimal structures, Solidthinking Inspire has been used for topological optimization and ParaGen is used for size and shape optimization. A gap was noticed in this research between the production and generating of optimized structures. Traditional framework cannot be used to produce these kind of structure, because of the high amount of material (formwork) and the necessary labour hours. Fabric formwork can provide an alternative. Current traditional fabric formwork is combined with additional (wooden) falsework. In this research we described a process of producing optimal structures without additional falsework by applying rigidizing inflatables. The scale models, we produced with fabric formwork and concrete, showed us, a smooth
surface and a high accuracy to the digital models. The scale models are tested with structural analysis programs (Staad PRO and GSA) and physical tests (pressure bench). The digital geometry with the highest stiffness from the structural analysis program, indicates similar results in the physical tests. The combination of structural optimization and production method with rigidized inflatables is promising. Applications of rigidized inflatables are found in contemporary architecture such as emergency relief and military purposes. More research to validate and optimize the production technique is necessary.

10 ACKNOWLEDGEMENTS

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REFERENCES

SINGLE-LAYER SPHERICAL GEODESIC DOMES INTERACTING WITH A NETWORK OF INFLATABLE BEAMS

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Key words: Spherical Lattice Shells, Single-layer Networks, Inflatable Structures, Mechanics Solids Interacting, Composites.

Summary. In this paper we propose an example of a thin geodesic spherical dome composed by a lattice of inflatable interacting beams. The geometry of the lattice is determined by searching for the minimum variation of the characteristic dimensions of the elements that make up the dome (length of the bars and area of the panels). A first illustration of the mechanical response of the structure is given, with reference to the loads that usually are assumed to act on this type of shell.

1 INTRODUCTION

In single layer spherical lattice shells, the simpler and more widespread arrangement of the structural elements consists in placing the bars along the sides of a regular triangular mesh drawn on the surface. The reasons in favour of this type of network are to be found primarily in its high structural performance; in fact, these shells allow for structures characterized by high stiffness, by conveniently exploiting the very low in-plane compliance possessed by each single triangular mesh.

Other factors that are usually considered to be of secondary importance, can gain a certain importance in the choice of the type of lattice. Among them, the possibility of using prefabrication techniques for the realization of the various structural elements is certainly the most important (Makowski, [1]). The shells lattices, as happens for all spatial structures, are constituted, in fact, from a large number of different parts (joints, bars, cladding panels). To this regard, to be able to standardize as much as possible the size of each component allows for a significant reduction of the costs of implementation (Tarnai, [2]).

In this perspective, the use of cylindrical inflatable beams instead of the usual metallic elements provides numerous benefits ([3], [4]). First of all, it is possible to achieve a reduction, which can be considerable in some cases, of the total weight of the structure. Secondly, the pre-tension that is established in the beams during their inflation phase assures
that each panel of the shell is in a state of traction, thereby reducing the risk that the beams may undergo to phenomena of loss of stability of the equilibrium configuration. For these reasons, the shells obtained from the assembly of spherical lattice inflatable beams seem to have a good chance of being profitably used for building structures, both permanent (for these applications, the use of materials called “rigidifiable” looks promising) and temporary.

When using inflatable beams, one has to reconsider the whole organization of the design and realization of these structures. In particular, the presence of inflatable elements, which acquire stiffness by virtue of their internal pressure, and the assembly of the structure, for which no external equipment is needed (self-deployment structures), represent as many peculiar characteristics to be properly taken into account from the beginning of the design process (Comte et al. [5]).

In this work we describe an example of a spherical geodesic dome to be implemented through a network of interacting inflatable beams. The geometry of the lattice is determined by trying to minimize the variation of the characteristic dimensions of the elements that make up the dome (essentially, the length of the beams and the area of the panels). The mechanical response of the structure is discussed briefly.

2 SPHERICAL GEODESIC DOMES

The use of domes as roof structures is well documented since the ancient times. Beautiful examples of domes can be found in the buildings erected by early civilizations. Without aiming to provide a compendium of the history of the domes in civil construction, we only observe that the choice of the materials and of the particular type of structure has constantly been evolving. In a relatively short time, the typology passed from the initial thin spherical masonry domes, sometimes spherical and with slightly variable or constant thickness, requiring remarkable centring, up to the latest and thin reticular metallic single-layer spherical domes, which do not require any kind of centring. The transition between the lattice domes with hierarchical elements and the current homogeneous ones, without any hierarchy between elements (such as, for example, the lamellar domes), has been even more rapid.

Parallel to the typological evolution, the materials used have also been changing. Driven by the need to reduce as much as possible the cost of maintenance, the domes designers quickly passed from common structural steel to stainless steel, up to the present and widespread use of elements in aluminium alloys. Following this route, we arrive in the present day to the use of fiber-reinforced composite materials (textile materials).

2.1 The issue of the strength and the stability of lattice domes

In the modern lattice metal domes, with no hierarchy between elements of the same type and operating almost exclusively in extensional regime, the issue of the resistance of the different structural components (joints, bars and panels) is of secondary importance. At least during the last two centuries, the characteristics dimensions of the thin spherical domes have remained almost unchanged, as well as the intensity of the live loads (actions of wind and snow). On the other hand, the strength of the materials has increased considerably and it must be considered that, in the presence of an extensional stress state, the structural elements can carry out their resisting function at best. It therefore follows that, nowadays, other aspects are
to be considered as fundamental for the search of an optimal structural solution.

The major issue that rules the choice of the particular lattice domes is without any doubt the stability of equilibrium. In fact, the increase in the strength of the new materials, and the small changes or even the lowering of the loads imposed on the structure, turned out in a progressive decrease in the size of the resistant elements. On the one hand this has resulted in a substantial economy in the use of materials, but on the other, being also decreased the stiffness of the elements, the structures have become more susceptible to buckling than in the past. The loss of stability can occur at the local level, if it involves single bars or panels, whenever to the latter are also assigned resisting functions, in addition to coating functions, or at the global level, if the phenomenon concerns one or more joints and the bars directly connected to them.

2.2 The spherical geodesic lattice domes

The search for technical solutions corresponding to a reduction of the costs of construction of the dome pushes very often to a standardization of the dimensions of the structural components. In this sense, Füller scored an extremely important result with the “reticular geodesic dome”. The idea of Füller is to find a simple law that allows drawing a triangular mesh on a sphere (spherical grid). The method chosen is the projection on the spherical surface of the thirty sides of an icosahedron, concentric to the sphere. On each face of the original icosahedron is then traced a net with sides parallel or perpendicular to those of the same face. In both cases, the plane lattice thus obtained is projected radially from the centre of the icosahedron on the circumscribed sphere. This method of division has been the subject of a famous patent filed by the same Füller in December 1951 (Füller, [6]).

The result of this second subdivision and the subsequent projection is that the dimensions of the sides, as well as those of the faces, in which the sphere is now divided are no longer equal to each other, although such differences are still technically acceptable. The elements (bars/panels) placed near the vertices of the icosahedron will have minimal length/area, while those located near the centres of gravity of the faces might be larger; these regions are therefore equipped with lower stiffness.

The mesh of each spherical geodesic dome is chosen in such a way to reduce the variance of the length of the bars, as far as possible, by exploiting various symmetries. The most important are listed below.

a) “binary symmetry” or rotational symmetry with respect to each axis passing through the midpoint of two diametrically opposite sides of the icosahedron. The lattice overlaps with itself when it is rotated by 180° around each of those axis. This symmetry greatly simplifies the design of the spherical grid since there are 15 binary axes of symmetry.

b) “ternary symmetry” or rotational symmetry around each axis passing through the centre of gravity of two faces of the icosahedron that are perpendicular to it and diametrically opposed. The lattice overlaps with itself when it is rotated by 120° around each of those axis. Such symmetry is the more evident, since it allows to simply checking which elements of each face correspond to each other and, therefore, have same dimensions. Since there are 10 symmetry axes ternary, standardized
production of the elements is strongly favoured.

c) “quinary symmetry” or rotational symmetry around each axis passing through two diametrically opposite vertices of the icosahedron. The lattice overlaps with itself when it is rotated by 72° around each of those axis.

The presence of these symmetries not only decreases the number of different components (joints, bars, panels), but, above all, facilitates the automatic generation of the entire lattice, once provided the value of the radius of the sphere and of the parameters of tessellation. By this way, several spherical lattices can be compared with each other allowing to choose the one that suits best the design requirements of the dome.

3 A LATTICE OF INFLATABLE BEAMS

The technical solution for domes, which we intend to illustrate in its main features in this paper, is characterized by the use of inflatable elements as beams and membranes for the panels. This choice is motivated primarily by the opportunity to obtain by this way domes considerably lighter than those made with metallic beams, and that can be quickly built and removed without having to resort to cumbersome centring or lifting devices. These characteristics represent as many advantages in the case of temporary constructions or whenever the dome has to be erected very quickly, as it occurs, for example, following a calamitous event. Other properties of some interest, which we will discuss in more detail in the following, concern the ultimate behaviour near collapse that is reasonably expected for this type of structures. The inflatable beams, as well as panels, are, in fact, in a state of pre-tension, induced by the internal pressure in the beams, which exerts a beneficial action on their load-bearing capacity and on the stability of the equilibrium configuration. Moreover, even once the limit load that corresponds to the onset of a collapse mechanism for the dome is reached, the dome will return in its initial configuration, occupied before the application of the load, if the load is removed (reversible shakedown).

4 A FIRST APPLICATIVE EXAMPLE: AN OPEN GEODESIC DOME

In order to illustrate by an example the proposed technical solution, let us consider the open geodesic dome shown in Figure 1.

The structure consists essentially of a thin hemispherical dome interacting with a triangular lattice of beams arranged in a single layer. The radius $R$, both of the membrane and the mean surface of the lattice of beams, is equal to 10.0 m.

The chosen geodesic dome is a simple layer one, characterized by a tessellation of type “alternating parallel” $\{3,5+\}_{10.0}$ having a triangulation number $T = 100$. If the lattice would cover the entire sphere, the number of nodes $V$ would be equal to $V = 10T + 2 = 1002$; moreover, $E = 30T = 3000$ and $F = 20T = 2000$ would represent, respectively, the number of edges and the faces of the polyhedron (we recall that $F + V = S + 2$, according to Euler's formula for the uniform polyhedra). For reasons of space, here we omit the details that would justify the above-mentioned relations; for more details about the classification of the tessellation of a geodesic dome and about the geometric relationships existing between the triangulation number and the number of vertices of the polyhedron, we refer to (Ligarò, [7]). In the specific case, the dome being a semi-sphere, the vertices (nodes) reduce to $V = 526$, the
edges (i.e., the bars) to $E = 1525$, and the panels (faces) to $F = 1000$.

Both the bars and the panels are made of the same composite material. Each bar is a thin walled cylinder (diameter $d = 12.0$ cm, thickness $t = 0.1$ cm, average length $h = 125$ cm). The panels are plane triangular nearly equilateral membranes with thickness $t = 0.1$ cm. The composite material is formed by glass fibers (S-Glass) and a matrix thermoplastic resin (PVC), 70% in volume. In each thin structural element (i.e., wall tube or panel) a plane stress condition is assumed, the constitutive relation for the material is shown below:

$$
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} = 
\begin{bmatrix}
1/E_1 & -\nu_{12}/E_1 & 0 \\
-\nu_{12}/E_1 & 1/E_2 & 0 \\
0 & 0 & 1/G_{12}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} = \mathbf{D}\mathbf{s},
$$

(1)

where $E_1 = E_2 = 6110$ kN/cm$^2$, $G_{12} = 333.5$ kN/cm$^2$, $\nu_{12} = 0.274$. The composite material is assumed to have a thermal expansion coefficient $\alpha = 1.38 \times 10^{-5}$ C$^{-1}$ and a specific weight $\gamma = 1.961 \times 10^5$ kN/cm$^3$.

In the following, the distribution of stresses in the beams and the panels of the geodesic dome are shown for different load conditions. For what concerns the initial optimization phase of the structure we refer to (Ligarò, [7]); the static analysis is carried out by using a common commercial finite element software, assuming a linear elastic behaviour of the structure. In particular, we consider the four elementary load conditions listed below:

- inflation of beams;
- structure’s self weight;
- snow load;
• wind load.

The final internal pressure of all the inflated beams is set equal to 2 bar. The static actions due to snow and wind were evaluated according to the technical regulations currently in force in Italy [8]. In this regard, it is assumed that the building is in a location at the sea level in the province of Pisa, in Tuscany.

4.1 The inflation phase

In the solution adopted, the dome is built by simply inflating the beams that make up the lattice up to reach an internal pressure of 0.02 kN/cm². The distribution of the axial force in the beams and that of the minimum principal stress in the triangular panels is shown in Figure 2.

![Figure 2: a) axial force in the inflated beams (min = 0.54 kN; max = 0.87 kN); b) minimum principal stress in the shell elements (min = 0.03 kN/cm²; max = 0.28 kN/cm²).](image)

The figure clearly shows that the regularity of the tessellation allows for obtaining a state of pre-tension in the beams, which is characterized by a variability contained within more than acceptable limits (the minimum axial force is equal to 60% of the maximum). The pre-traction is almost uniform for triangular panels also, except for the layer that is directly in contact with the fixed constraints placed at the base of the hemisphere.

With reference to this first phase, it has also to be noted that the average value of the tensile stresses in the panels is equal to about 0.46 kN/m, and that the work required for the inflation of the structure is equal to about 8480 kNm. By way of example, we observe that the work of inflation would be considerably less (of the order of 1000 kNm) if a uniform thin shell membrane would be put under pressure from the inside of the dome, while keeping the constant the average stress in the panels. In other words, for a fixed stress level in the material, the proposed solution allows storing a much larger amount of elastic energy in the elements of the dome, compared to the so-called pneumatic constructions.

4.2 The dome self-weight

The stresses in the structure that are produced by the combined action of the dome self-weight and the pre-traction state induced by inflating of the beams are shown in Figure 3.
Given the extreme lightness of the dome, the effect of the weight of the structure produces, as was expected, only slightly appreciable changes in the distribution of the stresses with respect to the previous case. It has also to be noted that the small value of the thickness of the panels ($t = 0.1$ cm) inevitably leads to consider the typical problem of stress concentration (localized reinforcement), as commonly happens near the edges of any tensile structure.

4.3 The snow load

If the stresses assessed in the previous section are added to those due to snow load, we obtain the results shown in Figure 4. As regards the magnitude of the load, this was evaluated according to the NTC2008. In particular, it was assumed that the building is placed at the sea level, in the province of Pisa (therefore, it falls within the zone III), and that the exposure coefficient is equal to unity. Under these conditions, the design value of the snow load is equal to 0.48 kN/m².

It is noted that the snow load, when it reaches the design value, causes the onset of compressive stresses in the inflated beams. Since in the model we have adopted the beams are not able, by hypothesis, to support any compressive stress, it follows that already at this load level the structure will exhibit a mechanical response of non-linear type, although scarcely
perceptible. In this regard, we observe, in fact, that the percentage of the beams subject to an appreciable compression (say higher, in absolute value, to 0.1 kN) is equal to 4%. The same percentage would drop to one per cent if the magnitude of the snow load were reduced by 10%. Even the triangular panels are subject to tensile stresses almost everywhere, except the narrow band directly in contact with the constraints placed at the base of the dome.

4.4 The wind load

If the stresses evaluated in section 4.2 (weight + inflation) are added to those produced by the wind, we obtain the results shown in Figure 5. As regards the magnitude of the load, this was evaluated according to the NTC2008. In particular, it is assumed that the construction is in Tuscany, and therefore in the area 3, that the category of exposure is the third and that the roughness class of the soil is B (the one typical, for example, of the urban areas). Under these conditions, the design value of the wind load is equal to 0.46 kN/m². To determine the pressure on the generic element of the dome, the design value of the uniform distributed load is multiplied for the coefficients of exposure and shape, $C_e$ and $C_p$. The value attributed to these coefficients depends on the particular inclination of the cover: the corresponding values of $C_e$ and $C_p$ were calculated for each panel before performing the static analysis.

As it was expected, the wind is the more engaging load for the dome structure. The first observation, which clearly emerges from Figure 5, is the presence of some compressed beams and panels. This result is clearly inconsistent with the particular type of structure we have chosen to use. The inflatable beams can withstand only very small compressive stresses that, in any case, are negligible compared to tensile stresses. As a consequence, an incremental stress analysis should be performed: once a beam becomes unstressed it should not be considered in the subsequent load increments. By this way, it is reasonable to believe that the distribution of stresses would show some differences, although not so large, with respect to that shown in Figure 5. In particular, the lattice of beams, as well as the set of panels, would present an inactive part, where the elements are unstressed, and an active (tense) part. Moreover, the tractions will be greater than those calculated in the linear scheme that we have adopted.

Figure 5: a) axial force in the inflated beams (min = -0.17 kN; max = 1.65 kN); b) minimum principal stress in the shell elements (min = -0.07 kN/cm²; max = 0.11 kN/cm²).
A second observation, contrarily to the first one, makes reasonable assuming less noticeable differences between the linear and nonlinear solution. In the dome region exposed over the wind, the pressure may be represented as a system of inward pressing forces on the nodes of the lattice. The beams converging at any one of these nodes will be subjected to a state of compression. However, as soon as the first compressive stresses appear, the stiffness of the inflatable beams reduces considerably until it becomes evanescent, and the node becomes free to move in the direction of the force acting on it. The displacement of the node continues until the local curvatures of the dome reverse their sign and the beams become stretched (anticlastic curvature), thus regaining their stiffness (Pomeroy, [9]). In other words, by taking into account the above-described phenomena, the compression obtained by the linear analysis may be considered, in some sense, as “apparent”, since they could be substituted by tensile stresses, at least in some cases, if a nonlinear analysis would be executed.

Finally, we observe that the dome might not be perfectly airtight for strong winds. In these cases, the intensity of the resulting wind pressure on the dome reduces considerably. The severity of the state of stress produced by the wind suggests the opportunity, however, to make some changes in the overall organization of the structure, by adopting suitable measures able to reduce the effects of such actions. In this regard, a first simple solution might be to differentiate the intensity of the inflation pressure according to the position of the beams.

5 CONCLUSIONS

- In this work we shown a first example of spherical geodesic dome made by a network of medium pressurized inflatable beams, interacting with an elastic membrane shell.
- The design problem of the dimensioning of the elements that make up the dome has requested, to be solved, the analysis of different topics. A first basic item is represented by the choice of the particular network of beams. Here we have chosen a suitable tessellation (alternating parallel) defined by the number of triangulation. The geometry of the lattice was found by trying to make minimum, as much as possible, the variance of the length of the bars.
- The main characteristic feature of this study is the proposal to use light inflatable beams made of composite material instead of the usual metal elements. This choice allows obtaining constructive solutions of lower weight, which may be built-up and removed quickly, without having to resort to cumbersome centring or lifting devices. The inflatable beams, as well as the panels, are in a state of pre-traction, induced by the internal pressure in the beams, which exerts a beneficial action on the load bearing capacity and on the onset of buckling phenomena.
- Even if the limit load for the dome is reached, and a collapse mechanism is activated, the dome will return to its initial configuration once the load is removed. Finally, the lattice of inflatable beams is considerably less vulnerable to incidental damage, compared to the solutions of the pneumatic type. The final example has shown the possibilities offered by this type of technical solution.
REFERENCES

STRESS ANALYSIS OF INFLATED POLYHEDRA FOR THE 32-PANEL SOCCER BALL

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Key words: Soccer ball, polyhedron, inflation of membrane, dynamic relaxation

Abstract. This paper presents a stress analysis of a membrane modelling the 32-panel soccer ball. The most popular soccer ball type and three variations are considered. The discretized mesh of the stress-free polyhedron-shaped membrane is subjected to internal pressure and the sphericity and the stress distribution of the models are compared.

1 INTRODUCTION

In spite of that soccer has been one of the most influential sports games for decades, scientific analysis on the geometry of the soccer ball has received significantly less attention. Since its origin in the 19th century the history of modern soccer witnessed a vast variety of balls shapes. They mostly consisted of flat panels of leather sewn together and inflated by means of a bladder. The 32-panel icosahedral ball type has become the most widely used model since the 1970’s and is still one of the models sold in the largest numbers worldwide. Though materials used for fabrication turned from leather to synthetics, its composition remained an assembly of twelve pentagonal and twenty hexagonal panels representing one of the Archimedean solids, the truncated icosahedron. It served as the official ball for several world championships.

A recent approach in soccer ball design introduced curved panels thermally bonded in order to create a more precise approximation of the sphere (e.g. Teamgeist or Jubilani), however, the 32-panel ball has remained the most popular ball.

Early attempts to create a round ball of flat pieces based on heuristic constructions lacking probably any scientific foundations. As the game gained increasing importance
ball shapes based on regular and semiregular polyhedra were created requiring geometrical background for the analysis. There have been a number of approaches for the definiton of the roundness of spatial solids from different fields of science. Among them the most widely used term considered in mathematics is the so-called isoperimetric quotient ($IQ$), introduced by Polya [1] in its present form:

$$IQ = 36\pi \frac{V^2}{A^3}$$

where $V$ and $A$ denote the volume and the surface area of the body, respectively. The scaling factor $36\pi$ is introduced so that $IQ$ yields 1 for the unit sphere. A central problem in mathematics of spatial bodies is the isoperimetric problem, which states: among a set of bodies of given surface area, which one encloses the largest volume (largest $IQ$), i.e. which one approximates the sphere the most closely?

In a previous study the authors of this paper addressed the issue of roundness of the 32-panel icosahedral soccer ball [2], using a set of geometrical properties to quantify the roundness of a spatial solid introduced previously by the first author [3]. This paper considers four different types of 32-panel soccer ball geometry, all commercially available: the truncated icosahedron, which is an Archimedean solid, the Geo 0.84 model by Nike, the equal panel area ball by Puma, and the Hyperball® by Prof. P. Huybers [4]. Our aim is to simulate the inflation of a polyhedron-shaped membrane representing the soccer ball. Discretized geometric models of all types are created and subjected to internal pressure. The key parameters of the analysis are the density of the finite mesh, the applied pressure, and the material stiffness of the membrane. The roundness of the inflated shapes are evaluated and compared. The next three sections outline the model for the computations, the most important results, and the major conclusions.

2 MODELLING

2.1 Geometrical model

The initial (undeformed) shape of the ball is defined by the truncated icosahedron or its variations. The first model is the Archimedean truncation of the icosahedron representing the most popular ball configuration. The Archimedean truncation yields regular pentagons and hexagons, i.e. all edges are of the same length. The second model is Geo 0.84 by Nike where the ratio of edges between hexagons and edges of the pentagon is approximately 0.839. This configuration is obtained by a nonregular truncation of the icosahedron resulting in larger pentagons. It is claimed that such truncation results in the optimal stress distribution in the inflated structure. The third model (Puma) is a truncation of the icosahedron in a way that it yields pentagons and hexagons of equal surface area. The fourth model is not purely a truncation of the icosahedron but a more sophisticated form even though it is made of 32 panels. The geometry of the Hyperball is derived from the isodistant truncation of the icosahedron (i.e. when all panels are at
András Lengyel and Krisztián Hincez

Figure 1: Triangular mesh models for four ball types: (left to right) the truncated icosahedron (Archimedean), Nike Geo 0.84, Puma equal face area ball, Hyperball.

the same distance from the centre of the inscribed sphere). This polyhedron is modified by further truncation of the common edges of hexagons forming rectangular faces. Both truncations are defined in an isodistant way meant to produce the roundest shape. The key feature of this model is the reduction of the large number of faces by dividing the rectangles to triangles and trapezoids, which then can be joined with the neighbouring pentagons and hexagons. (A detailed description is given in [4].) Sketches of the four models are given in Figure 1.

In order to have a realistic approximation of the sphere, all polyhedron faces are divided into a number of triangles. The triangulation has been performed in a way to preserve the icosahedral symmetry of the surface. All pentagons, hexagons, and rectangles are first divided into five, six, and four equal triangles, respectively by drawing radial lines to the vertices from the centroids of the faces, which are then further divided into smaller triangles by placing points along the edges at equal distances. In the case of the fourth model, the ratio of the sides of some of the triangles were inconveniently large, therefore an alternative triangulation has been applied in parts of the domain, also preserving symmetry.

2.2 Structural model

The surface of the polyhedron is regarded as a thin membrane of isotropic material characterized by its in-plane stiffness and Poisson’s ratio. The triangular elements may develop stresses and strains in the plane determined by the position of their vertices and free rotational displacements may occur at the edges. Internal pressure inside the closed membrane is represented by a distributed load of equal intensity on all elements.

We apply the dynamic relaxation method (DRM) [5] to determine the equilibrium shape of the structure. It is an iterative technique widely used for tensile structures. Fictitious lumped masses are assigned to the vertices of the mesh. In each time step of the iteration, the position, the velocity, and the acceleration of each vertex is computed, and the equations of motions are numerically solved. The final deformed shape is obtained when the vertices are balanced and have zero velocity.
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Figure 2: Isoperimetric quotient $IQ$ for the four ball types (1: the truncated icosahedron (Archimedean), 2: Nike Geo 0.84, 3: Puma equal face area ball, 4: Hyperball) against mesh density parameter $N$.

3 RESULTS

All initial (uninflated) models are scaled to have volume equal to that of a 100 mm radius sphere. Once DRM yields the final equilibrium positions of all nodes in the mesh, geometrical and mechanical evaluation of the deformed shape is performed. The isoperimetric quotient $IQ$ is computed via the surface area $A$ and volume $V$. Values of $IQ$ show convergence with respect to the density of the mesh. The density is characterized by the number $N$ of the sections along the edges of any face of the polyhedra. (E.g. $N = 3$ is applied for the generation of shapes in Figure 1.) It is found in a series of test computations that $IQ$ does not vary significantly for $N > 12$ (see Figure 2) therefore density $N = 12$ has been applied for further analysis.

The best isoperimetric quotient has been obtained for the standard model and for Geo 0.84, followed by the equal face area model, and the Hyperball.

The stiffness of the membrane and the applied pressure have been set to obtain realistic inflated shapes. FIFA Inspected quality for soccer balls requires balls to have circumference 68 to 70 cm among various specifications. We set normal stiffness $S = 40 \text{kN/m}$, Poisson’s ratio $\nu = 0.2$, and internal pressure $p = 1.0 \text{atm}$ for the analysis.

Stresses for all four models have been computed, see Figures 3 to 6. (In this paper the term ‘stresses’ refer to specific forces in sections of the membrane in units of force/length as customary for tensile structures and not in units of force/area as in continuum mechanics.)
Figure 3: Distribution of maximum (left) and minimum (right) principal stresses in the membrane for model 1. Blue and red colours denote high and low stress levels, respectively.

Figure 4: Distribution of maximum (left) and minimum (right) principal stresses in the membrane for model 2. Blue and red colours denote high and low stress levels, respectively.

Figure 3 shows the distribution of principal stresses for model 1. The maximum principal stresses (left) are large in the middle of the hexagonal panels and slightly smaller in the pentagonal faces. Strips of large stresses connect the centres of neighbouring faces indicating the directions of characteristic stretching in the membrane. Local low stress regions are formed around the vertices of the panels. Minimum principal stresses (right) range over a wider interval. Peaks are measured in the middle of the hexagonal panels whereas peaks in the pentagonal faces are slightly smaller. Stresses along the edges are approximately uniform with medium intensity, and the smallest values are again obtained around the vertices. It indicates that these regions are moderately affected by inflation.
Stresses of models 2 (see Figure 4) and 3 (see Figure 5) are characterisically similar to those of model 1. The pentagonal faces remain regular but the symmetry of the hexagonal faces reduces to \(D_3\). With larger pentagons, the ratio of the long and the short edges of hexagons increases. Distribution of stresses corresponds to the symmetry of the shape of the structure. It is found that peaks of the maximum principal stresses occur in the strips perpendicular to long edges while stresses at the centres of panels and in strips near short edges decrease. Distribution of minimum principal stresses varies moderately. As the size of the pentagonal faces increase relative to that of the hexagonal ones, the peaks in the middle of all faces increase or decrease with the surface area, accordingly, compare models 1 to 3.

The layout of model 4 (see Figure 6) is characteristically different from the other three regarding the number of faces and the ratio of edges. Three sides of each hexagons are significantly smaller than the rest resulting in a significantly different stress distribution. Regions around the vertices are joined in pairs, and the hexagonal patterns become practically triangular. Largest values of the maximum principal stress occur in the hexagonal and the rectangular faces forming a contiguous region of nearly uniform stresses surrounding isolated pentagonal parts of smaller stress. It indicates that the loadbearing is primarily provided by the frame of hexagons and rectangles. (In the actual fabrication rectangles are divided and joined with the neighbouring faces.) The minimum principal stresses are more evenly distributed than in the previous cases. Smallest values again occur around the vertices.

4 CONCLUSIONS

Maximum and minimum stresses as well as isoperimetric quotients for all inflated models are summarized in Table 1. Ranking of the models with respect to \(IQ\) in decreasing
order is Nike Geo 0.84, the standard truncated icosahedron, the Puma equal face area ball, and the Hyperball. It is found that the largest $\sigma_1$ stresses are practically in inverse correspondence with roundness: $\sigma_{1,\text{max}}$ decreases with $IQ$ increasing except that Hyperball has rank 3. It suggests that lower peak stresses are expected with better roundness. Also, with increasing $IQ$, the smallest $\sigma_1$ values in the membrane increase closing the gap between maximum and minimum, i.e. more evenly distributed stress field is expected. Principal stresses $\sigma_2$ vary more, though it can also be concluded that among the truncated icosahedral models the gap between the maximum and minimum decreases with $IQ$ increasing. It is interesting to note that Hyperball performs significantly better in this respect, which can be attributed to the double truncation to smoothen the peaks of the polyhedron. Overall, Geo 0.84 and the standard ball produce the best roundness values and are the most efficient in reducing the largest stresses and their deviation. The special geometric configuration of the Hyperball proves capable of involving vertex regions into the loadbearing.

ACKNOWLEDGEMENTS

The research reported here was partially supported by OTKA grant no. K81146 awarded by the Hungarian Scientific Research Fund. This paper was also supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

REFERENCES


Table 1: Comparison of four ball models with respect to the largest and the smallest values of principal stresses $\sigma_1$ and $\sigma_2$, and the isoperimetric quotient $IQ$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma_{1,max}$</th>
<th>$\sigma_{1,min}$</th>
<th>$\sigma_{2,max}$</th>
<th>$\sigma_{2,min}$</th>
<th>$IQ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard TI</td>
<td>6.350</td>
<td>4.769</td>
<td>6.082</td>
<td>3.310</td>
<td>0.999710726</td>
</tr>
<tr>
<td>Nike Geo 0.84</td>
<td>6.279</td>
<td>4.819</td>
<td>6.032</td>
<td>3.274</td>
<td>0.999725590</td>
</tr>
<tr>
<td>Puma equal face area</td>
<td>6.399</td>
<td>4.666</td>
<td>6.107</td>
<td>3.109</td>
<td>0.999639643</td>
</tr>
<tr>
<td>Hyperball</td>
<td>6.366</td>
<td>4.419</td>
<td>5.943</td>
<td>3.627</td>
<td>0.999519980</td>
</tr>
</tbody>
</table>


THEORY AND EXPERIMENT RESEARCH ON
THE STATIC CAPABILITY AND DYNAMIC PROPERTY OF
INFLATABLE BEAMS

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Key words: Inflatable Beam, Load Bearing Capacity, Dynamic Property, Experiment.

Summary. The paper analysis static load bearing capacity and dynamic properties of two examples of inflatable beams, the membrane material selected for the first example is the ETFE coated fabric, and the membrane material of the second example is airship envelop fabric, Uretek3216L. The load-displacement curve of inflated beam in bending is obtained and compared with the experimental data. The deflections calculated of ETFE inflatable beams show coincidence with measured curves, but the deflection calculated of airship envelop inflatable beam has deviations between calculated and experimental results. The dynamic properties of ETFE inflatable beams showed good coincidence with experimental results, but the first frequency of airship envelop inflatable beam have difference with experimental results.

1 INTRODUCTION

The paper analysis static capability and dynamic property of two examples of inflatable beams, the membrane material selected for the first example is the ETFE coated fabric, and the membrane material of the second example is airship envelop fabric, Uretek3216L. The wrinkles are easy to be formed when a tip load is applied on the inflated beams, the implicit scheme can lead to severe instabilities due to the lack of stiffness in the fabric, explicit time schemes overcome this difficulty, but they need a huge number of time steps to obtain a realistic stable final shape. The simulation of inflated beams by the FEM adopting these two methods.

2 TECHNIQUE FOR INFLATABLE BEAMS

2.1 Inflating simulation

In this work the influence of the pressure and volume coupling on structure behaviour is studied, a technique with a layer fluid element connecting all nodes of the membrane is implemented into a finite element model to take into account the influence of gas volume
variation, with corresponding change in pressure for enclosed gas on the stiffness of inflatable membrane structures\cite{1}.

2.2 Imperfection planting

Initial imperfection planting in inflated beam can make the bending analysis more smooth, the top 10 order bulk modes were selected to form the imperfection of inflated beam, the imperfection of the inflated beam is selected as 20\% of the thickness of fabric.

2.3 Contact simulation

The membrane body will mutually contact when the inflated beam was bended to a certain extent, in addition, the end part of beam will contact with the fixed support component. The aim of simulating contact is to confirm the contact area and to calculate the contact pressure. Coulomb friction was used to describe the friction model between contact surfaces in both implicit scheme and explicit method, the friction coefficient is set as zero.

3 ETFE INFLATABLE BEAM

The dimensions of the ETFE inflatable beam are shown in Figure 1 (unit is mm), the thickness of ETFE coated fabric is 250 \( \mu \)m. The ETFE coated fabric is isotropic material, the elastic modulus of ETFE is selected as \( E_{\text{Warp}} = E_{\text{Weft}} = 0.68\text{kN/mm}^2 \), and the Poisson’s ratio is selected as 0.38. The nonlinear load bearing capacity of inflated beam was analyzed when a homogeneous load was applied on loading belt, three points are selected to record load-displacement curves of them, positions of three points are also shown in Fig.1.

![Fig.1: The etfe inflated beam and point layout](image)

3.1 Static load bearing capacity analysis

The wrinkles are easy to emerge when bending load is applied on the inflated beams, the implicit scheme can lead to severe instabilities due to the lack of stiffness in the fabric, explicit time schemes overcome this difficulty, but they need a huge number of time steps to obtain a realistic stable final shape, the simulation by the FEM of inflated beams adopting these two methods. FEM calculations were conducted at air pressures values of 3000 and 5000pa, the load-displacement behavior of ETFE inflated beam adopting implicit scheme is shown in Fig.2, and the load-displacement behavior of the ETFE inflated beam adopting
explicit scheme is shown in Fig.3.

![Fig.2: Analytical load-displacement behavior of the ETFE inflated beam adopting implicit scheme](image1)

![Fig.3: Analytical load-displacement behavior of the ETFE inflated beam adopting explicit scheme](image2)

Seen form Fig.2 and Fig.3, the calculation is not convergent when the ETFE inflated beam was bended at certain degree adopting implicit schemes, but the calculation can continue to destination when adopting an explicit scheme. The load-displacement behavior before the beam is unstable using an explicit scheme show a correspondence with that using an implicit scheme.

### 3.2 Experiment of static load bearing capacity

The full-scale ETFE inflated beam was fabricated and the deflections of three points under bending are measured. An experiment of ETFE inflated beam under bending is shown in Fig.4. The experimental displacement-force curves of three points for air pressure of 3 and 5kPa are shown in Fig.4
The deflection calculated of ETFE inflatable beams show excellent coincidence with measured curves comparing Fig.2, Fig.3 and Fig.5.

3.3 FEM results of dynamic properties

The frequencies and modes of beams are conducted by FEM for two cases, viscosity acting force of outer air is not considered for one case, while viscosity acting force of outer air is considered for another case. The frequencies of ETFE inflated beams were calculated using added air mass when considering viscosity of outer air, added mass of outer air is calculated through equation (1), which is proposed by Yu through test of cylinder structure[2].

\[ m_a = \frac{1.9 \rho \pi D^2 L}{4} \]  

where, \( D \) is diameter of the cylinder, \( L \) is length of the cylinder, \( \rho \) is density of outer fluid.

The FEM results of first order frequency for two cases at pressure values of 3, 4 and 5kPa are shown in Table 1, there are about 30% differences between values in two cases. The first order mode of the ETFE inflated beam is shown in Fig.6.
Table 1: FEM results of the 1st order frequency

<table>
<thead>
<tr>
<th>Inner Pressure Value (Pa)</th>
<th>The 1st order frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not considering added air mass</td>
</tr>
<tr>
<td>3000</td>
<td>8.91</td>
</tr>
<tr>
<td>4000</td>
<td>8.93</td>
</tr>
<tr>
<td>5000</td>
<td>8.97</td>
</tr>
</tbody>
</table>

Fig. 6: FEM result of the first order mode of the ETFE inflated beam

3.4 Test of dynamic properties

The frequencies and modes of beams are tested using laser vibrometer, the test results of first order frequency value at pressure values of 3, 4 and 5kPa are shown in Table 2, and the tested first order mode is shown in Fig. 7.

Table 2: Test results of the 1st order frequency

<table>
<thead>
<tr>
<th>Inner Pressure Value (Pa)</th>
<th>The 1st order frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>7.38</td>
</tr>
<tr>
<td>4000</td>
<td>7.50</td>
</tr>
<tr>
<td>5000</td>
<td>7.58</td>
</tr>
</tbody>
</table>

Fig. 7: Test result of the first order mode of the ETFE inflated beam

It can be seen that the test results of the 1st order frequency is very close to the FEM results of the 1st order frequency when considering added mass of outer air through comparing values in Table 1 and Table 2, the differences within 5% between them. The test results of first order frequency also have 20% differences with the FEM results of the 1st order frequency when not considering added mass of outer air. The tested first order mode of the ETFE inflated beam is coincided with the FEM result comparing Fig. 6 and Fig. 7.
AIRSHIP ENVELOP INFLATABLE BEAM

The dimensions of the airship envelop inflatable beam are shown in Figure 1 (unit is mm), the thickness of ETFE coated fabric is 380 µm. The airship envelop fabric is non-isotropic material, the elastic properties of the airship envelop fabric were measured in house with our biaxial test rig\(^{[3]}\), the E-modulus in warp is \(E_{\text{Warp}} = 0.947 \text{kN/mm}^2\), the E-modulus in weft is \(E_{\text{Weft}} = 0.895 \text{kN/mm}^2\), and the Poisson’s ratio is 0.36. The nonlinear load bearing capacity of inflated beam was analyzed when a homogeneous load was applied on loading belt, three points are selected to record load-displacement curves of them, positions of three points are also shown in Fig.8.

4.1 Static load bearing capacity analysis

The wrinkles are easy to be formed when bending is applied on the inflated beams, the simulation by the FEM of inflated beams adopting implicit and explicit methods. FEM calculations were conducted at air pressures values of 3000 and 5000 Pa, the load-displacement behavior of airship envelop inflated beam adopting implicit scheme is shown in Fig.9, and the load-displacement behavior of the airship envelop inflated beam adopting explicit scheme is shown in Fig.10.
Seen from Fig.9 and Fig.10, the calculation is not convergent when the airship envelop inflated beam was bended at certain degree adopting implicit schemes, the calculation can continue to destination when adopting an explicit scheme. The calculated results using an explicit scheme do not tally with that using an implicit scheme.

4.2 Experiment of static load bearing capability

The full-scale inflated beam was fabricated and the deflections of three points under bending are measured. An experiment of airship envelop inflated beam under bending is shown in Fig.11. The experimental displacement-force curves of three points for air pressure of 3 and 5kPa are shown in Fig.12.
The deflection calculated of airship envelop inflatable beam has deviations between calculated and experimental results comparing Fig.9, Fig.10 and Fig.12.

4.3 Theoretic analysis of dynamic properties

The frequencies and modes of beams are conducted by FEM for two cases, viscosity acting force of outer air is not considered for one case, while viscosity acting force of outer air is considered for another case. The frequencies of ETFE inflated beams were calculated using added air mass when considering viscosity of outer air.

The FEM results of first order frequency for two cases at pressure values of 3, 4 and 5kPa are shown in Table 3, there are about 37% differences between values in two cases.

<table>
<thead>
<tr>
<th>Inner Pressure (Pa)</th>
<th>The 1st order frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not considering added air mass</td>
</tr>
<tr>
<td>3000</td>
<td>15.30</td>
</tr>
<tr>
<td>4000</td>
<td>15.36</td>
</tr>
<tr>
<td>5000</td>
<td>15.39</td>
</tr>
</tbody>
</table>

4.4 Test of dynamic properties

The frequencies and modes of beams are tested using laser vibraometer, the test results of first order frequency value at pressure values of 3, 4 and 5kPa are shown in Table 4.

<table>
<thead>
<tr>
<th>Inner Pressure (Pa)</th>
<th>The 1st order frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>10.13</td>
</tr>
<tr>
<td>4000</td>
<td>10.38</td>
</tr>
<tr>
<td>5000</td>
<td>10.50</td>
</tr>
</tbody>
</table>

The test results of first order frequency have 10% differences with the FEM results of the 1st order frequency when considering added mass of outer air, and the test results of first
order frequency have 50% differences with the FEM results of the 1st order frequency when not considering added mass of outer air. The tested first order mode of the ETFE inflated beam is coincided with the FEM result.

5 CONCLUSIONS

- The load-displacement curve of inflated beam in bending is obtained and compared with the experimental data. The deflection calculated of ETFE inflatable beams show excellent coincidence with measured curves, but the deflection calculated of airship envelop inflatable beam has deviations between calculated and experimental results, which indicate that the static capability of inflatable tube is sensitive to the elastic parameters of fabric, and the material nonlinearity of the airship envelop fabric is the major factor[4].

- The structural vibration behaviors of inflated beams are analyzed, the first order frequency of inflated beams reduced 25%-40% when taking into account of added mass of outer air, so the influence of outer air on inflated beams is not be overlooked. The first frequency and mode of ETFE inflated beam showed good correlation with experimental results, but the first frequency of airship envelop inflated beam has difference with experimental result.

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APPLICATION OF THE FLEXOMETER-TEST ON MEMBRANE-FABRIC USED FOR RETRACTABLE MEMBRANE STRUCTURES

STRUCTURAL MEMBRANES 2013

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Key words: Flexometer-Test, Retractable roofs, Test-procedures

Summary: In this paper, the Flexometer-Test is described and applied to four different membrane materials. As part of this it is proven that the procedure is very suitable to simulate the stress a retractable roof puts on the fabric. Finally, the effect on these four samples is shown and evaluated through tensile strength and air tightness tests.

1 INTRODUCTION

It is a well known fact that tents were one of the first kinds of housing mankind has developed. The ancient tents were small, but had the great advantage that they could easily be taken apart and mounted again elsewhere.

Nowadays people still like to be flexible towards weather conditions or other natural influences. Therefore stadiums and theatres are often designed with retractable roofs. This kind of roof can be opened or closed with the help of electric engines and hydraulics. Its technique has created big challenges for the structural designers and it is still lively discussed which fabric fits best for retractable roofs.

Furthermore a standardized test procedure that recreates the conditions membrane material is subjected to when it is frequently folded and unfolded still has to be found.

As part of my Bachelors-Thesis research has been done in the leather and textile industry to find such a test. In the so called “Flexometer-Test”, the core of my research, a piece of cloth is folded in two directions at the same time. This creates a unique fault in the fabric that is vastly different from the one caused by simple folding and unfolding. Applying this test to various common products has lead to interesting results. Additionally, a first classification of these materials is proposed to help designers in evaluating the suitability of membrane material as a constantly moving element in a structure.

2 RESEARCH

During preparation for my thesis on retractable membrane roofs an extraordinary damage was found, shown in the following pictures. This particular failure of the material is very rare and could not be found in static structures.
Therefore, it can be concluded that constant folding and unfolding of membrane material has a great impact on its lifespan.

Most projects today undergo a certain amount of testing to make sure they will function over the intended time span. This may be done through mock-ups or additional tests of folding etc. Unfortunately, the demands and the methods are project-specific and there is no uniform basis on which membrane-fabrics could be categorized to make it easier for designers to choose the right product for the job.

Therefore, the task was set to apply a new and unique stress on different types of commonly used architectural membrane fabrics. Ideally, that test should give a more realistic portrayal of the real life situation and give a deeper understanding on what actually damages the membrane to decide where the design of retractable roofs could be improved.

To find a test that mimics the real situation well, one has to look at the damaging mechanism first. This table is a shortened version of the one, depicted in my thesis. It has been pooled from sources \(^1,^2\). Biological, chemical and physical influences have been dropped:

**Table 1: Influences on retractable roofs**

<table>
<thead>
<tr>
<th>Status of the membrane</th>
<th>Always</th>
<th>Closed</th>
<th>Opening/closing</th>
<th>Open</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical</td>
<td>Self weight</td>
<td>Loading</td>
<td>Abrasion</td>
<td>Abrasion</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Moving &amp; still kinks, folds and double folds</td>
<td>Moving &amp; still kinks, folds and double folds</td>
</tr>
</tbody>
</table>

Every retractable roof has three stages: Closed, moving and open. A closed roof, meaning that the membrane is under tension and spans over its maximal size, is no different from any other static membrane roofing. Therefore, it does not contain kinks, creases or folds. So the critical stages are the moving and parking of the roof. When looking at an open roof where the fabric has been raffled to its center-point, it can be found that kink folds form. The stress that is forced onto the material is enormous and there can be no doubt that this crumpling is what reduces the webbing's strength.
When looking for a test procedure that would model that kind of behaviour, my research began in the database of standardized material tests of the textile industry, that contains almost 1000 entries. After ruling out all colour-, tightness- or abrasion-themed tests and solely focussing on crumpling, folding etc. a small group of tests remained. When investigating in other industrial branches, for example convertible roofs for cars, it could be found that most products were tested on a varying series of this same, small group of experiments. With little expectations on finding more procedures no further research was done. Besides there was already a test that protruded from the others. It appeared to mimic the exact stress of moving kink folds the way they emerge in retractable membrane roofs. The so called “Flexometer”- or “Bally”- Test therefore became the basis for the rest of my research. To my knowledge, this test was never applied to synthetic membrane material.

3 THE TESTING PROCEDURE

The Flexometer-Test appeared to be more suitable than simple folding and unfolding procedures or the crumpling tests because the damage inflicted on samples, closely resembled the images of damaged retractable roofs. Moreover, does its mechanism reduce crumpling to its most damaging element, which is moving kink folds. This way it creates a very controllable and so reproducible stress onto the sample. This is not the case with the other crumpling-tests.

The machine consists of a fixed lower part and a moving upper part. The moving clamp performs a twist of 22.5° around axis A at 100±5 cycles per minute, whereas the lower clamp remains immobile.

In the standard this description of the test is given: A sample is folded and fixed in the upper clamp. The remaining part of the probe is folded in the opposite direction and fixed in the lower clamp. When the upper clamp moves, a running crease is created within the material. The sample is periodically checked for damage.
This way a kink fold is created and moved over the same area, causing tremendous stress and therein resulting slow dismemberment of webbing and coating, or destruction of one or both of these components.

The standard suggests the following amounts of cycles after which the sample should be checked for damage.

**Table 2:** Folding cycles, according to DIN EN ISO 32100

<table>
<thead>
<tr>
<th></th>
<th>200</th>
<th>315</th>
<th>500</th>
<th>800</th>
<th>1250</th>
<th>2000</th>
<th>3150</th>
<th>5000</th>
<th>8000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1250</td>
<td>2000</td>
<td>3150</td>
<td>5000</td>
<td>8000</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>12500</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125000</td>
<td>200000</td>
<td>315000</td>
<td>500000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>
The numbers are based on the so-called "Renard"-Series with its factor m equal to 10. So the term is the tenth root of ten to the power of X. Unfortunately, it is not stated why this series of numbers was chosen.

\[ R_{10} = (10^{\frac{1}{10}})^X \]

To comply with the standard as much as possible, these suggested numbers of cycles were adopted by me. The same applies for the climatic conditions. All tests were carried out at approximately 23°C and 50% relative humidity.

After building a mock-up of the machine, the first tests were done and the concept of the machine's mechanics proved to be as expected. The intended damage could be seen in several small test strips, but not in all. Therefore, the first conclusion could be made that the test also works on membrane material but not reliably. Additionally, the strips appeared to be very small and posed the question if such a small damage would cause significant reduction of strength. The pictures taken from actual failures showed a much larger affected area.

So when assessing the first results, the decision was made to build another mock-up in a 2:1 scale from the proposed measurements of the standard. Combined with rounded edges, great results could be achieved with the machine and a larger fault proved to be reproducible with great reliability.

![Figure 5: Comparison: Flexometer-sample & actual damage](image)

To determine the loss of structural integrity as well as tensile strength, additional tests had to be imposed on the material before and after the treatment in the Flexometer.

The loss of strength was set to be measured with a simple monoaxial tensile strength test that stretches a sample with a preset, constant speed until it ruptures. The force at which the strip fails is the measured value of tensile strength. To account for scattering, several tests had to be done and the 5%-fractile of the results could be compared. With three of these results it would be possible to draw a graph of tensile strength relating to cycles in the Flexometer.

For the loss of tightness towards penetration through rainwater it was proposed to use a test of water-tightness that unfortunately was not available on short notice. Therefore an air-tightness test was applied that, combined with comprehensive microscopic investigation, proved to be sufficient enough to derive acceptable conclusions from.

On this basis the first tests could be performed
4 TESTING PHASE

The company “Eccon”, located in Austria, which is also greatly involved with retractable roofs, fabricated a scaled Flexometer machine for me, according to my specifications. With it, it was possible to test several materials for their resistance against kink folds.

With this machine my research could go into its practical phase. Having been provided with various materials such as PTFE-coated glass fibre webbing, Silicone-coated glass fibre webbing, PVC-coated PES webbing and PTFE-coated PTFE webbing (Tenara), sponsored by "Verseidag", "Koch" and "Sefar", it was possible to get a direct comparison between the most commonly used combinations of types of coatings and weaves.

However, at this point it was still unclear how many times the material would have to be folded to see an effect in each of the fabrics. Additionally, in case that number was found, how should be determined whether it corresponded with the stress of real structures.

To answer this question it has to be decided when the creasing causes the greatest damage in real life situations. On one hand, it can be argued that kink folds form when the roof is moving. As they appear, they move within the fabric and may cross each other, creating even greater stress. The engines are computer-controlled and perform exactly the same motion with each opening or closing of the roof. So one could imagine that the creases always form the same way and slowly worsen the condition of the weave with every time it is moved. The basis of that argument is that once the roof has come to rest in its garage, there is no further movement. But on the other hand, one could say that wind, as well as changing of temperature or vibrations in the structure could induce swaying of the garage and the parked membrane within. The consequence would be a constantly moving crease over a confined area, resulting in multiple times the damage of creasing during opening and closing.

While both concepts are based on the same failure, they both bear a fundamentally different consequence for the outcome of the testing. That is because, if one assumes that creasing only happens during the moving phase, the result is that there are roughly going to be about 500 times that the folding and unfolding occurs. That equals to 20 cycles per annum over an estimated service life of 25 years. Whereas there is no way of accurately telling how many times the material is going to sway and how many times the fold will move during the closed stage. But it can be safely assumed that the movement is going to be a lot more often.

After having figured out how the material has to be fixed in the machine so it does not get damaged through handling or unwanted kinking, first tests had to be carried out to determine the inflicted damage. The most important question at the time was how many cycles the machine has to perform to inflict damage on the material.

To document the effectiveness of the procedure and to show that it can also be used on all kinds of membrane material, several different products were fixed in the machine. The Flexometer was then switched on and started running through the whole spectrum of proposed cycles from the standard. After every listed number of foldings the effects were documented for later use. After 31500 cycles even the last fabric showed a significant change in the webbing or the surface and the test was finished.

Under a heat-detecting camera, it could be seen that the movement causes great heat within the material. That is through friction between the webbing and the coating. It slowly dismembers the components and causes the composite to disintegrate.
The next aspect that had to be figured out was in which direction to the folding, the tensile strength was to be tested. Considering that membrane structures usually have a biaxial strain the first idea was that the tested strip had to be stretched orthogonally to the direction of folding. Especially if one looks at the number of threads, damaged through the Flexometer, there is a much larger area covered in the direction of the folding. So there would not be only one to three damaged threads, but up to 50. It was anticipated that the reduction of strength was going to be far more significant in that direction. This proved to be mainly true. But fixing the samples properly, as well as finding a shape for the samples that allowed them to fit in the Flexometer, as well as the tearing-machine posed great challenges that could not be overcome properly. This was also observable in the test-results which showed large scattering. Ripping the samples in the same direction that the folding had happened was far simpler and also lead to satisfying results. It was therefore the favoured method to give a first insight on what the test does to the material. Further research on how to tear the samples in a $90^\circ$ angle to the folding may lead to better results and probably even a different outcome of the tests, but were not part of my thesis.
With the acquired information the following final test series was set to document how the materials react to the treatment in the Flexometer. Unfortunately, the area of PVC-coated PES fabric and silicone-coated glass fibre material given to me was not large enough to cut out 5 samples for each test.

Table 3: Samples, used for the final test

<table>
<thead>
<tr>
<th>Material</th>
<th>Reference (0 cycles)</th>
<th>3150 cycles</th>
<th>12500 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTFE/PTFE – warp</td>
<td>5 Samples</td>
<td>5 Samples</td>
<td>5 Samples</td>
</tr>
<tr>
<td>PTFE/PTFE – weft</td>
<td>5 Samples</td>
<td>5 Samples</td>
<td>5 Samples</td>
</tr>
<tr>
<td>PES/PVC – warp</td>
<td>4 Samples</td>
<td>3 Samples</td>
<td>3 Samples</td>
</tr>
<tr>
<td>PES/PVC – weft</td>
<td>5 Samples</td>
<td>4 Samples</td>
<td>4 Samples</td>
</tr>
<tr>
<td>Glass/PTFE – warp</td>
<td>5 Samples</td>
<td>5 Samples</td>
<td>5 Samples</td>
</tr>
<tr>
<td>Glass/PTFE – weft</td>
<td>5 Samples</td>
<td>5 Samples</td>
<td>5 Samples</td>
</tr>
<tr>
<td>Glass/Silicone – warp</td>
<td>5 Samples</td>
<td>5 Samples</td>
<td>5 Samples</td>
</tr>
<tr>
<td>Glass/Silicone – weft</td>
<td>0 Samples</td>
<td>0 Samples</td>
<td>0 Samples</td>
</tr>
</tbody>
</table>

Despite the inconsistence of sample-quantities it was possible to draw a graph, showing cycles and corresponding tensile strength. This way the decline of strength is easily observable as well as comprehensible.

As mentioned before there was no time to inquire the loss of water-tightness in depth and therefore only one test for each material and number of cycles could be conducted. Additionally, the test, used for this data did not work as planned and what can be seen under a microscope is not backed by the data. More information on that is given in the following section.
After carrying out the tests, the following results could be obtained:

**Table 4:** All test results & statistical analysis

<table>
<thead>
<tr>
<th></th>
<th>PTFE/PTFE</th>
<th>PES/PVC</th>
<th>Glass/PTFE</th>
<th>Glass/Silicone</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cycles</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3150</td>
<td>12500</td>
<td>0</td>
</tr>
<tr>
<td>Sample1 [N/mm]</td>
<td>110</td>
<td>109</td>
<td>100</td>
<td>92</td>
</tr>
<tr>
<td>Sample2 [N/mm]</td>
<td>109</td>
<td>109</td>
<td>103</td>
<td>95</td>
</tr>
<tr>
<td>Sample3 [N/mm]</td>
<td>112</td>
<td>110</td>
<td>106</td>
<td>91</td>
</tr>
<tr>
<td>Sample4 [N/mm]</td>
<td>113</td>
<td>110</td>
<td>105</td>
<td>93</td>
</tr>
<tr>
<td>Sample5 [N/mm]</td>
<td>112</td>
<td>108</td>
<td>101</td>
<td>-</td>
</tr>
<tr>
<td><strong>arithmetic average [N/mm]</strong></td>
<td>111</td>
<td>109</td>
<td>103</td>
<td>92</td>
</tr>
<tr>
<td><strong>Variance [N/mm]</strong></td>
<td>1.64</td>
<td>0.45</td>
<td>5.78</td>
<td>2.89</td>
</tr>
<tr>
<td><strong>Standard deviation [N/mm]</strong></td>
<td>1.28</td>
<td>0.67</td>
<td>2.40</td>
<td>1.70</td>
</tr>
<tr>
<td><strong>Standard deviation [%]</strong></td>
<td>1%</td>
<td>1%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td><strong>5%-Fractile [N/mm]</strong></td>
<td>109</td>
<td>108</td>
<td>98</td>
<td>89</td>
</tr>
<tr>
<td><strong>Loss of strength [%]</strong></td>
<td>0%</td>
<td>1%</td>
<td>10%</td>
<td>0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Warp</th>
<th>Weft</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cycles</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3150</td>
</tr>
<tr>
<td>Sample1 [inch]</td>
<td>40</td>
<td>9</td>
</tr>
<tr>
<td>Sample2 [inch]</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Sample3 [inch]</td>
<td>1016</td>
<td>216</td>
</tr>
<tr>
<td>Sample4 [inch]</td>
<td>1016</td>
<td>1016</td>
</tr>
<tr>
<td>Sample5 [inch]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Despite all effort, faults did occur in the running crease especially with the delicate glass-weaves. This lead to strikingly different results in strength and so the results had to be dropped from interpretation and are marked in the table above.

The therein resulting 5%-fractiles of all strength-values give information on how the material reacts on the Flexometer-procedure.

5 INTERPRETATION & CONCLUSION

Visualizing the obtained data as a function of cycles in the Flexometer in graphs looks as follows:

![Graph](image)

Figure 8: Folding cycles and corresponding 5%-Fractile results
As expected, the fibreglass webbing ruptures after a few cycles. After 3150 cycles, the tensile strength of the material has decreased over 40%. As one can see under a microscope, the filament is completely destroyed where the folding has taken place. The remaining strength comes from the intact fibres left and right to the crease. If strain was to be applied under a 90° angle, the strength measured would be only tear strength. The material can be seen as broken and should not be used. The radius of the redirection, caused by folding, is too sharp for the material and it breaks. It is not suitable for retractable roofs; the coating does not make a difference at this. Both the PTFE as well as the silicone coated webbing behave the same way.

The PTFE as well as the PES webbing behave differently. Both decrease very little in strength, which shows that this materials are more suitable for retractable roofs than fabric from glass fibres. When looking at the decrease in strength one finds that the PES fabric overall loses more load bearing capacity than the Tenara. It seems that of all tested products, the fabric made from PTFE-monofile fibres is the one best suitable for retractable roofs. When looking at the air tightness, a different outcome is imposed. Where the Tenara loses resistance against air penetration after few cycles, the PES performs nicely. As mentioned before, the tightness test was performed in a rush for deadline reasons, so there was not enough time to have a sufficient number of samples. Therefore, the results are not definite or representative. But when looking at the pictures taken under a microscope the reason for the large scattering can be found.

The PTFE coating almost instantly starts to have micro-cracks in the coating. Tiny cracks, only micrometers wide appear on both sides of the yarn. Therefore a passage through the material is formed, wide enough for air to pass through. Whether it is big enough for a capillary action to occur is unknown. But it is obvious that dirt and mould are given a rough surface on which to attach to with ease. The reason why the PVC-coating performs so well is also unveiled under a microscope. The coating gets continuously thinner through the Flexometer. But the surface remains intact until it finally ruptures and leaves a large hole. As soon as this happens, the entry into the load-bearing part of the composite material is open and all sorts of foreign objects as big...
as a pinhead are able to enter the membrane. Further studies on this matter should give a better understanding on which coating is more suitable to withstand folding.

Figure 10: Transmitted light microscopic image and details of two sample’s surfaces (PES/PVC: weakened spots marked; PTFE/PTFE: missing coating marked, thread visible)

In conclusion, it can be said that the Flexometer test is very suitable for the task of determining the resistance of membrane material against kink folds. If agreed on common requirements, it could be used to categorize available products, which ultimately will lead to a better feasibility and reliability of retractable roofs. However, the appropriate number number of cycles still has to be discussed.

The potential of the test is still not exhausted. For testing under extreme climates may lead to a significantly different outcome and conclusion about the material's suitability. The same is true for artificial aging before folding.

Looking at the results of the tightness tests it can be said that it does not work the way it was attempted in this study. If one was to stretch the material with a service load after folding it, the coating may fail sooner, giving fluid a chance to enter the webbing. Under this point of view a test for wicking is probably more significant than tightness, where both layers of coating have to fail to achieve a reduction of the measured values.

REFERENCES
FAILURE OF POLYETHYLENE THIN FILM MEMBRANE STRUCTURES

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Key words: StratoFilm, J-integral, Viscoelasticity, Free Volume Model

Abstract. The failure of balloons made of Linear Low Density PolyEthylene (LLDPE) is investigated. The chosen film is 38 µm thick StratoFilm 420, currently used for the NASA Super-Pressure balloons [1]. The visco-elastic behaviour of the film has been extensively studied and is already accounted for in the balloon design [2, 3, 5]. The next step in the development of accurate predictive tools for super-pressure balloons requires models that capture the transition from visco-elastic and visco-plastic behaviour to fracture.

It is shown that realistic estimates of failure of LLDPE membrane structures can be obtained from visco-elastic simulations based on the non-linear visco-elastic model of the balloon film proposed by Kwok [5], supplemented with a fracture resistance criterion derived from the experimentally-based J-integral.

1 INTRODUCTION

The use of Linear Low Density PolyEthylene (LLDPE) films in NASA superpressure balloons has motivated extensive studies of their viscoelastic behavior in the small and large strain regimes. However, their viscoplastic behavior and ultimate failure have remained relatively unexplored, making it difficult to quantify the failure margins of structures built from such films. Currently balloon designers are forced to the conservatism of point-based stress failure criteria. The objective of the present study is to gain insight into the viscoplastic tearing of LLDPE films and to develop quantitative models that in future will enable rational estimates of load margins against viscoplastic failure by tearing.
Essential background for the present study are the small-strain nonlinear models [2, 3] and the large strain viscoelastic models [4, 5] for LLDPE films. A stress limit based on a 2% strain offset is currently used as failure criterion in LLDPE balloon design, but such an approach is over-conservative in the case of localized stress peaks rise around geometry or stress singularities. Point-wise failure criteria neglect the stress redistribution that occurs near a stress peak and thus can significantly underestimate the reserve strength of a structure. A global approach that allows the stresses to redistribute until macroscopic yielding or tearing of the film occur is required to estimate the full strength.

Failure of polymers is usually approached by means of time-dependent yield criteria expressed in terms of stress components or energy. The time-to-failure of the polymer is captured by these criteria [6, 7, 8]. But no specific study of the time-to-failure of LLDPE film has been carried out. Tielking [9] carried out a series of complex tests on wide rectangular, semi-biaxial samples to obtain the relationship between crack amplitude and the \( J \)-integral.

Recent advances in experimental techniques, particularly in 3D digital image correlation, as well as large-strain constitutive modeling of LLDPE thin films have made it possible to obtain the critical values of the \( J \)-integral from direct strain measurements on a wide range of sample geometries. With this approach it is possible to obtain the complete relationship between crack amplitude and the \( J \)-integral from a single test, and hence it is possible to study strain rate effects in a direct way. It is hoped that these advances, presented in the present paper, will open the way to making direct connections between time-to-failure and crack propagation in thin films.

2 FAILURE OF POLYMERS

Uncrosslinked polymers, which include LLDPE, are used mainly above the glass transition temperature (\( T_g \)), to avoid brittle behavior. Hence, under normal operating conditions they can show significant time dependent deformation and plasticity. Their failure behavior also depends on time and temperature. For example, a test sample may be well below the breaking load measured at room temperature in a short-duration test, but it may fail when loaded for a longer period of time. Also, at lower temperature its strength increases.

The prediction of failure for such materials has been modeled by defining a function (failure criterion) that relates stress, strain and several other time-dependent parameters to a time-dependent yield stress.

One approach [18, 19] considers the function:

\[
f = \frac{1}{2} \sigma_{ij} \sigma_{ij} - \left( A + B \exp \left( -C \sqrt{\left( \epsilon_{ij}^V - \epsilon_{ij}^E \right) \cdot \left( \epsilon_{ij}^V - \epsilon_{ij}^E \right)} \right) \right)^2 = 0,
\]

where \( \sigma_{ij} \) is the stress tensor, \( \epsilon_{ij} \) is the strain tensor and the superscripts \( V \) and \( E \) denote viscous and elastic components. Note that Eq. 1 is defined in terms of a Mises equivalent
stress (first term) and a time-dependent yield stress (second term). $A$, $B$ and $C$ are material parameters.

Alternatively, it is assumed that failure is linked to the stored strain energy reaching a critical level, without accounting for viscous or plastically dissipated energy [20, 21, 22]. Furthermore, only the deviatoric component of the strain energy is considered, not the dilatational energy. Since, the time-dependent stress and strain components can be estimated from a finite element analysis, implementation of the latter approach requires only a single failure parameter to be defined and hence it is simpler to implement this approach than the three-parameter criterion in Eq. 1.

An alternative approach to the failure of polymer films relates failure to the propagation of a crack [9]. In fracture mechanics the $J$-integral is a useful tool to analyze problems involving crack propagation in inelastic materials. The $J$-integral is the integral of the energy release rate on a contour that surrounds the crack tip. It is path independent and in elastic fracture mechanics it is equal to the energy release rate, i.e. the fracture energy per unit surface of crack [23], Fig. 2. It can be calculated from:

$$J = \int_{\Gamma} \left( W_n - T \frac{\partial u}{\partial x} \right) ds,$$

where $\Gamma$ is the chosen contour, $W$ is the strain energy, $n_1$ is the component of the normal strain in the direction normal to $\Gamma$, $T$ is the stress normal to $\Gamma$, $u$ is the movement of the crack tip, $x$ is defined parallel to the crack and $ds$ is an infinitesimal element along $\Gamma$.

When a critical level of the $J$-integral is reached the crack starts to propagate. The $J$-integral is usually plotted as a function of the crack length increase, $\Delta a$, Fig. 1. $J_c$ is the critical value of $J$ when the crack size starts to increase. The slope of the $J$-curve beyond $J_c$ indicates the resistance of the material to crack propagation.

In elastic materials, plane-stress state, the toughness, $K$, is directly related to the energy release rate, $G$, and to $J$, by

$$\frac{K^2}{E} = G = J$$

In the case of inelastic materials there are difficulties in considering the toughness because some energy is dissipated, but the $J$-integral still gives a general method to determine the energy release associate with the crack propagation, and can be determined by using the stress-strain relation far from the crack-disturbed area.

Tielking [9] carried out unidirectional load tests on 76 mm long and 254 mm wide samples of 20 µm thick StratoFilm. Because of the large width to length ratio of these samples, the effect of their edge deformation on crack propagation is negligible. The $J$-integral could be evaluated indirectly from the equation [24]:

$$J = \frac{1}{B} \frac{\partial W_T}{\partial a},$$

$$J = \int_{\Gamma} \left( W_n - T \frac{\partial u}{\partial x} \right) ds,$$
Figure 1: Schematic diagram of $J$-integral.

Figure 2: Integration path around the crack tip.
where $B$ is the thickness of the sample, $W_T$ is the total work of the loading mechanism and $a$ is the half length of the crack. The $J$-integral can be determined by a multiple loading-unloading procedure using this equation; each point of the $J - \Delta a$ diagram can be determined from the difference between the loading-unloading energies and the change in $\Delta a$.

3 FAILURE BY VISCOELASTIC TEARING: PRELIMINARY TESTS

To obtain an initial understanding of the failure of StratoFilm 420 three sets of preliminary tests were carried out on 75 mm long and either 6 or 12 mm wide laser-cut dogbone samples, with the machine direction of the film aligned with the longer dimension of the sample. These tests investigated the difference in behavior between pristine film vs. film damaged by introducing a pinhole or a small slit.

The tests were carried out in an Instron 3119-506 environmental chamber at temperatures between 203 K and 263 K, using an Instron 5569 electromechanical materials testing machine. The ultimate strength, $f_u$, and ultimate extension, $u_u$, were determined from the peak in the load-extension diagram generated by the Instron BlueHill software. The full set of results is presented in Table 1. Note that in the case of samples that failed after extensive plastic deformation $f_y$ and $f_u$ refer to the original cross-section and hence do not reflect the true stress state in the necked sample. Representative snapshots from each set of tests are shown in Fig. 3(a-d) and the load-extension plots for four representative tests are presented in Fig. 3(e).

The first set of tests was carried out on samples that had no visible initial damage. Tests at two different strain rates, $1.33 \times 10^{-3} \text{ s}^{-1}$ and $2.66 \times 10^{-3} \text{ s}^{-1}$, and two different temperatures, 263 K and 223 K, on 6 mm and 12 mm wide samples showed similarly large stretching (100-300 %) followed by the formation of a neck and failure of the sample. In all of these tests there was a significant amount of plastic deformation, as evidenced by the milky appearance of the sample (crazing). At the higher temperature (263 K) the full length of the inner part of the samples developed uniform crazing. At the lower temperature (223 K) the crazing started at one end of the sample (in the case shown in Fig. 3(b) crazing started at the bottom) and propagated through the full length of the sample before necking began. The ultimate stress was higher at the lower temperature and at the higher strain rate. This type of behavior is described with good accuracy by the energy-based failure criteria in Sec. 2, apart from the need to account for the large elongation before failure. The observed dependence of the ultimate stress on temperature is also well described by this approach.

The second set of tests was carried out on samples that had been initially damaged by introducing a pinhole in the middle. The idea for this test, which was done only at the lower temperature of 223 K, was that the pinhole might provide an initiation point for a crack that tears through the sample before a large amount of crazing occurs. However, extensive crazing was still observed before failure in this set of tests. Crazing always started at a point away from the pinhole, usually near one of the ends of the sample, and
Table 1: Results of preliminary failure tests. Width [mm], Temperature [K], Humidity [%], Rate [mm/s], yield stress ($f_y$) [N/mm$^2$], breaking stress ($f_u$) [N/mm$^2$], ultimate extension ($u_u$) [mm].

<table>
<thead>
<tr>
<th></th>
<th>Test 1: no hole</th>
<th>Test 2: pinhole</th>
<th>Test 3: slit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>12 12 6 12 6 12</td>
<td>12 12 6 12 6 12</td>
<td>12 12 12 12 12</td>
</tr>
<tr>
<td>Temp.</td>
<td>263 263 263 263 223 223 223 223 223 223</td>
<td>223 223 223 203</td>
<td></td>
</tr>
<tr>
<td>Hum.</td>
<td>75 85 30 40 40 35 40 40 40 40 40</td>
<td>40 40 20 20 20</td>
<td></td>
</tr>
<tr>
<td>Rate</td>
<td>0.1 0.2 0.1 0.2 0.1 0.2 0.1 0.2 0.1 0.2 0.1 0.01 0.001 0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_y$</td>
<td>16 15 14 24 26 15 26 26 26 26 22 26 26 34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_u$</td>
<td>17 16 24 15 26 17 22 24 22 26 26 26 34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_u$</td>
<td>190 210 210 150 100 110 60 35 60 35 3 3 3 2.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The tests on initially undamaged samples showed a large amount of visco-plastic deformation. Lower temperatures caused the strength to increase, as expected from the failure criterion approach in Section 2. However, doubling the strain rate did not increase the strength as had been expected from the failure criterion. Introducing a pinhole or a slit in the sample did not change the overall strength but it substantially decreased the ultimate elongation.

These results were useful in planning further, more detailed experiments. Since all three sample configurations had provided approximately the same ultimate strength, the choice between pristine, damaged by a pinhole, or damaged by a slit was made on the basis of which configuration is most suited to producing the highest resolution in the strain field near the crack that ultimately tears through the sample. This strain field can be used to evaluate the $J$-integral around the crack. Here the key factor is the limited viewing field over which Digital Image Correlation systems can produce high-resolution strain fields. In order to image a narrow region of the sample, and to avoid that this region moves out of the viewing field during the test, the third sample configuration was selected.
Figure 3: Snapshots from four tests: (a) no hole, T=263 K; (b) no hole, T=223 K; (c) pinhole, T=223 K; (d) slit (a=2 mm), T=223 K. (e) Force-extension diagrams for the four tests.
4 TEST APPARATUS AND SAMPLE CONFIGURATION

The strain in the film was measured with the Correlated Solutions Vic-3D 2010 Digital Image Correlation (DIC) system. The use of three-dimensional DIC allowed us to capture the effects of out of plane deformations, including wrinkling of the film. The test configuration can be seen in Figure 4; the cameras were Point Gray GRAS-50SSM-C with Pentax 75 mm F/2.8 lenses, set up with a field of view of approximately 50 mm. The test samples were lightly sprayed with black paint to provide a random speckle pattern with average size of 0.25 mm; they were captured at a rate of 1 s. The images were processed with Vic-3D using a correlation subset of $29 \times 29$ pixels and the strain field was computed from an 8-tap B-spline interpolation of the displacement field [25]. The crack length, $2a$, defined as the distance between the crack tips, was measured manually from the images. The pixel distance was measured and it was converted to the actual value using the initial 2 mm length of the slit as a calibration length.

Two types of tests were carried out: unidirectional tests on 12.7 mm (half inch, ASTM D-412 A) dogbone samples, and bidirectional tests on spherical bubbles obtained by inflating a circular sample with diameter of 125 mm, clamped around the edge. Both sets of samples contained a 2 mm wide slit in the middle, made with a scalpel. In the first test the machine direction of the film was aligned with the loading direction. The second test used an air pressure box with a 125 mm diameter hole, see Figure 5. StratoFilm samples were clamped over the box and the box was connected to an Omega IP610-030 pressure regulator. The applied pressure was measured with an Omega DPG409-015G electronic pressure gauge. The deformed shape of the test sample was measured with DIC and the strain field near the crack was obtained with Vic-3D using the same settings described above. Air leakage through the slit was prevented by means of an inner layer of wrap foil liner. According to a preliminary FEM analysis of the inflation of the film, the maximum stress occurs in the middle of the bubble and hence the slit for the crack analysis was placed there.

All tests were performed at 253 K, starting half an hour after closing the door of the environmental chamber and setting the controller at this temperature.

5 J-INTEGRAL DIAGRAMS

The $J$-integral defined in Eq. 2 requires the strain energy and the stress components to be known along the chosen contour $\Gamma$. Since the integral is path independent, it is best to choose $\Gamma$ to be as far as possible from the crack tip, to keep the strains smaller and achieve greater accuracy in the stress calculation. Note that the integral should start from the edge of the crack, but standard DIC cannot measure strains close to a free edge.

The $J$-integral has been calculated along an approximately elliptical path with semi-axes of $\sim 3$ mm and $\sim 4$ mm, see Figure 6. The calculation has been repeated for each time step, using time-smoothed strain energy and stress values, and for each of the three tests that had been carried out.
Figure 4: Instron thermal chamber with DIC cameras.

Figure 5: Section of pressure box for bidirectional tests.
Figure 6: Strains around the crack tip. The rounded rectangle at the center has been excluded from the strain calculation.

The standard $J$-integral vs. crack propagation diagrams for the three tests are shown in Figs 7-9. Each figure shows the values of the $J$-integral around the left crack tip (A) and the right crack tip (B), and both the elastic strain energy and the total work are also plotted.

The results of the tests have been corrected to remove the effects of the transverse deformation of the sample, which causes an overall change in the crack length without any movement of the crack tip, Figure 10. The average transverse deformation was measured parallel to the crack, and the actual size of the crack was corrected with this deformation value.

Figures 7-9 show that the difference between elastic strain energy and the total work is small. This is not surprising because all of the tests were carried out at low temperature and they lasted no longer than half an hour.

The differences between the $J$-integrals for the A and B sides in the uniaxial tests, Figures 7-8, are larger than for the bubble test, Figures 9, because the loading arrangement for uniaxial tension is more prone to asymmetry effects. The basic characteristics of the $J$-$\Delta a$ diagram for the three tests are as follows: in all three tests the blunting period lasts until $\Delta a \sim 0.08$ mm, and the values of $J_c$ are 500, 2000, and 1700, $J/m^2$ respectively.

Comparing Figure 1 to Figure 7, note that the initial part (AB) of the diagram corresponds to the formation of the plastic zone around the tip of the crack which causes a change in the crack length, see Figure 11. Then, the crack begins to extend and finally it
Figure 7: Variation of $J$-integral with crack size, for uniaxial test at 253 K and $1.33 \times 10^{-4}$ s$^{-1}$.

Figure 8: Variation of $J$-integral with crack size, for uniaxial test at 253 K and $1.33 \times 10^{-5}$ s$^{-1}$. 
Figure 9: Variation of $J$-integral with crack size, for bubble test at 253 K.

Figure 10: Effects of transverse deformation (a) reduction in overall width of sample and (b) change of crack size during uniaxial test at 253 K and $1.33 \times 10^{-4} \text{ s}^{-1}$. 
propagates from C onwards. Figures 7 and 8 show a more complex variation before the critical value of $J$ is reached.

Figure 12 shows a plot of the longitudinal average strain and the measured stress (obtained by dividing the stress by the initial cross-sectional area) during the full duration of a particular uniaxial test. A third plot shows the computed average stress during the same test, whose value was obtained from the longitudinal stress across a cross-section of the sample, derived from the strain field measured with DIC and converted to stress using the constitutive model. The time at which the crack began to propagate in this test is shown. It can be seen from these plots that the stress estimate is significantly higher than the measured stress, and it is about 30% too high when the critical crack amplitude is reached. Hence it can be concluded that our estimate of $J_c$ for this test would also be higher than the actual value by a corresponding amount.
6 Discussion and Conclusion

The first question addressed in this study was the selection of a test sample configuration to study the failure behavior of LLDPE thin films. A comparison between the force-extension diagrams of pristine dogbone samples of StratoFilm 420 vs. samples with either pinholes or 2 mm wide transverse slits has produced comparable strength values. The third configuration has the advantage that the region of greatest interest in the sample is known from the beginning of the test, and hence high-resolution imaging of this region is possible, thus achieving greater accuracy in the measurement of the strain field in the region where failure occurs. For this reason, the 2 mm wide slit configuration was selected for further study.

Once this choice had been made, uniaxial tests on dogbone samples were carried out at two different strain rates, and a further test was carried out on a pressure-loaded circular diaphragm. Testing dogbone samples has the advantage that it is easier to set up and the overall strain rate can be easily controlled. Testing a pressurized diaphragm has the advantage that the stress distribution more clearly resembles the operation conditions of StratoFilm in a balloon structure.

Having calculated the variation of the $J$-integral on either side of initially 2 mm wide slits, in both test configurations we found that the critical increase in crack amplitude is $\sim 0.08$ mm and the critical value of the $J$-integral is in the range 500-2000 J/m$^2$ at a temperature of 253 K. We also found that the value of $J_c$ increased by a factor of 4, from $\sim 500$ to $\sim 2000$ J/m$^2$ when the strain rate was increased by an order of magnitude, from $1.33 \times 10^{-4}$ s$^{-1}$ to $1.33 \times 10^{-5}$ s$^{-1}$. Lastly, we found that the free volume constitutive model based on Rand[2] and Kwok[5] tends to over-predict the average stress in a dogbone sample, by around 30% at the strain of $\sim 4\%$ at which the critical value of $J$ was reached.

Based on the results of the present study, a more extensive investigation of the $J$-integral around 2 mm wide slits in dogbone specimens of StratoFilm 420 will be required, including a range of temperatures and strain rates, in order to develop models for the energy dissipation near a propagating crack. Such models could be used to analyze the combinations of pressure and time that lead to failure in a structure made of StratoFilm 420. Also, a refinement of the large-strain constitutive model for StratoFilm 420 would be desirable.

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