Program

Monday

1. Variational formulation in linear solid mechanics (RLT)
   Strong, weak and variational forms of BVP in linear elasticity.

2. FEM technology for 1D problems (MB)
   Axisymmetric 1-d elasticity.
   Euler-Bernoulli and Timoshenko beam models.
   Locking numerical evidence.

3. FEM technology in solids problems (MB)
   Isoparametric elements and numerical integration.
   Incompressibility / near incompressibility.
   Hybrid and mixed FE.
   Enhanced strain FE.

4. Structural finite elements (MB)
   Dimensional reduction.
   Plate and shell models.
   Finite elements for thin-walled structures.

5. Introduction to FEAP and problem solution (TUTORIAL)
   Tutorial on FEAP command language.
   Tutorial on programming in FEAP environment.

Thursday

16. Isogeometric modeling and analysis (AR)
    Introduction to splines and NURBS.
    Basics of isogeometric analysis.
    Simple investigations.

17. Isogeometric modeling and analysis (GS)
    Mathematical properties of isogeometric fields.
    Complex geometries: trimming and multipatch.
    Locking-free isogeometric elements.

18. Isogeometric modeling and analysis (RLT)
    Implementation details for displacement and mixed methods.
    Interpolation using extraction operators.
    Examples applications for solids and shells.

19. Nonlinear dynamics problems (AR)
    Explicit vs. implicit integration schemes.
    Central difference, Newmark, and generalized alpha-methods.
    High order approximations in structural vibration and dynamic problems.

20. Tutorial on isogeometric analysis (TUTORIAL)
    Simple in-house Matlab codes.
    Isogeometric problem solution in FEAP.

Friday

21. Constraints (RLT)
    Formulation using Lagrange multiplier and penalty type methods.
    Tied interface and contact problems.
    Spatial approximations for finite element and IgA.

22. Particle, meshless, and collocation schemes (AR)
    An introduction to meshless methods.
    Smoothing particle hydrodynamics and other approaches.
    Some recent developments on particle methods.
    Isogeometric collocation methods.

23. Fluid Dynamics and Fluid Structure Interaction (MB)
    Phenomena of fluid flow, incompressible Navier-Stokes equations.
    Computational modeling of fluids.
    Basic remarks on coupled problems, phenomena of fluid structure interaction, solution algorithms for FSI problems.

Multi-scale problems (RLT)

Homogenization methods.
Scale bridging using representative volume elements (FE2).
Parallel implementation details.
Example applications.

The course will be held in Pavia. Possible location could be the conference room of IMATI – CNR (Institute of Applied Mathematics and Information Technologies) in Via Ferrata 3, 27100 Pavia, Italy.
The main objective of this course is to provide engineers, graduate students, and researchers with a review of numerical techniques and solution algorithms for nonlinear mechanics. It presents the current state-of-the-art in finite element modeling of nonlinear problems in solid and structural mechanics and illustrates difficulties (and possible solutions) appearing in a number of applications.

Different sources of nonlinear behavior are presented in a systematic manner. Special attention is paid to nonlinear constitutive behavior of materials, large deformations and rotations of structures, contact and instability problems with either material (localization) or geometric (buckling) nonlinearities, which are needed to fully grasp weaknesses of structural design.

The course will also provide insight both on advanced mathematical aspects as well as into the practical aspects of several computational techniques, such as the finite element method, isogeometric analysis, meshless techniques, virtual element method.

The objective is thus to provide the participants with a solid basis for using computational tools and software in trying to achieve the optimal design, and/or to carry out a refined analysis of nonlinear behavior of structures. The course finally provides a basis to account for multi-physics and multi-scale effects, which are likely to achieve a significant breakthrough in a number of industrial applications.

**TUTORIALS AND COURSE MATERIAL**

Tutorials are organized as a final section each day and are meant not as a standard lecture but as an interactive part of the course. In fact, tutorials are based on addressing simple problems to be solved on the fly as a basis for an interactive discussion between the teaching body and the course attendees. We strongly encourage students to bring their own laptops and we plan to distribute files, so that students can run examples, interact, and participate lively to the tutorials. Depending on the specific topic, the tutorials will be managed by one or more of the teachers and they will be based on using different software. Special emphasis will be given to FEAP personal version (http://www.ce.berkeley.edu/projects/feap/feappv) or simple “in-house” codes written in Matlab or Maple.

The course material will consist of electronic copies of lecture materials and survey papers. Copies of Finite Element Analysis Program (FEAP) computer codes, written by Prof. Robert L. Taylor at UC Berkeley, and the complete volume of notes will be made available to all attendees.

**LECTURERS**

**Franco Brezzi (FB).** Professor of Mathematical Analysis since 1976 and of Numerical Analysis since 2008. Awarded as ISI Highly cited researcher in Mathematics, his scientific contribution counts more than 150 papers in international journals, 5 books and many book chapters. His scientific interests are mainly concentrated in the field of Numerical Methods for Partial Differential Equations. In particular, from the point of view of methodological tools, he works mainly on Finite Element Methods (of various kinds). From the applicable point of view, he is mostly interested in problems, arising from various Engineering fields, such as Structural Mechanics, Fluid Mechanics, and Electromagnetics.

**Robert L. Taylor (RLT).** Professor in the Graduate School, Department of Civil and Environmental Engineering, University of California, Berkeley. His main research areas cover several areas of computational mechanics and material modeling, contact problems, solution algorithms and software development. He is well known for his co-authored books on the finite element method (with O.C. Zienkiewicz et al.) and development of the finite element program FEAP.

**Ferdinando Auricchio (FA).** Professor of Mechanics of Solids at the University of Pavia, Italy. FA main research topics span over constitutive modeling of innovative materials, biomechanics and finite element methods. He is currently leading one of the five strategic project for the whole University of Pavia, project entitled “3D@UniPV: Virtual Modeling and Additive Manufacturing (3D printing) for Advanced Materials”.

**Manfred Bischoff (MB).** Professor and head of the Institute of Structural Mechanics at the University of Stuttgart. Winner in 2000 of the EUROMECH European Young Scientist Award and in 2008 of the IACM Young Investigator Award, elected in 2012 fellow of the International Association for Computational Mechanics (IACM). His main research topics are nonlinear computational structural mechanics and dynamics, modeling and analysis of shells, finite element technology, structural optimization, contact problems, isogeometric analysis, computational material modeling.

**Alessandro Reali (AR).** Professor of Mechanics of Solids at the Department of Civil Engineering and Architecture of the University of Pavia, and ‘Fischer’ Fellow at the Institute of Advanced Study of the Technical University of Munich. His main research interests are isogeometric analysis, advanced constitutive modeling, mixed finite elements, and strong-form (particle and collocation) methods. He is an ISI “Highly Cited Researcher” and the recipient, among other honors, of the ERC “Ideas” Starting Grant, of the IACM “Argyris” Award, and of the ECCOMAS “Zienkiewicz” Award.

**Giancarlo Sangalli (GS).** Professor of Numerical Analysis at the Mathematics Department of University of Pavia, GS has worked on multiscale numerical methods, domain decomposition methods and, more recently, on isogeometric methods, with application in solid, fluid mechanics, and electromagnetism. In particular, he has contributed to the analysis of isogeometric methods in several directions, ranging from basic elliptic problems, to eigenvalue analyses, complex geometry parametrizations (e.g., T-splines), and efficient implementation (quadrature, linear solvers).